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A NON-GAUSSIAN MODEL OF CONTINUOUS
ATMOSPHERIC TURBULENCE FOR USE
IN AIRCRAFT DESIGN

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1976

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|--|--|--|--|--|--------------------|
| 1. Report No. NASA CR-2639 | | 2. Government Accession No. | | 3. Recip  | |
| 4. Title and Subtitle "A Non-Gaussian Model of Continuous Atmospheric Turbulence for Use In Aircraft Design" | | | | 5. Report January 1976 | |
| | | | | 6. Performing Organization Code 0061499 | |
| 7. Author(s) Paul M. Reeves, Robert G. Joppa, and Victor M. Ganzer | | | | 8. Performing Organization Report No. | |
| | | | | 10. Work Unit No. | |
| 9. Performing Organization Name and Address University of Washington Seattle, Washington | | | | 11. Contract or Grant No. NGR-48-002-085 | |
| | | | | 13. Type of Report and Period Covered | |
| 12. Sponsoring Agency Name and Address National Aeronautics & Space Administration Washington, D.C. 20546 | | | | 14. Sponsoring Agency Code | |
| | | | | 15. Supplementary Notes | |
| 16. Abstract Although it is recognized that atmospheric turbulence is a non-gaussian process, mathematical models of turbulence for simulator work have assumed gaussian processes due to the lack of a practical non-gaussian model. This report <ol style="list-style-type: none"> 1) suggests a non-gaussian model of atmospheric turbulence, 2) presents a statistical analysis of this model, 3) shows that the statistical properties of this model are more nearly in agreement with observed atmospheric turbulence properties than are those of gaussian models, and 4) derives methods by which the model can be used to study linearized aircraft responses. <p>The model presented here is restricted to the regions of the atmosphere where the turbulence is "steady" or "continuous", and the assumptions of homogeneity and stationarity are justified. Also spatial distribution of turbulence has been neglected, so the model consists of three independent, stationary stochastic processes which represent the vertical, lateral, and longitudinal gust components.</p> <p>The non-gaussian and gaussian models are compared with experimental data, and it is shown that the gaussian model underestimates the number of high velocity gusts which occur in the atmosphere, while the non-gaussian model can be adjusted to match the observed high velocity gusts more satisfactorily. Neither model represents velocity increment distributions very satisfactorily, although a possible solution to this difficulty is presented for the non-gaussian model.</p> <p>Application of the proposed model to aircraft response is investigated, with particular attention to the response power spectral density, the probability distribution, and the level crossing frequency. A numerical example is presented which illustrates the application of the non-gaussian model to the study of an aircraft-autopilot system.</p> <p>Listings and sample results of a number of computer programs used in working with the model are included.</p> | | | | | |
| 17. Key Words (Suggested by Author(s)) Aircraft Atmospheric turbulence Atmospheric turbulence modeling Autopilot Gusts Turbulence | | | 18. Distribution Statement UNCLASSIFIED-UNLIMITED STAR Category 02 | | |
| 19. Security Classif. (of this report) UNCLASSIFIED | | 20. Security Classif. (of this page) UNCLASSIFIED | | 21. No. of Pages 239 | 22. Price* 6.00 |

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SYMBOLS AND NOTATION

In the following list, the subscript α will denote letter subscripts, n will denote integer subscripts, and specific numerical subscripts will be denoted by i followed by a list of possible values.

Symbols

| | |
|--------------|--|
| a | random function of time |
| A_n | random variable |
| \bar{A} | random vector (Eq. 180) |
| \tilde{A} | covariance of \bar{A} (Eq. 182) |
| b | random function of time |
| B_n | random variable |
| \bar{B} | random vector (Eq. 181) |
| \tilde{B} | covariance of \bar{B} (Eq. 183) |
| c | random function of time |
| C | correlation function |
| C_n | random variable |
| \tilde{C} | matrix |
| d | random function of time |
| D_i | $i = 1, 2, 3$. constants |
| D_n | random variable |
| e | random function of time |
| $E\{\cdot\}$ | expected value operator |
| f | frequency |
| F | cumulative probability of exceedance or cumulative probability distribution (Eqs. 139, 140, 141) |

| | |
|-----------------------|--|
| g | random function of time |
| $h(\cdot)$ | random function of time |
| $h(\cdot, \cdot)$ | kernel function (Eq. 147) |
| h_x | impulse response function |
| H_x | transfer function |
| i | $\sqrt{-1}$ |
| j | integer index |
| k | random function of time |
| k_x | constant |
| K_0 | modified Bessel function of second type and order zero |
| L_x | scale length |
| m | integer index |
| M | upper limit of index m |
| n | integer index |
| N | upper limit of index n |
| N_i | $i = 1, 2, 3, 4, 5$. constants |
| N_x | level crossing frequency, number of crossings with positive slope per unit distance (Eq. 81) |
| \hat{N}_x | special case of level crossing frequency (Eq. 109) |
| $p_{x_1, x_2, \dots}$ | joint probability density function of the random variables x_1, x_2, \dots |
| \hat{p}_x | standardized probability density function (Eq. 31) |
| \hat{p}_{x_1, x_2} | special case of joint density function (Eq. 108) |
| P_x | probability distribution function (Eq. 10) |
| P_x^c | cumulative gust velocity distribution function (Eq. 11) |
| q | integral function (Eqs. 149, 150) |

| | |
|------------------|--|
| r | response time history |
| R_x | probability distribution parameter of the turbulence model (Fig. 13) |
| s | Laplace transform variable |
| t | time |
| t_c | truncation time limit (Fig. 24) |
| u | longitudinal gust velocity |
| U | mean true airspeed of aircraft |
| v | lateral gust velocity |
| w | vertical gust velocity |
| x | variable |
| \bar{x} | vector of variables |
| y | variable |
| \bar{y} | vector of variables |
| z | variable |
| α | variable |
| $\alpha(\cdot)$ | function of correlation Eq. (28) |
| β | variable |
| γ_x | wavelength representing viscosity effect (Eqs. 82, 83, 84, 113, 114) |
| δ | variable |
| Δ_x | increment of x (Eq. 22) |
| ϵ_{ise} | goodness-of-fit criterion (Eq. 139) |
| ϵ_{log} | goodness-of-fit criterion (Eq. 141) |
| ϵ_{max} | goodness-of-fit criterion (Eq. 140) |

| | |
|--------------------------|--|
| η_x | wide band gaussian white noise time history |
| λ_n | eigenvalue of response |
| Λ_n | eigenvalue of response time derivative |
| ν_x | wavelength representing viscosity effect (Eq. 115) |
| ξ | interval of real line or region of plane |
| π | 3.1415... |
| σ_x | standard deviation of x |
| τ | time lag |
| $\phi_{x_1, x_2, \dots}$ | joint characteristic function of variables x_1, x_2, \dots |
| $\hat{\phi}_{x_1, x_2}$ | special case of characteristic function (Eq. 103) |
| Φ_{xxx} | power spectral density of random function x |
| χ_n | eigenfunction of vehicle response |
| $\dot{\chi}_n$ | eigenfunction of vehicle response time derivative |
| ψ_n | eigenfunction of vehicle response |
| $\dot{\psi}_n$ | eigenfunction of vehicle response time derivative |
| Ω | spatial frequency |
| Ω_c | spatial frequency limit (Fig. 23) |

Notation

| | |
|------------------------|---|
| $\int_{x_1}^{x_2} (N)$ | denotes N integrations between limits x_1 and x_2 |
| \dot{x} | time derivative of x |
| \bar{x} | column vector of elements $x_1, x_2, \text{etc.}$ |
| \bar{x}^T | transpose of vector \bar{x} |
| A | matrix A |
| \hat{x} | standardized variable or special case of x |
| max | maximum value |

ACKNOWLEDGMENT

The authors wish to express their sincere appreciation for the advice given by Professor J. Vagners of the Department of Aeronautics and Astronautics, Professor R. D. Martin of the Department of Electrical Engineering, and Professor J. A. Businger of the Department of Atmospheric Sciences and Geophysics.

The support of this research by the National Aeronautics and Space Administration under NASA Grant number NGR-48-002-085; and the assistance of the NASA Technical Officer for this grant, Mr. R. L. Kurkowski of NASA Ames Research Center, is also gratefully acknowledged.

SUMMARY

This report considers the problem of modeling continuous atmospheric turbulence for the purposes of aircraft design. The discussion is limited to the representation of turbulence by three stationary, independent stochastic processes which are physically interpreted as the longitudinal, lateral, and vertical gust components at the vehicle center of gravity.

The gaussian model now in wide use is reviewed. A comparison of this model with experimental data shows that it underestimates the number of high velocity gusts which occur in the atmosphere. Furthermore, it cannot reproduce observed velocity increment distributions.

A class of non-gaussian processes is proposed as a turbulence model. Though previous publications have described the application of this model to flight simulator work, this report analyzes the model in greater detail, and is the first publication to apply it to analytical calculations. A comparison with experimental turbulence data is presented, and it is concluded that the non-gaussian model is superior to the gaussian model for the purposes of representing high velocity gusts. However, in the form presented, the new model does not improve upon the gaussian model in so far as the modeling of velocity increments is concerned.

The problem of applying the non-gaussian model to the calculation of vehicle response statistics is investigated. The specific statistics of interest in this report are the response power spectral density, probability distribution, and level crossing frequency. The first of these is easily handled by well known methods. The calculation of the second and third quantities, however, requires the development of an eigenfunction-eigenvalue expansion technique. A numerical example is presented and a number of computer programs useful for studying the model are included.

INTRODUCTION

The effects of atmospheric turbulence have been a continuing concern to the aircraft designer since the earliest days of powered flight. Typical turbulence related problems which must be solved during the design of any aircraft are:

- 1) determination of ultimate structural strength required to sustain peak loads induced by turbulence
- 2) effects of turbulence on the fatigue life of the structure
- 3) performance of control systems in turbulence
- 4) handling and ride qualities in turbulence.

In an effort to provide the designer with some practical means of solving these and other problems, a number of statistical models of turbulence have been developed over the years. These models attempt to describe, in terms of as few parameters as possible, those characteristics of turbulence which are most important for various aspects of the design problem. Several of the most widely used of these models will be described below. In general they fall into two complementary classes, discrete and continuous, both of which are in use at the present time.

Discrete Models

Historically, the discrete model of turbulence was the first to be developed beginning with the work of Rhode and Lundquist in 1931 (Ref. 1). This model was intended to describe only the extreme gusts encountered by an aircraft over its operational lifetime. Because of its emphasis on extreme gusts, the discrete model is typically used to estimate ultimate strength requirements. As its name implies, the discrete model treats

turbulence as a series of isolated gusts. The principal assumptions used in deriving the discrete model are (Ref. 2, 3):

- 1) For purposes of aircraft design, atmospheric turbulence can be modeled as a collection of isolated gusts randomly distributed along the flight path of the aircraft.
- 2) These gusts have random magnitudes but fixed shape.
- 3) The aircraft is a deterministic linear system with sufficient damping that each encounter with one of these discrete gusts results in a single significant response peak.

Typical responses which may be studied by means of the discrete model are peak structural loads imposed by turbulence, extreme vertical accelerations of the aircraft, etc. Two discrete gust shapes which have been employed in the past are a ramp 30.48 meters (100 feet) in length used in the United Kingdom (Ref. 4) and a one-minus-cosine shape used in the United States (Ref. 5).

Since the gust shape is fixed and the vehicle is assumed to respond in a well damped linear manner, the relationship between gust magnitude and the peak value of response for a given aircraft is simply a constant of proportionality depending upon the flight condition (i.e., airspeed, gross weight, altitude, etc.) at the time of gust encounter. Thus, by recording response peaks and corresponding flight conditions, one can compute the magnitude of the discrete gust which caused each peak. These magnitudes, known as "equivalent gust velocities," have been extensively measured for various types of aircraft based on recordings of vertical accelerations by counting accelerometers. Statistics on the frequency of their occurrence have been compiled by many authors (e.g., Ref. 6).

The discrete model can be used to predict the responses of a proposed aircraft design by the following method (Ref. 7). First, using the experimentally determined distribution of equivalent gust velocities and the dynamic characteristics of the proposed vehicle, work backward through the procedure described above to obtain the distribution of vertical accelerations for the vehicle. Then assume a factor of proportionality relating these accelerations to the response magnitude, and finally convert the distribution of accelerations into the required response distribution.

Even though much data is available on the distribution of equivalent gusts, the discrete model is not satisfactory for all aspects of the aircraft design problem. For example, its application in areas such as structural fatigue, control system performance, ride qualities, or even extreme responses which involve lightly damped modes is highly questionable. The reason for this is that the assumptions on which the model is based are seldom realized in practice. In most instances turbulence does not occur as discrete gusts but as a continuous random disturbance. Furthermore, the conversion from measured responses to equivalent gust velocities neglects most of the dynamic characteristics of the vehicle, particularly the structural modes which tend to be lightly damped. As a result, success in calculating the response statistics of a proposed aircraft by means of the discrete gust model depends largely upon the proposed aircraft having very nearly the same response characteristics as the vehicle with which the original data were collected. Although this fortuitous circumstance may exist in some cases, it cannot always be assumed. Thus, although the necessity of evaluating the responses of a proposed aircraft to discrete gusts is still recognized as an important part of the design procedure (Ref. 8), the use of the discrete model to calculate most response

statistics has largely given way in recent years to the use of turbulence models which attempt to take both the dynamic characteristics of the vehicle and the continuous nature of turbulence into consideration. These are known as "power spectral density," "power spectral," or (perhaps more correctly) "continuous" models. It is this type of model which is of primary interest in the present report.

Continuous or Power Spectral Models

As will be seen shortly, the underlying idea of this type of model is that atmospheric turbulence can be represented by a continuous stochastic process which acts as a disturbing influence on the vehicle. The name "power spectral" has frequently been associated with these models because their development originated from studies of the power spectral density of atmospheric turbulence (Ref. 9). In actuality, these models involve not only the power spectral density of turbulence, but also an assumed probabilistic structure which is consistent with the power spectrum. Hence, to call them "power spectral" models is to stress one aspect and neglect the other. Thus, although long usage has firmly established the name "power spectral" (and indeed the name will often be used in this report), the reader should be aware that power spectral models incorporate not only a power spectrum but also a probability structure.

Because a turbulence model will be of little value to aircraft design if it cannot be used to predict vehicle responses, the type of stochastic process employed in the model must be one for which it is possible to calculate these responses with a minimum of difficulty. Because of this constraint, it is usually assumed that turbulence can be represented by a gaussian process. In this report, continuous models which incorporate

this assumption will be referred to as continuous gaussian models or gaussian power spectral models.

Gaussian Power Spectral Models

As mentioned above, power spectral models represent turbulence as a continuous disturbance. The aircraft is imagined to fly through large regions of turbulent air. (The adjective "large" is used here to mean that the time required for the aircraft to pass through one of these regions is very much greater than the response time constants of the vehicle.) It is usually assumed that within each region the turbulence is homogeneous and stationary, and is characterized by intensity and scale length parameters. More formally stated, the principal assumptions made in applying this type of model in its most simple form are (Refs. 3, 10):

- 1) Each encounter of an aircraft with continuous atmospheric turbulence can be modeled as a deterministic linear system (the aircraft) perturbed by three independent, stationary stochastic processes. These processes represent the longitudinal, lateral, and vertical gust components occurring at the vehicle center of gravity as it moves through the gust field.
- 2) The power spectral density of each random process belongs to a family of spectral shapes characterized by two parameters, the gust velocity standard deviation (σ) and the gust scale length (L). The scale length is a deterministic function of altitude and the standard deviation is a random variable which changes for each encounter of

the aircraft with a turbulent region of air. Within each turbulent region σ is assumed constant. Both L and σ may take on different values for each of the three gust components.

- 3) Each of the three gust components is a gaussian process.

The assumptions of the gaussian model make it possible to calculate the statistics of any vehicle response for each region of turbulence as functions of the parameters L and σ of that region. The statistical quantities of most frequent interest are:

- 1) Power spectral density -- a description of the magnitude of the Fourier components present in the response.
- 2) Probability distribution -- a description of the probability that a given response magnitude is exceeded.
- 3) Level crossing frequency -- the expected number of times per unit distance of flight that a given level or response magnitude is exceeded.

Specific methods by which these quantities can be calculated will be discussed in a later part of this report; for the present it will merely be noted that the assumptions of vehicle linearity and the gaussian nature of turbulence permit their evaluation with a minimum of difficulty. These results will, as noted above, be dependent upon the assumed values of L and σ along with the characteristics of the vehicle dynamics. Another way to express this dependence is to say that the response statistics are "conditioned" on these parameters. Note that under the assumptions of the model only one of these parameters, σ , is random.

The response statistics which are to be expected over the lifetime of the vehicle for a given flight condition (i.e., fixed altitude and

vehicle dynamics) can be computed by means of the gaussian power spectral model if the distribution of the random variable σ is known. Distributions of σ have been estimated based on extensive measurements of atmospheric turbulence. However, these distributions are not usually known for all altitudes of interest. Also, data presented in reference 11 shows that the scale length of turbulence is actually a random variable even at a fixed altitude; and the importance of neglecting this effect is not known. Furthermore, as will be discussed in later sections of this report, there is much evidence that atmospheric turbulence is not a gaussian process. As a result, the gaussian power spectral model is not entirely suitable for calculating lifetime statistics in a purely formal probabilistic manner. The procedures used for this purpose typically combine many of the ideas of the gaussian power spectral model with the response statistics of existing aircraft in a method not unlike that used with the discrete model (e.g., Ref. 12).

The power spectral model is used primarily to evaluate response statistics for selected flight conditions in continuous turbulence. For this application both the scale length and standard deviation of the turbulence as well as the vehicle characteristics are fixed at values representative of flight conditions which could reasonably be expected to occur in service. Thus the aircraft is imagined to be flying through an infinitely large region of stationary, homogeneous turbulence. The response statistics calculated for this case are then examined to determine whether or not the design is satisfactory. This approach is especially useful for control, handling, and ride qualities studies; and is the type of application for which the turbulence model developed in this report is intended.

For this restricted usage, sufficient data are available to develop a much more realistic model, and the three assumptions made above bear reconsideration. The first assumption stated that atmospheric turbulence could be represented by three independent, stationary stochastic processes. Physically, this is equivalent to requiring that: 1) the turbulence is stationary and homogeneous; and 2) the dimensions of the vehicle are much smaller than the scale lengths of the three gust components. These two conditions are, of course, not always satisfied and much research has been devoted to relaxing them. References 13 through 16, for example, describe how the power spectral model can be extended to account for such effects as the spatial distribution of the gust velocity field over the surface of the aircraft, correlation of the gust components, and the non-stationarity of the turbulence. Generalizations of this type will not be considered in this report because of the increased complication they would introduce. The reader should, however, be aware that such improvements are possible. For the simplified model described in this report the first assumption of the gaussian power spectral model will be considered valid.

The second assumption of the power spectral model concerned the specification of a family of power spectral densities for each gust component which depended upon only two parameters, L and σ . This assumption has been investigated at low altitude (Ref. 11) and found to be valid if both L and σ could be chosen freely for each sample. Representative values of the scale lengths to be used in the model can be expressed as a deterministic function of altitude by simply selecting the mean L measured at each altitude. The power spectral shapes usually assumed are those

proposed by von Karman (Ref. 17) or Dyrden (Ref. 18). Thus at low altitudes it is possible to satisfy the second assumption.

At high altitudes the validity of this assumption is not so certain, the problem being made more difficult by a lack of data. Those results available (e.g., Refs. 19, 20) show that spectra often behave like $\Omega^{-5/3}$ over the full range of wavelengths measured, (Ω denotes spatial frequency in radians per meter.) This would indicate that scale lengths are either very long or nonexistent at these altitudes. In practice this difficulty might be overcome while retaining the validity of the second assumption by using the von Karman or Dryden spectra and merely choosing a value of L longer than the longest wavelength to which the vehicle in question will respond. The standard deviation could be selected so that the power spectral density is properly scaled in the range of wavelengths to which the vehicle does respond.

The second assumption of the power spectral model is thus valid at low altitudes, but becomes more questionable as altitude increases. The lack of a scale length at high altitudes may perhaps be overcome by choosing a very long scale length as described above, or the treatment of high altitude turbulence as a self-similar process in the manner to be described below may be another solution.

The third assumption of the power spectral model, that turbulence is a gaussian process, is known to be incorrect (Refs. 21, 22). Since this fact is of central interest in this report, it will be discussed in some detail.

The non-gaussian nature of atmospheric turbulence makes itself apparent in two ways which are of importance to aircraft design. Compared to a gaussian process, turbulence is characterized by:

- 1) an increased number of high velocity gusts
- 2) an increased number of large gust velocity increments.

These two effects will be discussed separately even though they are obviously somewhat related.

High gust velocities are of importance to aircraft because it is this type of disturbance which tends to produce significant rigid body motions of the vehicle. These large gusts displace the vehicle from its equilibrium flight path, disconcerting the passengers and requiring the pilot to take corrective control action. As mentioned above, experimental data indicates that the gaussian turbulence model significantly underestimates the frequency of occurrence of these high gust velocities.

Reference 23 reports the examination of a large number of turbulence samples, each recorded at low altitude over a flight path distance of approximately 37 kilometers. The average number of velocity peaks in each sample was found to be 495, while the magnitude of the highest peak in each sample was consistently found to be greater than five times the standard deviation of the sample. This frequency of occurrence of such high gust velocities is several orders of magnitude greater than predicted by the gaussian assumption. Furthermore, peak gust velocity cumulative probabilities of exceedance observed in atmospheric turbulence (Refs. 11, 24) behave like $\exp(-x)$ rather than $\exp(-x^2)$ as predicted by the gaussian assumption. This again indicates a serious underestimation of high velocity gusts by the gaussian model.

The second non-gaussian characteristic of atmospheric turbulence which is important to aircraft design is the occurrence of a greater number of large gust increments than predicted by the gaussian model. The increment of a process $u(t)$ is defined to be a running difference of the form

$u(t) - (t - \tau)$, where τ is the constant time lag of the increment. Increments of $u(t)$ are necessarily gaussian if u itself is gaussian. Note that a large increment is not necessarily associated with high values of $u(t)$, only large changes of value. Large velocity increments, especially with short time lags, are of importance in causing loads on the aircraft structure because they contain high frequency components which tend to excite the elastic modes of the vehicle while having a minimal effect on its gross motions. Published evidence (Refs. 22, 25, 26) indicates that the gaussian model underestimates these large increments.

Gaussian Patch Model

The data described above which indicate that atmospheric turbulence is non-gaussian are typically composed of time histories recorded over a flight path distance of more than 30 or 40 kilometers. The apparently non-gaussian characteristics of these data which have been discussed above are sometimes explained within the framework of stationary gaussian processes by introducing a slightly modified form of the gaussian power spectral model known as the "quasi gaussian" or "gaussian patch" model (Ref. 21). The central idea of this model is that long, apparently non-gaussian samples of turbulence can be divided into a number of shorter gaussian segments with different intensities. The assumptions on which this model is based are the same as those of the conventional gaussian model described above, except that now each encounter of the aircraft with a large region of turbulence is imagined to be a number of encounters with smaller, independent patches of stationary, homogeneous turbulence. Though these patches are smaller than the turbulent region

which they make up, they are nevertheless still assumed to be large in the sense that the vehicle response can be assumed stationary within each patch, and transient effects between patches can be neglected. The scale length of the turbulence in all patches is assumed to be the same, but the intensity is allowed to vary randomly from patch to patch. This random intensity is usually restricted to only two discrete values (Ref. 27), although more values can be admitted if required.

Because of the above mentioned assumptions, the order in which the vehicle encounters patches of differing intensities is immaterial, and all nonstationary effects can be neglected insofar as vehicle responses are concerned. Thus the response of an aircraft flying through a large region of turbulence is assumed divisible into a number of shorter, independent time histories, each of which is stationary and gaussian. The intensity of each of these shorter response time histories corresponds to one of the two discrete values of turbulence intensity which are allowed.

Under the assumptions of the gaussian patch model, measurements of the gust velocity and response standard deviations based on time averages over very large turbulent regions are invalid because the assumption of stationarity (and therefore ergodicity) is incorrect for time histories involving more than a single patch. For example, the turbulence intensity estimated by time averaging would be a weighted average of the true intensities which were encountered. Given that the patch model is correct, it is possible to estimate the true patch intensities by a simple curve fitting technique based on measured level crossing frequencies (Ref. 27).

The important result of assuming the gaussian patch model to be correct is that probability distributions and level crossing frequencies

measured in large (30 km or larger) regions of turbulence will not appear gaussian because they will be based on samples from gaussian processes with differing intensities. If the gaussian patch model could be shown to correctly explain the non-gaussian characteristics of atmospheric turbulence it would greatly simplify the aircraft design problem because the conventional gaussian model described previously could still be used to model encounters with single patches of turbulence, and encounters with multiple patches could be described in terms of ensembles of stationary, independent gaussian processes. Thus the gaussian patch model would allow the apparently non-gaussian characteristics of atmospheric turbulence to be modeled without recourse to a non-gaussian turbulence model. Furthermore, responses of vehicles to the model could be expressed as collections of stationary gaussian processes and would therefore not require a nonstationary analysis.

Unfortunately, evidence indicates that the assumptions of the patch model are incorrect. Reference 27, for example, has examined the patch length implied by this model, and concluded that the most intense patches are only two or three kilometers in length. Because these patches are so short, the assumption of stationary vehicle responses becomes very questionable. This problem will be discussed in greater detail in a later section of this report.

It will also be shown later in this report that, if the assumptions of either the gaussian or the gaussian patch models are correct, then the standardized density function of the turbulence increments must be identical to that of the turbulence itself and independent of the time lag (at least for small time lags). However, references 22, 25, and 26 present data obtained at altitudes from sea level to 18,000 meters

(60,000 feet) showing that this is not the case, particularly at low altitudes. Thus it appears that neither the gaussian model nor the gaussian patch model can account for the frequent occurrence of large velocity increments in atmospheric turbulence.

To summarize the above comments, it appears that the first assumption of the gaussian power spectral model, regarding the representation of turbulence as a three component stochastic process, has been heavily researched and many improvements have been made. These possible improvements will not be considered in this report because of the increased complication they would introduce. The second assumption, regarding the use of a family of power spectral densities dependent only on the variables L and σ appears to be valid at low altitudes and can very probably be used at high altitudes for the purposes of modeling flight through turbulence. The third assumption, concerning the representation of atmospheric turbulence as a gaussian process, is not correct; and attempts to remedy the situation by introduction of the gaussian patch model do not appear to be justified.

Thus research on the subject of a non-gaussian turbulence model is a promising area in which to make a significant improvement in aircraft design procedures. The remainder of this report will concentrate on this aspect of turbulence modeling.

Associated Current Research

The only other current work in this area to the author's knowledge is that due to Jones (Refs. 3, 28) who is developing a model of turbulence based on the concept of turbulence as a self-similar process in the sense of Mandelbrot (Refs. 29, 30, 31). A process $u(t)$ is self-similar

if it has the property that transformations of the form $h^k u(ht)$ (for any value of h and some fixed value of k) do not change its statistical properties.

Two important implications of self-similarity from the standpoint of turbulence modeling are:

- 1) a self-similar process has an intermittent structure.
- 2) the power spectral density of a self-similar process must behave like $\Omega^{-\nu}$ for all Ω , where ν is related to the constant k of the self-similar transformation.

The first of these is an observed characteristic of atmospheric turbulence (e.g., Ref. 32) and is a very desirable property of a turbulence model. The second characteristic is not true of low altitude turbulence but, as described above, is easier to justify at high altitudes where measurements of power spectra have indicated just such behavior.

In reference 28, Jones has suggested a discrete model of turbulence which employs both random magnitudes and random lengths of the assumed gust shape. These two variables are related so that the level crossing frequency exhibits a distribution of the form $\exp(-x)\sqrt{x}$, which is reasonably consistent with observed level crossing data. This discrete model is proposed as a means of investigating the well-damped modes of the vehicle response.

Jones also discusses the possibility of a power spectral model of atmospheric turbulence which utilizes the idea of self-similarity. This model is essentially the gaussian patch model discussed above, but the self-similarity assumption is used to derive a relationship between patch length and patch intensity such that the extreme gust velocities of the model exhibit a level crossing frequency of the form $\exp(-x)/\sqrt{x}$.

Again this form is consistent with observed data. Jones proposes the self-similar power spectral model as a means of investigating lightly damped modes of the aircraft response.

The Non-Gaussian Model

The turbulence model proposed in this report, unlike that of Jones, will consider all modes of the aircraft response simultaneously, and will apply to turbulence with finite scale lengths. The principal purpose is to provide a non-gaussian turbulence model for use in representing typical encounters of aircraft with continuous atmospheric turbulence.

The research reported here is an extension of that published in references 33 and 34, which have described the development and application of a "patchy" non-gaussian turbulence model for use in flight simulators. The contribution of the present report is the further analysis of this model and the development of analytical techniques for applying it to vehicle response studies.

Since the present problem is more difficult than that discussed in references 33 and 34, the model treated in this report has been simplified to consider only the three gust components acting at the vehicle center of gravity. Reference 34 indicates at least one technique which can be used to expand this model so as to include the effects of gusts distributed over the surface of the vehicle.

The non-gaussian model proposed here differs from the gaussian power spectral model described previously in that it assumes atmospheric turbulence to be modeled by a certain class of stochastic processes which are, in general, non-gaussian. Since this class contains gaussian processes as a subclass, it follows that the proposed model is not

entirely distinct from the gaussian model, but can be viewed as a generalization which includes the gaussian model as a special case. (The name "non-gaussian" applied to this model is perhaps too general since the model is restricted to only a subclass of all non-gaussian processes. However, this name is conveniently short; and within this report there is no danger of confusion.)

The principal assumptions of the non-gaussian model as it is described here are (compare with those of the gaussian model above):

- 1) Each encounter of an aircraft with continuous atmospheric turbulence can be modeled as a deterministic linear system (the aircraft) perturbed by three independent, statistically stationary stochastic processes. These processes represent the longitudinal, lateral, and vertical gust components at the vehicle center of gravity as it moves through the gust field.
- 2) The power spectral density of each random process belongs to a family of spectral shapes characterized by two parameters, the gust velocity standard deviation (σ), and the gust scale length (L). The scale length is a deterministic function of altitude and σ is a random variable which changes for each encounter of the aircraft with a turbulent region of air. Within each turbulence region σ is assumed constant. Both L and σ may take on different values for each of the three gust components.
- 3) Each of the three gust components is a non-gaussian process of the form $u(t) = a(t) b(t) + c(t)$, where a , b , and c are independent, stationary gaussian processes.

The probability structure of the proposed model is implicit in the third assumption.

Outline of Report

The remainder of this work is divided into eight sections and an appendix. The principal topics treated in each section are:

Review of the Gaussian Model. Equations describing the gaussian power spectral model and its applications to vehicle response calculations are reviewed. This section also defines several statistical quantities which are used throughout the remainder of the report.

Validity of the Gaussian Model. This section shows that the assumption of a gaussian process is inconsistent with measured turbulence data.

Formulation of the Non-gaussian Model. The general ideas leading to the non-gaussian model proposed in this report are presented.

Analysis of the Non-gaussian Model. The specific form of the turbulence model is developed along with expressions for the power spectral density, probability distribution, and level crossing frequency of the model. The increment distribution of the model is also discussed.

Validity of the Non-Gaussian Model. This section presents probability distributions and level crossing statistics obtained for various low altitude flight conditions, and compares these with statistics predicted by the gaussian and non-gaussian models.

Calculation of Non-Gaussian Response Statistics. Methods for calculating the power spectral density, distribution function, and level crossing frequencies of aircraft responses to the non-gaussian turbulence model are developed.

Numerical Example. An example is presented showing the calculation of power spectral density, distribution function, and level crossing frequencies of the altitude error allowed by a simple autopilot.

Conclusions and Suggestions for Further Research. A brief summary of results is presented along with descriptions of a number of areas in which additional research is needed if the model described here is to become a useful tool for aircraft design.

Appendix A: Computer Programs. Listings and sample cases are presented for a number of computer programs used in working with the non-gaussian turbulence model.

Appendix B: Tabulated Functions. Tabulated values of the probability density, probability distribution, and level crossing frequency of the non-gaussian model are presented.

REVIEW OF THE GAUSSIAN MODEL

The gaussian model considered in this section is intended to represent large regions of homogeneous and stationary turbulence. The three principal assumptions on which it is based are:

- 1) Each encounter of the aircraft with continuous atmospheric turbulence can be modeled as a deterministic linear system (the aircraft) perturbed by three independent stationary stochastic processes. These processes represent the longitudinal, lateral, and vertical gust components at the vehicle center of gravity as it moves through the gust field.
- 2) The power spectral density of each random process belongs to a family of spectral shapes characterized by two parameters, the gust velocity standard deviation (σ) and the gust scale length (L). The scale length is a deterministic function of altitude and the standard deviation is a random variable which changes from encounter to encounter.
- 3) Each of the three gust components is a gaussian process.

Also, the statistical quantities of major interest to aircraft design are:

- 1) power spectral density
- 2) probability distribution
- 3) level crossing frequency.

The purpose of this section is to familiarize the reader with the definitions and notation of this report as well as to acquaint him with the turbulence model which is now in wide use. The following discussion will show how the statistical quantities listed above may be calculated first

for the gaussian model itself and then for the response of a linear system to the model. In addition, a result concerning the distribution of gust velocity increments and a simply physical interpretation of the model will be presented.

Power Spectral Density of the Turbulence Model

Two forms of power spectral densities are presently in common use, those proposed by von Karman (Ref. 17) and those by Dryden (Ref. 18). These spectra are compared in figure 1 below. The von Karman spectra (Eqs. 1, 2, and 3) are known to provide a more accurate representation of measured turbulence spectra because they properly model the $\Omega^{-5/3}$ behavior observed at high frequencies (Refs. 11, 35). However, the fractional exponents present mathematical difficulties which are avoided by use of the Dryden spectra (Eqs. 4, 5, and 6). The Dryden spectra will be assumed in this report.

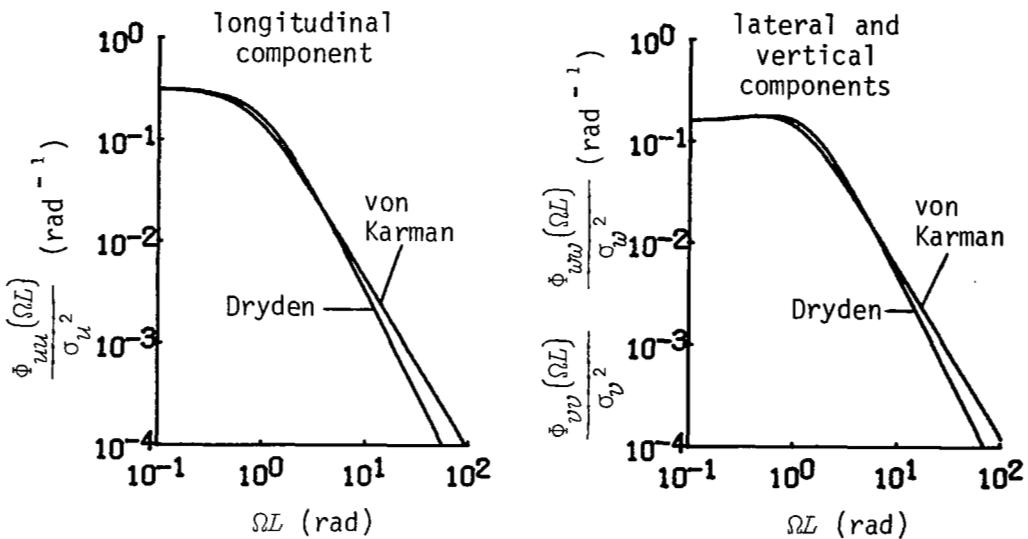


Figure 1.--Comparison of von Karman and Dryden power spectral densities. The Dryden spectra are assumed in this report.

$$\Phi_{uu}(\Omega) = \frac{L_u \sigma_u^2}{\pi} \frac{1}{[1 + (1.339 L_u \Omega)^2]^{5/6}} \quad (1)$$

$$\Phi_{vv}(\Omega) = \frac{L_v \sigma_v^2}{2\pi} \frac{[1 + \frac{8}{3} (1.339 L_v \Omega)^2]}{[1 + (1.339 L_v \Omega)^2]^{11/6}} \quad (2)$$

$$\Phi_{ww}(\Omega) = \frac{L_w \sigma_w^2}{2\pi} \frac{[1 + \frac{8}{3} (1.339 L_w \Omega)^2]}{[1 + (1.339 L_w \Omega)^2]^{11/6}} \quad (3)$$

$$\Phi_{uu}(\Omega) = \frac{L_u \sigma_u^2}{\pi} \frac{1}{[1 + (L_u \Omega)^2]} \quad (4)$$

$$\Phi_{vv}(\Omega) = \frac{L_v \sigma_v^2}{2\pi} \frac{[1 + 3(L_v \Omega)^2]}{[1 + (L_v \Omega)^2]^2} \quad (5)$$

$$\Phi_{ww}(\Omega) = \frac{L_w \sigma_w^2}{2\pi} \frac{[1 + 3(L_w \Omega)^2]}{[1 + (L_w \Omega)^2]^2} \quad (6)$$

The reader should be aware that these are "two sided" spectra defined for both positive and negative values of Ω . The standard deviation of each gust component is given by an expression of the form

$$\sigma^2 = \int_{-\infty}^{\infty} \Phi(\Omega) d\Omega \quad (7)$$

Probability Distribution of the Gust Velocity

The model discussed here is gaussian by definition, thus each component has a conditional probability density function of the form

$$p(x|\sigma) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right]. \quad (8)$$

The notation $(\cdot|\sigma)$ is used here to indicate the dependence or "conditioning" of the density function upon σ . In general, the σ in equation 8 corresponds to the σ 's appearing in equations 4 through 6 above, and is different for each gust component. Figure 2-a below shows a graph of this density function for positive values of x .

An equivalent description of the gust velocity probability distribution is given by the distribution function. The distribution function is related to the density function by integration.

$$P(x|\sigma) = \int_{-\infty}^x p(y|\sigma) dy \quad (9)$$

Figure 2-b shows the gaussian distribution function plotted on a probability scale. Note that it is a straight line on this scale.

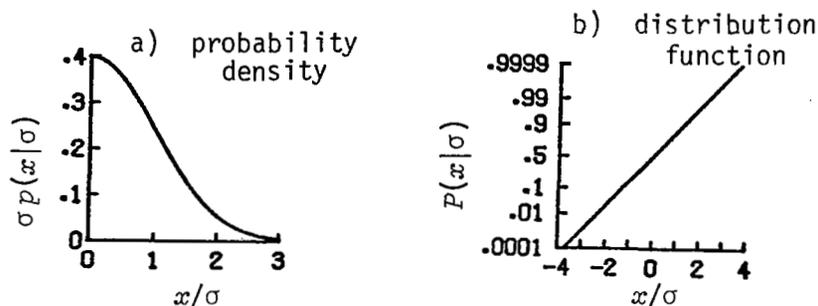


Figure 2.--Probability density and distribution functions of the gaussian turbulence model.

It will be of interest later to determine the distribution which would be obtained by combining data from a number of samples of the gaussian turbulence model. Note that the density function, equation 8, depends only on the standard deviation of the gust velocity. Thus, if the probability distribution of σ is known, the average or unconditional density function of the gust velocity can be found by integration

$$p(x) = \int_0^{\infty} p(x|y) p_{\sigma}(y) dy. \quad (10)$$

It should be noted that $p(x)$ is not a gaussian density.

In order to conform to conventional usage, the gust velocity distribution function for a number of data samples will be expressed as a complementary distribution of the absolute value of the gust velocity

$$P^c(x) = 1 - \int_{-x}^x p(y) dy. \quad (11)$$

$P^c(x)$ is the probability that the absolute value of the gust velocity is greater than x , and will be referred to as the cumulative gust velocity distribution in this report. The reader should be aware that P^c decreases monotonically even though the term "cumulative" is used to describe it.

Level Crossing Frequency of the Gust Velocity

The level crossing frequency of a gaussian process is given by the well known result due to Rice (Ref. 36). Rice's equation (divided by $2U$ to give the expected number of crossings with positive slope per unit distance of flight) applied to the longitudinal gust component gives the following expression for the conditional level crossing frequency.

$$N_u(x|\sigma_u, \sigma_u^*, U) = \frac{\sigma_u^*}{2\pi \sigma_u U} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_u}\right)^2\right] \quad (12)$$

The quantity σ_u^* in this equation denotes the standard deviation of the first derivative of the u -gust time history, and U is the mean true air-speed of the aircraft. Similar equations apply to the lateral and vertical components of the model.

A problem arises in the application of equation 12 to the turbulence model described here because σ_u^* does not exist for a process with the Dryden power spectral density (Eq. 4), nor will the analogous standard deviations exist for the other components of the model. The reason for this is clear when the relationship between σ_u^* and the power spectral density of u is considered. The theory of continuous stochastic processes (Ref. 37) requires that

$$\sigma_u^{*2} = U^2 \int_0^{\infty} \Omega^2 \Phi_{uu}(\Omega) d\Omega. \quad (13)$$

But the u -gust spectrum (and for that matter the lateral and vertical gust spectra) behaves like Ω^{-2} for large values of Ω . Therefore, the integral in equation 13 does not converge and σ_u^* does not exist. This result stems from the fact that the model has not accounted for viscous dissipation, which causes the true power spectrum of turbulence to decay more rapidly for wavelengths shorter than about one centimeter. These extremely short wavelengths are of no importance when vehicle responses are being calculated, and are justifiably neglected when the model is used for this purpose. However, in the next section of this report it

will be of interest to compare the level crossings of the model to those measured experimentally, and it is important to show that a level crossing frequency can be defined for the model without altering its characteristics as far as response calculations are concerned.

In practice this problem is sometimes overcome by truncating the power spectral densities at some convenient frequency. Another method is the addition of high frequency poles to the spectral forms as shown in equation 14 for the u -gust spectrum.

$$\Phi_{uu}(\Omega) = \frac{\sigma_u^2}{\pi} \frac{(L_u + \gamma_u)}{[1 + (L_u \Omega)^2][1 + (\gamma_u \Omega)^2]} \quad (14)$$

These poles can be physically interpreted as representing the effects of viscous dissipation, and γ_u can be thought of as representing the wavelength at which this dissipation becomes important. As will be seen shortly, however, the precise value chosen for γ_u will not be important for the purposes of this report.

Using equation 14 for Φ_{uu} allows one to compute the standard deviations required in Rice's equation. In particular,

$$\sigma_u^* = U \sigma_u (\gamma_u L_u)^{-1/2}. \quad (15)$$

Substitution of this result into equation 12 gives the following result for N_u .

$$N_u(x | \sigma_u, L_u, \gamma_u) = \frac{1}{2\pi(\gamma_u L_u)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_u}\right)^2\right] \quad (16)$$

Note that the zero crossing frequency is given by

$$N_u(0|\sigma_u, L_u, \gamma_u) = (2\pi)^{-1} (\gamma_u L_u)^{-1/2} \quad (17)$$

Thus the value of γ_u may be thought of as determining the zero crossing frequency of the u -gust component, and could be chosen so as to match equation 17 to some measured value.

The addition of high frequency poles to the vertical and lateral gust spectra in a manner similar to that of equation 14 will result in level crossing frequencies which differ from equation 16 by only a constant of proportionality.

$$N_v(x|\sigma, L, \gamma) = N_w(x|\sigma, L, \gamma) = (3/2)^{1/2} N_u(x|\sigma, L, \gamma) \quad (18)$$

Figure 3 shows a typical graph of level crossing frequencies for the gaussian model.

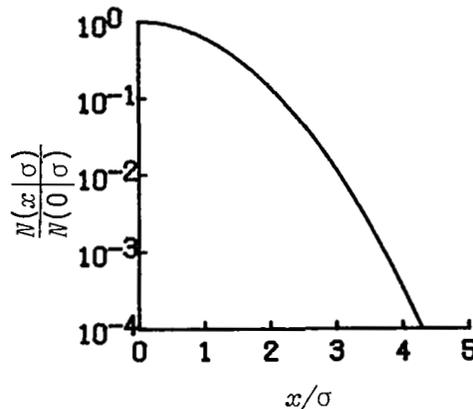


Figure 3.--Normalized level crossing frequency of the gaussian turbulence model.

It will be of interest later in this report to compute the level crossing frequencies which would be obtained by combining data from a large number of turbulence encounters. These theoretical results can

then be compared with experimentally measured data to determine the validity of the gaussian model. In past analyses of this type (e.g., Ref. 23) it is common to assume that the zero crossing frequency is constant for all data samples. That is,

$$N_u(0) = N_u(0|\sigma_u, L_u, \gamma_u) \quad (19)$$

for all values of σ_u , L_u , and γ_u . Thus the level crossing frequency for a large number of data samples is

$$N_u(x) = N_u(0) \int_0^{\infty} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right] p_{\sigma_u}(\sigma) d\sigma. \quad (20)$$

These level crossing curves are typically normalized with respect to $N_u(0)$ in order to obtain what is termed the normalized level crossing frequency or, perhaps more commonly, the cumulative probability of exceedance.

$$\frac{N_u(x)}{N_u(0)} = \int_0^{\infty} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right] p_{\sigma_u}(\sigma) d\sigma \quad (21)$$

Note that this result depends only upon the density function of σ_u .

Equation 21 was obtained from equation 16 by assuming that $N_u(0|\cdot, \cdot, \cdot)$ was a constant for all data samples. However, it is readily verified (and will be proved later in this report, Eq. 111, 112) that precisely the same conclusion will be reached if it is only required that σ_u be a random variable independent of both L_u and γ_u . In this event σ_u acts only as a scale factor of the process and can have no influence on the frequency with which it changes sign. Thus $N_u(0|\cdot, \cdot, \cdot)$ need not be a constant, but only a random variable independent of σ_u .

Expressions completely analogous to equation 21 can be obtained for the vertical and lateral gust components. Note that these results depend only upon the standard deviation of the gusts. In particular, the value of γ has no effect on the normalized level crossing frequency.

Distribution of Velocity Increments

Consider now the velocity increments of the gaussian turbulence model. Since identical results will be obtained for all three components, only the longitudinal gusts will be considered here. Define the u -gust increment to be

$$\Delta_u(t) = u(t) - u(t - \tau) \quad (22)$$

Since Δ_u is a linear transformation of a gaussian process, it must be gaussian. The mean and variance are readily calculated from equation 22.

$$E\{\Delta_u\} = 0 \quad (23)$$

$$E\{\Delta_u^2\} = \sigma_{\Delta}^2 = 2[\sigma_u^2 - C_{uu}(\tau)] \quad (24)$$

where C_{uu} is the autocorrelation function of u . Thus the probability density of Δ_u is given by

$$p_{\Delta}(x|\sigma_{\Delta}) = \frac{1}{\sigma_{\Delta}(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_{\Delta}}\right)^2\right], \quad (25)$$

and the average density of Δ_u computed from a number of samples of the gaussian model with differing intensities would be

$$p_{\Delta}(x) = \int_0^{\infty} \frac{1}{\sigma_{\Delta}(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right] p_{\sigma_{\Delta}}(\sigma) d\sigma. \quad (26)$$

A relationship will now be derived relating the average density of Δ_u to the average density of u itself. First write σ_{Δ} in the form

$$\sigma_{\Delta} = \sigma_u \alpha(\tau) \quad (27)$$

where

$$\alpha^2(\tau) = 2\left[1 - \frac{C_{uu}(\tau)}{\sigma_u^2}\right]. \quad (28)$$

Then the probability density of σ_{Δ} is related to the density of σ_u by

$$p_{\sigma_{\Delta}}(\sigma) = \frac{1}{\alpha(\tau)} p_{\sigma_u}\left[\frac{\sigma}{\alpha(\tau)}\right]. \quad (29)$$

Introduce the change of variable $\sigma' = \sigma/\alpha(\tau)$, substitute equations 27 and 29 into equation 26, and compare the resulting expression with equation 10 to obtain

$$p_{\Delta}(x) = \frac{1}{\alpha(\tau)} p_u\left[\frac{x}{\alpha(\tau)}\right]. \quad (30)$$

Now define the standardized density function to be

$$\hat{p}(x) = \sigma p(\sigma x) \quad (31)$$

and note that the average standard deviation of Δ_u is related to the average standard deviation of σ_u by

$$E\{\sigma_{\Delta}\} = \alpha(\tau) E\{\sigma_u\}. \quad (32)$$

It then follows from equations 30, 31, and 32 that

$$\hat{p}_{\Delta}(x) = \hat{p}_u(x) . \quad (33)$$

Equation 33 implies that the standardized density function of Δ_u is identical to that of u itself, and this result is independent of both the time lag τ and the distribution of σ_u .

Power Spectral Density of Vehicle Response

The power spectral density of some response $r(t)$ of a stable linear vehicle to a single component of the gaussian turbulence model is given by the well known relationship (Ref. 38) written here for the longitudinal gust component in terms of the spatial frequency Ω .

$$\Phi_{rr}(\Omega) = |H(iU\Omega)|^2 \Phi_{uu}(\Omega) \quad (34)$$

where $H(s)$ is the transfer function relation the response $r(t)$ to the input $u(t)$. Similar expressions hold for the other components of the turbulence model.

Probability Distribution of Vehicle Response

A linear response to a gaussian input is necessarily gaussian, thus the density function of any vehicle response $r(t)$ is given by

$$p_r(x) = \frac{1}{\sigma_r(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_r}\right)^2\right] \quad (35)$$

where σ_r is obtained from the power spectral density, equation 34, in a manner similar to equation 7. The distribution function of the response is given by an expression analogous to equation 9.

Level Crossing Frequency of the Vehicle Response

Since the vehicle response is necessarily gaussian, the level crossing frequency must be given by equation 12. In the case of a stable aircraft, the parameter γ introduced in the above discussion of the gust velocity level crossings need not be used because the vehicle will act as a low pass filter and the derivative of the response will always exist. The level crossing frequency of a vehicle response $r(t)$ to the u -gust component of the turbulence model is thus given by

$$N_r(x|\sigma_r, \sigma_r^*) = \frac{\sigma_r^*}{2\pi\sigma_r} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_r}\right)^2\right], \quad (36)$$

here σ_r and σ_r^* are determined from the power spectral density of the response (Eq. 34) by means of equations 7 and 13 respectively. Entirely analogous relationships hold for the lateral and vertical components of the model.

Physical Interpretation of the Gaussian Model

Each component of the gaussian turbulence model can be interpreted as wide band gaussian white noise passed through a linear filter as shown in figure 4.

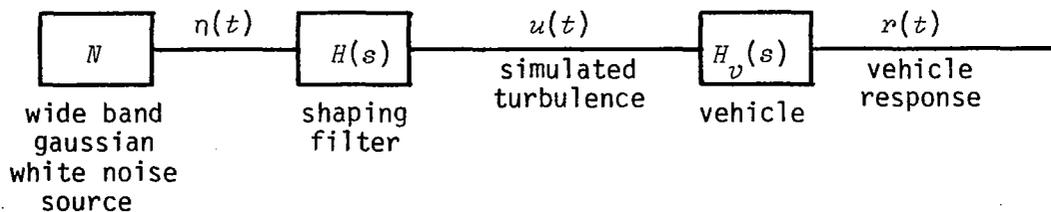


Figure 4.--Physical interpretation of a single component of the gaussian turbulence model and the response of a vehicle to this model.

The white noise source is assumed to generate gaussian noise with a power spectral density of unity over the range of frequencies passed by the shaping filter and the vehicle transfer function. That is,

$$\Phi_{\eta\eta}(\Omega) = 1.0 \quad (37)$$

The shaping filter transfer function is determined by means of equation 34 and the required turbulence power spectral densities discussed at the beginning of this section (Eqs. 4, 5, or 6).

VALIDITY OF THE GAUSSIAN MODEL

The gaussian turbulence model described in the previous section will now be compared with some experimentally measured gust statistics. It will be shown that neither the gaussian nor the gaussian patch turbulence model is able to reproduce these statistics.

Specific results to be presented are:

- 1) comparison of the gaussian probability distribution with gust velocity distributions estimated from single time histories of atmospheric turbulence
- 2) comparison of gust velocity distributions predicted by the gaussian model with distributions estimated from a large number of independent time histories
- 3) comparison of cumulative probability of exceedance distributions predicted by the gaussian model with distributions estimated from a large number of independent data samples
- 4) discussion of patch sizes implied by the gaussian patch model
- 5) comparison of measured velocity increment probability distributions with distributions predicted by the gaussian patch model.

Gust Velocity Distributions Estimated from Single Samples

Figure 5, originally published by Dutton (Ref. 21), presents estimated gust velocity distributions from two independent sources. In each case the gaussian distribution is indicated by a solid line. The curves

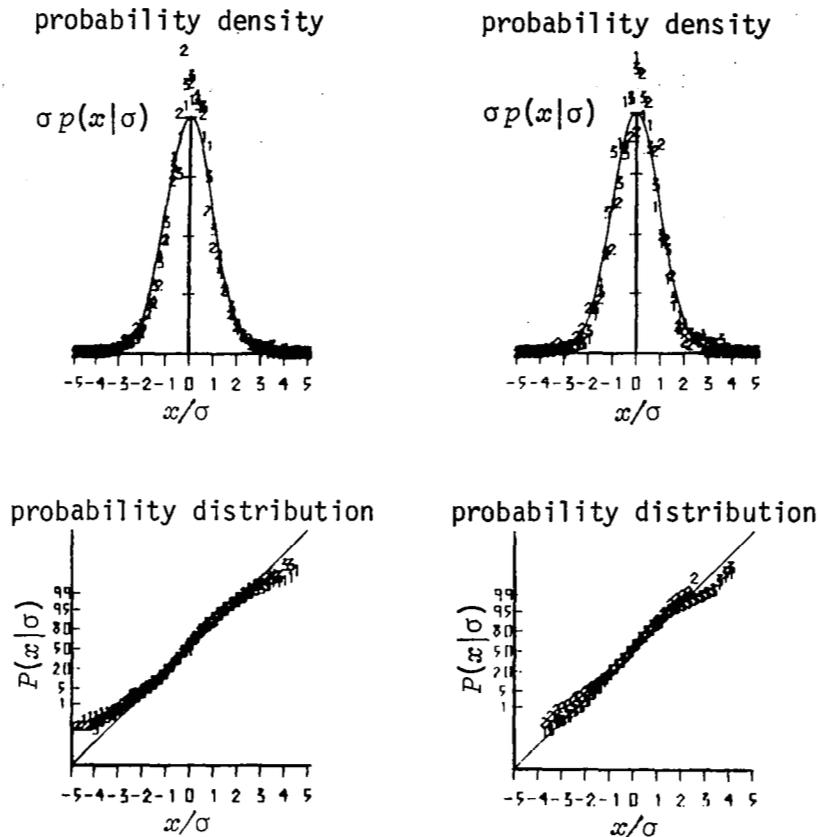


Figure 5.--Comparison of the gaussian turbulence model with gust velocity distributions estimated from single turbulence time histories.

on the left show three samples of the vertical gust component measured by G. K. Mather of the Canadian National Aeronautical Establishment (Ref. 39). These data were obtained at an altitude of approximately 9,000 meters (30,000 feet) over the Sierra Nevada mountains. The graphs on the right of figure 5 are from a single sample of severe turbulence encountered at an altitude of 18,000 meters (60,000 feet) during Project HI-CAT (Ref. 40, run number 264-16) sponsored by the United States Air Force. The numbers 1, 2, and 3 denote the vertical, lateral, and longitudinal gust components

respectively. The data of figure 5 clearly depart from the gaussian distribution at both small and large gust velocities,

It must be pointed out that these results alone do not disprove the hypothesis that atmospheric turbulence can be modeled as a locally gaussian process. Another possible explanation of the behavior illustrated in the figure is that the time histories from which the data were derived contained patches with differing intensities. This interpretation would agree with the gaussian patch model discussed in the introduction of this report. Further remarks regarding this model will be found in the fourth and fifth parts of this section.

Gust Velocity Distributions Estimated from Many Samples

Figure 6 presents two vertical gust velocity cumulative distributions obtained during the United States Air Force sponsored Project LO-LOCAT (Ref. 11). The data on the left were obtained during 5,000

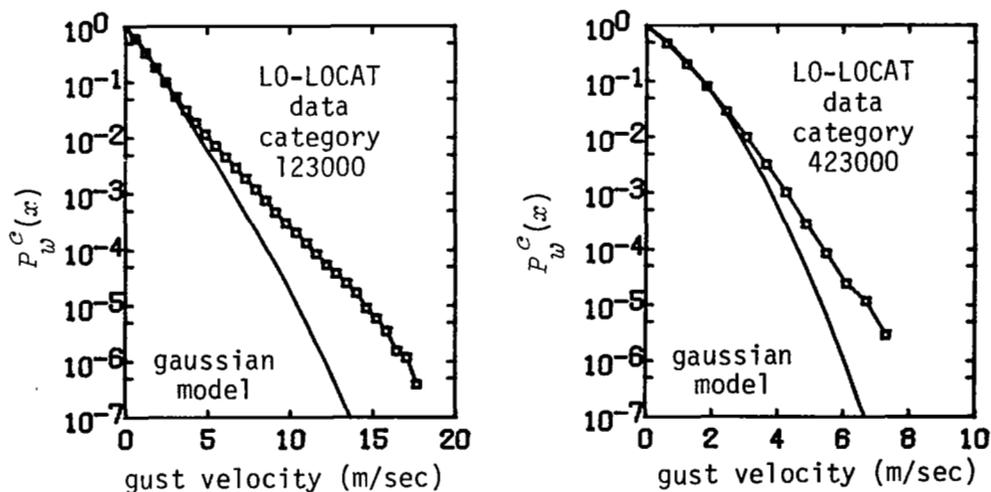


Figure 6.--Experimentally measured cumulative gust velocity distributions of atmospheric turbulence compared with those of the gaussian model.

kilometers (3,122 miles) of flight over high mountains at an average altitude of 230 meters (750 feet) above the surface in a neutrally stable atmosphere (data category 123000). The data on the right were collected during 2,000 kilometers (1,248 miles) of flight over plains at an altitude of 230 meters (750 feet) in neutrally stable conditions (data category 423000). In both cases the distribution predicted by the gaussian model has been calculated using equation 11 and the measured probability distributions of the standard deviations for each data category presented in reference 11. Note that in both cases the cumulative distribution estimated using the assumption of a gaussian process underestimates the occurrence of high gust velocities by substantial factors. Again, it should be pointed out that the apparently non-gaussian behavior of the data shown in figure 6 could be explained by the gaussian patch model.

Level Crossing Frequencies Estimated from Many Samples

Figure 7 presents two cumulative probability of exceedance distributions obtained during the LO-LOCAT project (Ref. 11). These data were

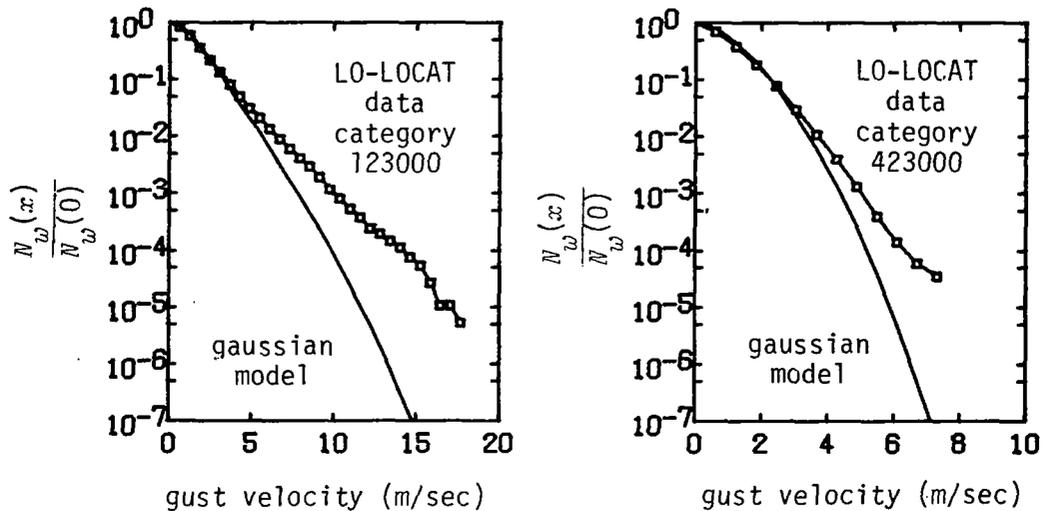


Figure 7.--Experimentally measured cumulative probability of exceedance distributions of atmospheric turbulence compared with those of the gaussian model.

derived from the same time histories which produced the data of figure 6, and a more detailed discussion of the test conditions will be found in the text describing that figure. The data on the left are from data category 123000 and those on the right are from category 423000. The distribution based on the gaussian model was derived using equation 21 and the distributions of gust velocity standard deviation presented in reference 11. Again the results based on the stationary gaussian turbulence model significantly underestimate the occurrences of high gust velocities.

Patch Sizes Implied by the Gaussian Patch Model

Figures 5, 6, and 7 have shown typical experimental observations which indicate that the stationary gaussian model underestimates the occurrences of high gust velocities. This apparently non-gaussian behavior can still be explained in terms of stationary gaussian processes by means of the gaussian patch model which was described in the introduction of this report.

If this model is a valid representation of atmospheric turbulence, the stationary gaussian model will provide a good representation of each turbulent patch and therefore will be a good model for aircraft design work. There is, however, evidence which indicates that the gaussian patch model is not valid.

A study of the patch size implied by the gaussian patch model carried out by Gould and MacPherson (Ref. 27) found that the more intense (and therefore the more important) patches are so small as to require a non-stationary analysis for vehicle response calculations. Figure 8 shows the relationship between patch size and intensity taken from reference 27. Intensity is measured in terms of vertical acceleration rms g 's of

the test aircraft. Note that patch size decreases rapidly with increasing intensity, and that the most intense patches are only two to three kilometers in length. An aircraft cruising at 200 m/sec true airspeed would pass through one of these patches in only 10 to 15 seconds. This interval is easily of the same order of magnitude as the significant response time constants of most vehicles, and encounters with such short disturbances would necessarily require a nonstationary analysis. It is also quite possible that the state of the vehicle at the time of initial entry into such short patches may have a significant effect on the response statistics, a result which would greatly complicate the analysis because the independence of patches could no longer be assumed.

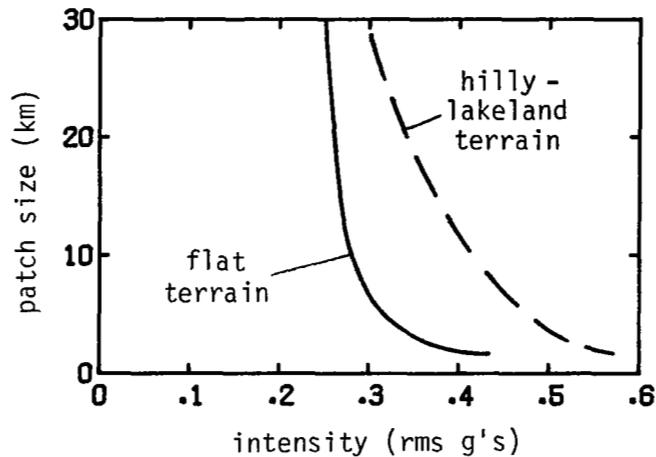


Figure 8.--Patch sizes implied by the gaussian patch model of atmospheric turbulence.

Thus, even though the gaussian patch model can explain the results shown in figures 5, 6, and 7, figure 8 indicates that the assumption of large patch sizes required to justify a stationary analysis of vehicle

responses is not valid. In addition to these results, the distribution of gust velocity increments provides further evidence that the gaussian patch model is invalid.

Distribution of Gust Velocity Increments

Figure 9, taken from reference 24, shows comparisons of gust velocity probability densities with corresponding velocity increment density functions. The solid line in each graph is the gaussian distribution and the dashed lines indicate the limits of the original data which can be found in reference 24. Each graph is based on several time histories which were chosen for their stationary character. In all cases the density functions have been standardized as defined in equation 31. The data on the left of the figure were obtained during the HI-CAT program (Ref. 40). The center graphs show data collected during flight through severe storm turbulence (Ref. 41). The data on the right were obtained at low altitude during the Barbados Oceanographic and Meteorological Experiment (Ref. 42). In

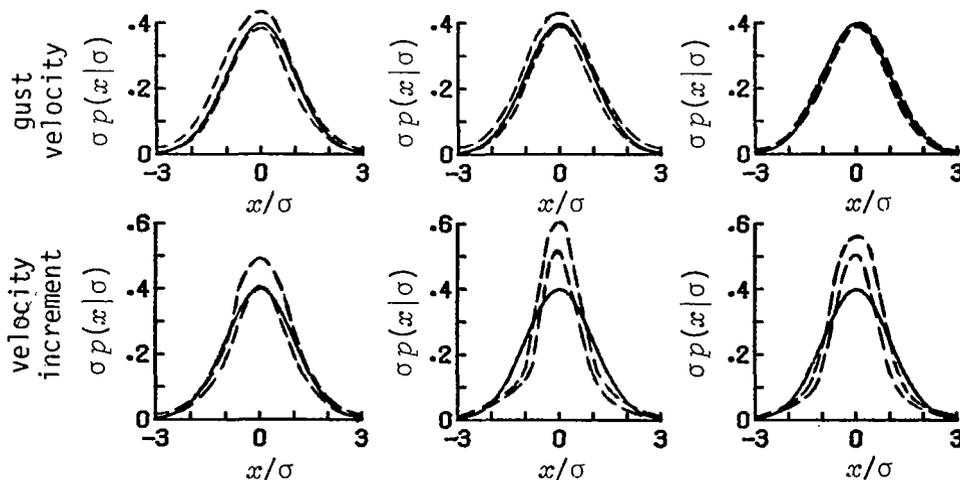


Figure 9.--Comparison of experimentally measured gust velocity distributions (top) with corresponding gust velocity increment distributions (bottom).

each case it is readily apparent that the increment distribution is not only non-gaussian but also quite different from the corresponding velocity distribution.

In the preceding section of this report (Eq. 33) it was shown that, if turbulence could be described as a collection of stationary gaussian patches, the standardized densities of the gust velocity and its increments were necessarily identical. Figure 9 clearly shows that this is not the case. Therefore, representation of atmospheric turbulence as a gaussian process is incorrect. Additional data presented in references 22 and 25 confirm this conclusion and also indicate that the gaussian model underestimates the occurrence of large gust increments.

Summary

This section has presented comparisons of the gaussian model with experimentally measured turbulence data from several independent sources. Examples of the gust velocity distributions and level crossing frequencies have indicated that the stationary gaussian model underestimates the occurrences of high gust velocities. The gaussian patch model, which could explain this behavior in terms of stationary gaussian processes, is not well suited to vehicle response calculations because it requires a nonstationary analysis of the most intense turbulence patches. It has also been shown that the distribution of velocity increments measured in turbulence is different from the distribution of velocity itself, a characteristic of turbulence which cannot be explained by the gaussian patch model. This last result constitutes a proof that neither the stationary gaussian nor the gaussian patch model is actually representative of the true structure of turbulence.

FORMULATION OF A NON-GAUSSIAN TURBULENCE MODEL

The preceding section has indicated that the gaussian turbulence model is not well suited for vehicle design work because it underestimates the occurrences of both high gust velocities and large gust increments. It is the purpose of this section to propose a non-gaussian turbulence model, and discuss the general line of reasoning leading to its formulation. Only a brief description will be presented here since a detailed analysis will be given in the following section.

The K_0 Model

The ideas which led to the turbulence model proposed here originated with the work of reference 33. The purpose of that report was to develop a "patchy" model of atmospheric turbulence for use with flight simulators. It is known that the gaussian model, generated by the system of figure 4, does not produce a realistic simulation of flight through turbulent air. Pilots complain of a lack of patchiness in the simulated turbulence (Ref. 43, 44). These patches are not associated with long term nonstationary changes of intensity in the sense of the gaussian patch model described previously, but are short bursts of activity which are sometimes only a few seconds in length.

No quantitative description of this type of patchiness has been given, but a qualitative measure can be obtained by observing the derivative of a turbulence time history. The top portion of figure 10 shows a comparison of derivatives from a sample of actual atmospheric turbulence and from the gaussian model. The gaussian time history was generated as shown in figure 4 and its power spectral density was chosen to match that of the actual turbulence as closely as possible. Observe that the

derivative of the true turbulence contains distinct bursts of activity which are totally lacking in the gaussian time history.

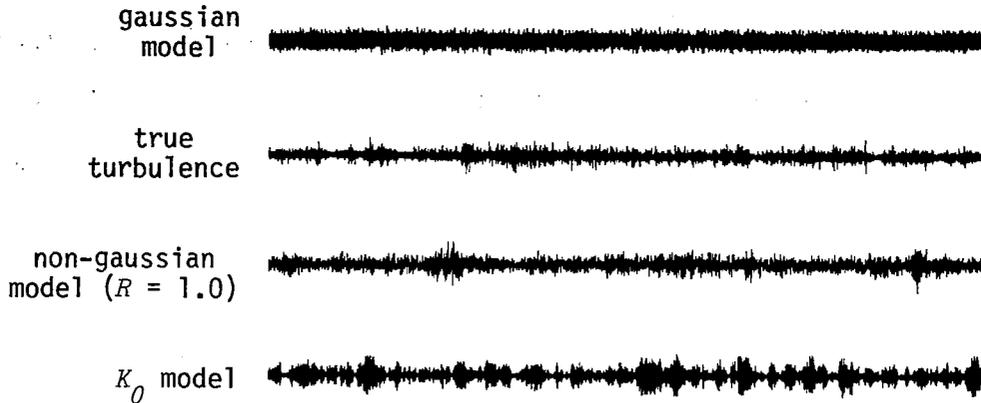


Figure 10.--The derivative of an atmospheric turbulence sample compared with derivative samples from several turbulence models. Sample length for these data is approximately 11 kilometers.

Reference 33 has suggested a product of independent gaussian processes as shown in figure 11 as a model of patchy turbulence. One of the two processes, say $a(t)$, can be imagined to represent a continuous gaussian time history without patches while $b(t)$ represents a modulating or "patch inducing" function. Reference 33 shows that the filter transfer functions

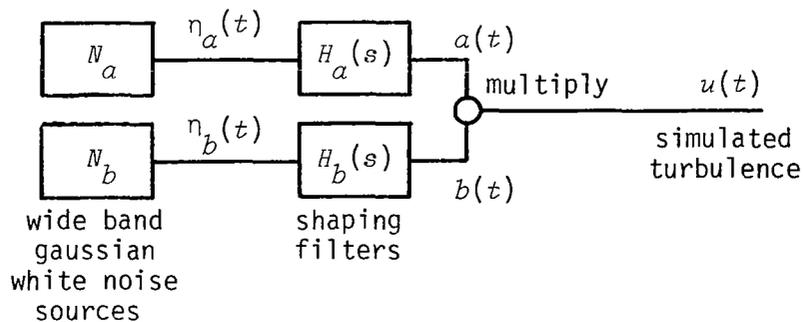


Figure 11.--Product of independent gaussian processes which has been suggested as a model of patchy turbulence.

H_a and H_b can be chosen so that the simulated turbulence time history has the Dryden spectral forms (Eq. 4, 5, or 6). The probability density of the simulated turbulence is proportional to a modified Bessel function of the second type and order zero, K_0 . For this reason the turbulence model represented by figure 11 is called the K_0 model. Figure 12 presents the probability density and distribution of this model along with those of the gaussian model.

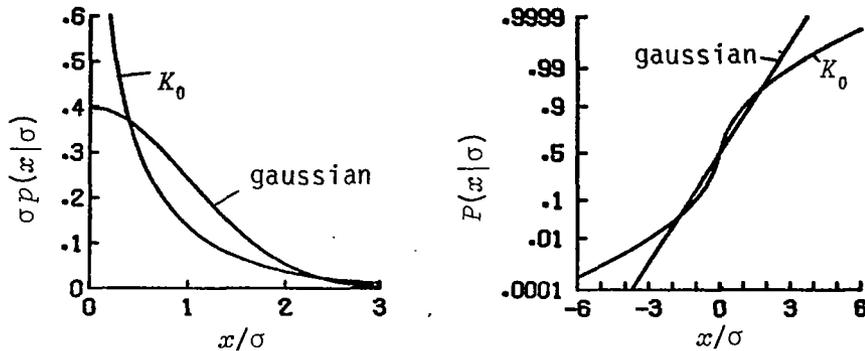


Figure 12.--Probability density and distribution functions of the K_0 turbulence model compared with those of the gaussian model.

A comparison of figure 12 with figure 5 on page 36 shows that the K_0 distribution departs from the gaussian distribution in a manner similar to that of actual turbulence. That is, the K_0 model is characterized by more small and large gusts than the gaussian model. However, it appears to be more severely non-gaussian than indicated by the experimental data.

The patchy characteristics of the K_0 model are shown at the bottom of figure 10. Note that they are much more severe than indicated by the true turbulence time history.

The Non-Gaussian Model

The fact that the K_0 and gaussian models seem to "bracket" the characteristics of atmospheric turbulence led, in reference 34, to the joining of these two models as shown in figure 13. For want of a better name this combination is called simply the non-gaussian turbulence model. It will be shown in the next section of this report that it is possible to choose the transfer functions H_a , H_b , and H_c of figure 13 so that the time histories $c(t)$ and $d(t)$ have identical power spectral densities of the Dryden form. When this choice is made the simulated turbulence will also have the Dryden spectral density. Furthermore, this result will be independent of the parameter R which appears in figure 13. Thus R does not affect the power spectral density of the non-gaussian model, it does however influence its probability distribution.

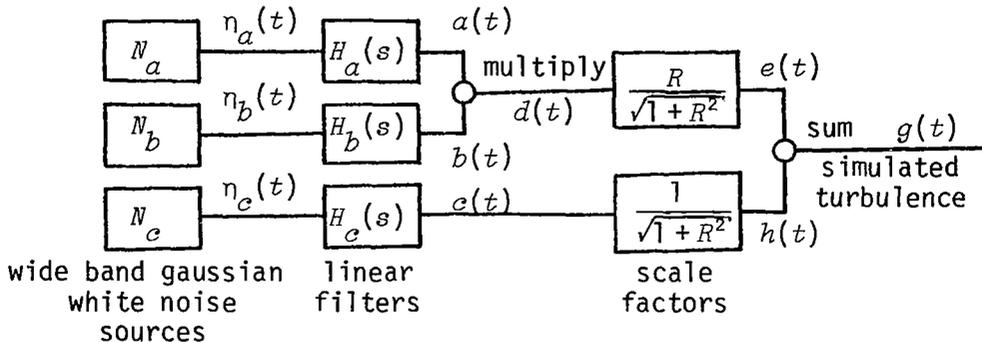


Figure 13.--Physical interpretation of a single component of the non-gaussian turbulence model. A similar interpretation applies to each of the three components.

To see the reason for this, suppose that the value of R is set to zero. Then the simulated turbulence will consist entirely of the gaussian time history, $c(t)$. As R is allowed to increase, more and more of the K_0 time history will be used. Finally, in the limit as the value of R

approaches infinity, the simulated turbulence will consist entirely of the K_0 process. Thus the parameter R is a control on the probability distribution and a continuum of distributions between gaussian and K_0 is available. Figure 14 shows the range of probability functions which can be obtained.

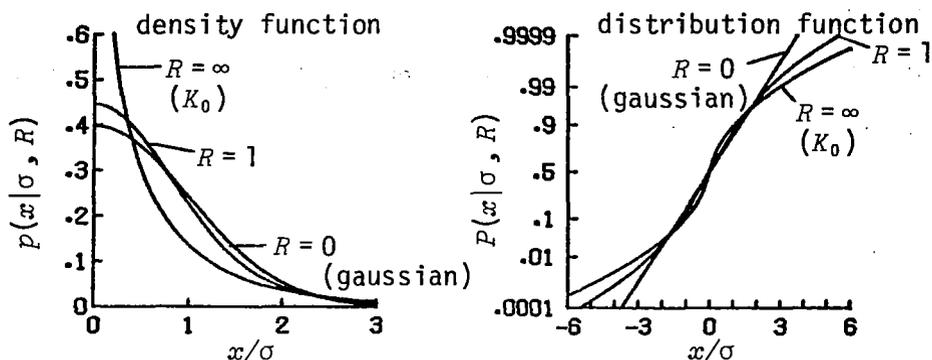


Figure 14.--Typical probability distributions attainable using the non-gaussian turbulence model.

As an indication of the patchy characteristics attainable using the non-gaussian model, figure 10 includes a sample of the derivative for the case R equal to unity. Note that it matches the characteristics of the true turbulence sample quite well.

Figure 13 is a physical interpretation of the non-gaussian model, and shows how it can be generated for numerical studies. The principal assumptions on which the model is based are the same as those of the gaussian model, except of course for the probability distribution. These assumptions are:

- 1) Each encounter of an aircraft with atmospheric turbulence can be modeled as a deterministic linear system (the aircraft) perturbed by three independent, stationary stochastic

- processes. These processes represent the longitudinal, lateral, and vertical gust components at the vehicle center of gravity as it moves through the gust field.
- 2) The power spectral density of each random process belongs to a family of spectral shapes characterized by two parameters, the gust velocity standard deviation (σ), and the gust scale length (L). The scale length is a deterministic function of altitude and σ is a random variable which changes from encounter to encounter.
 - 3) Each of the three gust components is a non-gaussian process of the form $u(t) = a(t)b(t) + c(t)$, where a , b , and c are independent, stationary gaussian processes.

The validity of the first two assumptions has been discussed in the introduction. The power spectral forms chosen for use in this report are those proposed by Dryden (Eq. 4, 5, 6). The third assumption is, of course, the central idea of the non-gaussian model. Its validity will be investigated in later sections of this report as the properties of the proposed model are compared with those of atmospheric turbulence.

ANALYSIS OF THE NON-GAUSSIAN MODEL

It is the purpose of this section to derive suitable expressions for the transfer functions used in the non-gaussian model, and to analyze the model's statistical properties. Specific results to be obtained for each component of the model are:

- 1) transfer functions
- 2) probability distribution
- 3) level crossing frequency
- 4) increment distribution.

Figure 13 presents the physical interpretation which can be applied to each of the model's three components. The notation of this figure will be used throughout the following development. The problem of vehicle response calculations will not be considered here, but will be taken up in a later section of this report.

Derivation of Transfer Functions

Recall from the introduction of this report that the turbulence models described here are to have the Dryden power spectral densities (Eq. 4, 5, 6). It will now be shown that it is possible to choose the filter transfer functions H_a , H_b , and H_c of figure 13 so that each component of the non-gaussian model has the appropriate spectral form. The method to be followed is:

- 1) Fourier transform the Dryden spectra to obtain the corresponding correlation functions
- 2) derive a general expression relating the correlation function of the non-gaussian model to the filter transfer functions

- 3) assume specific forms for the transfer functions involving unspecified constants
- 4) show that the constants can be chosen so that the correlation function of the non-gaussian model has the required Dryden form for each component.

In general, the correlation function is related to the power spectral density through a Fourier transform of the form

$$C(\tau) = \int_{-\infty}^{\infty} \Phi(\Omega) \exp(i U \Omega \tau) d\Omega . \quad (38)$$

Applying this transformation to the Dryden spectra, equations 4, 5, and 6 gives

$$C_{uu}(\tau) = \sigma_u^2 \exp\left(-\frac{U}{L_u} |\tau|\right) \quad (39)$$

$$C_{vv}(\tau) = \sigma_v^2 \left(1 - \frac{U|\tau|}{2L_v}\right) \exp\left(-\frac{U}{L_v} |\tau|\right) \quad (40)$$

$$C_{ww}(\tau) = \sigma_w^2 \left(1 - \frac{U|\tau|}{2L_w}\right) \exp\left(-\frac{U}{L_w} |\tau|\right) . \quad (41)$$

These equations complete the first step of the transfer function derivation. A general expression for the correlation function of the simulated turbulence time history of figure 13 will now be derived. By definition, this correlation function is

$$C_{gg}(\tau) = E\{g(t) g(t+\tau)\} . \quad (42)$$

From figure 13, $g(t)$ is

$$g(t) = a(t) b(t) R(1+R^2)^{-1/2} + c(t)(1+R^2)^{-1/2} . \quad (43)$$

Since a , b , and c are independent, zero mean random processes, the correlation function of $g(t)$ can be written as

$$C_{gg}(\tau) = C_{aa}(\tau) C_{bb}(\tau) R^2 (1+R^2)^{-1} + C_{cc}(\tau) (1+R^2)^{-1}. \quad (44)$$

Now consider $a(t)$ alone. It follows from the general relationship between the input and output power spectral densities of a linear system (Ref. 38) that the spectral density of a has the form

$$\Phi_{aa}(\Omega) = |H_a(iU\Omega)|^2 \Phi_{\eta_a \eta_a}(\Omega). \quad (45)$$

If it is assumed that the power spectral density of the process $\eta_a(t)$ is unity over the range of frequencies for which H_a is not essentially zero, then

$$\Phi_{aa}(\Omega) = |H_a(iU\Omega)|^2. \quad (46)$$

The correlation function of $a(t)$ can be found by Fourier transforming equation 46 as shown in equation 38.

$$C_{aa}(\tau) = \int_{-\infty}^{\infty} |H_a(iU\Omega)|^2 \exp(iU\Omega\tau) d\Omega \quad (47)$$

Similar results apply to $b(t)$ and $c(t)$ of figure 13. This result, along with equation 44 completes the second step of the transfer function derivation.

For the third step of the procedure, suppose that the following general forms are assumed for the transfer functions of the model.

$$H_a(s) = \frac{N_1}{1+D_1 s} \quad (48)$$

$$H_b(s) = \frac{N_2 + N_3 s}{(1 + D_2 s)^2} \quad (49)$$

$$N_c(s) = \frac{N_4 + N_5 s}{(1 + D_3 s)^2} \quad (50)$$

These particular expressions have been chosen because they will lead to useful results. There is no reason why other general forms could not be chosen which might also give good results. (See the discussion of increment distributions in this section and suggestions for further research in a later section of this report.)

Transforming equations 48 through 50 as shown in equation 47 yields the following correlation functions.

$$C_{aa}(\tau) = \frac{N_1^2}{2D_1} \exp\left(-\frac{|\tau|}{D_1}\right) \quad (51)$$

$$C_{bb}(\tau) = \left(\frac{N_3}{2D_2}\right)^2 \left\{ |\tau| \left[\left(\frac{N_2}{N_3}\right)^2 - \left(\frac{1}{D_2}\right)^2 \right] + \left[\frac{1}{D_2} + D_2 \left(\frac{N_2}{N_3}\right)^2 \right] \right\} \exp\left(-\frac{|\tau|}{D_2}\right) \quad (52)$$

$$C_{cc}(\tau) = \left(\frac{N_5}{2D_3}\right)^2 \left\{ |\tau| \left[\left(\frac{N_4}{N_5}\right)^2 - \left(\frac{1}{D_3}\right)^2 \right] + \left[\frac{1}{D_3} + D_3 \left(\frac{N_4}{N_5}\right)^2 \right] \right\} \exp\left(-\frac{|\tau|}{D_3}\right) \quad (53)$$

Substitution of these results into equation 44 will give the general form of the correlation function of the non-gaussian model.

If now the following choices are made for the arbitrary constants in equations 51, 52, and 53

$$\begin{aligned}
 N_1 &= 4\sigma_u \frac{L_u}{U} & N_2 &= 1.0 & N_3 &= \frac{2L_u}{U} \\
 N_4 &= \sigma_u \left(\frac{2L_u}{U}\right)^{1/2} & N_5 &= \sigma_u \left(\frac{2L_u^3}{U^3}\right)^{1/2} & D_1 &= \frac{2L_u}{U} \\
 D_2 &= \frac{2L_u}{U} & D_3 &= \frac{L_u}{U}
 \end{aligned} \quad (54)$$

the resulting correlation functions of the model become

$$c_{cc}(\tau) = c_{dd}(\tau) = c_{gg}(\tau) = \sigma_u^2 \exp\left(-\frac{U}{L_u}|\tau|\right) \quad (55)$$

which is the form of the u -gust correlation function, equation 39. Note that not only $g(t)$ but also $c(t)$ and $d(t)$ have the u -gust correlation function. The following result, which will be useful later in this section, is obtained directly from equation 55 by setting τ equal to zero.

$$\sigma_c^2 = \sigma_d^2 = \sigma_g^2 = \sigma_u^2 \quad (56)$$

If the constants of the filter transfer functions are chosen to be

$$\begin{aligned}
 N_1 &= \sigma_v (128)^{1/2} \left(\frac{L_v}{U}\right)^2 & N_2 &= 0.0 & N_3 &= 1.0 \\
 N_4 &= \sigma_v \left(\frac{L_v}{U}\right)^{1/2} & N_5 &= \sigma_v \left(\frac{3L_v^3}{U^3}\right)^{1/2} & D_1 &= \frac{2L_v}{U} \\
 D_2 &= \frac{2L_v}{U} & D_3 &= \frac{L_v}{U}
 \end{aligned} \quad (57)$$

the resulting correlation functions are

$$C_{cc}(\tau) = C_{dd}(\tau) + C_{gg}(\tau) = \sigma_v^2 \left(1 - \frac{U|\tau|}{2L_v}\right) \exp\left(-\frac{U}{L_v} |\tau|\right). \quad (58)$$

This is the desired form of the lateral gust correlation function, equation 40. Again, the time histories $c(t)$, $d(t)$ and $g(t)$ all have the same correlation function and

$$\sigma_c^2 = \sigma_d^2 = \sigma_g^2 = \sigma_v^2. \quad (59)$$

Finally, if the constants are chosen to be

$$\begin{aligned} N_1 &= \sigma_w (128)^{1/2} \left(\frac{L_w}{U}\right)^2 & N_2 &= 0.0 & N_3 &= 1.0 \\ N_4 &= \sigma_w \left(\frac{L_w}{U}\right)^{1/2} & N_5 &= \sigma_w \left(\frac{3L_w^3}{U^3}\right)^{1/2} & D_1 &= \frac{2L_w}{U} \\ D_2 &= \frac{2L_w}{U} & D_3 &= \frac{L_w}{U} \end{aligned} \quad (60)$$

the correlation functions are

$$C_{cc}(\tau) = C_{dd}(\tau) = C_{gg}(\tau) = \sigma_w^2 \left(1 - \frac{U|\tau|}{2L_w}\right) \exp\left(-\frac{U}{L_w} |\tau|\right). \quad (61)$$

This result is the Dryden form of the vertical gust correlation function. Again note that $c(t)$, $d(t)$ and $g(t)$ have the same correlation function, and therefore

$$\sigma_c^2 = \sigma_d^2 = \sigma_g^2 = \sigma_w^2. \quad (62)$$

This completes the derivation of transfer functions for the non-gaussian model. The results are summarized in table 1.

Table 1.--Transfer functions of the non-gaussian model

| Longitudinal Component | | |
|---|--|---|
| $H_a(s) = \frac{4\sigma_u \frac{L_u}{U}}{1 + \frac{2L_u}{U} s}$ | $H_b(s) = \frac{1}{1 + \frac{2L_u}{U} s}$ | $H_c(s) = \frac{\sigma_u \left(\frac{2L_u}{U}\right)^{1/2}}{1 + \frac{L_u}{U} s}$ |
| Lateral Component | | |
| $H_a(s) = \frac{\sigma_v (128)^{1/2} \left(\frac{L_v}{U}\right)^2}{1 + \frac{2L_v}{U} s}$ | $H_b(s) = \frac{s}{\left[1 + \frac{2L_v}{U} s\right]^2}$ | $H_c(s) = \frac{\sigma_v \left(\frac{L_v}{U}\right)^{1/2} \left[1 + \sqrt{3} \frac{L_v}{U} s\right]}{\left[1 + \frac{L_v}{U} s\right]^2}$ |
| Vertical Component | | |
| $H_a(s) = \frac{\sigma_w (128)^{1/2} \left(\frac{L_w}{U}\right)^2}{1 + \frac{2L_w}{U} s}$ | $H_b(s) = \frac{s}{\left[1 + \frac{2L_w}{U} s\right]^2}$ | $H_c(s) = \frac{\sigma_w \left(\frac{L_w}{U}\right)^{1/2} \left[1 + \sqrt{3} \frac{L_w}{U} s\right]}{\left[1 + \frac{L_w}{U} s\right]^2}$ |

Two additional features of the above derivation justify comment. First, the forms of the transfer functions assumed here have been selected so that the correlation functions of the model do not depend upon the parameter R . Thus R will not influence the power spectral density of the model. As was discussed in the preceding section, this is a very convenient result which allows R to act solely as a control on the model's probability distribution. However, it will become apparent presently that the particular choices made here will have undesirable effects in so far as the distribution of increments is concerned. Further comment on this subject will be found in the discussion of increment distributions to be found in this section, and also in the suggestions for further research in a later section of this report.

A second point concerning the above derivation is that the choice of parameters made in equation 54 is somewhat arbitrary. In particular, the quantities D_1 and D_2 could be chosen to be any pair of numbers satisfying the relationship

$$\frac{1}{D_1} + \frac{1}{D_2} = \frac{U}{L_u} \quad (63)$$

This arbitrariness has been used in Reference 45 to define a "patchiness parameter" which can be included as an additional variable of the model. This complication is not considered in the present work, but is mentioned as an indication of the further generality which can be introduced into the non-gaussian model.

Probability Distribution

Attention is now turned to some aspects of the probabilistic structure of the non-gaussian model. The first topic to be discussed will be the model's probability distribution. The following remarks apply equally well to all three components of the proposed model; for the sake of definiteness, however, only the longitudinal gusts will be explicitly discussed.

Consider again the physical interpretation of the non-gaussian model presented in figure 13. The simulated turbulence time history is

$$g(t) = e(t) + h(t), \quad (64)$$

where

$$e(t) = a(t) b(t) R(1+R^2)^{-1/2} \quad (65)$$

$$h(t) = c(t)(1+R^2)^{-1/2}. \quad (66)$$

The time histories $a(t)$, $b(t)$ and $c(t)$ are independent, zero mean, gaussian random processes. Thus each has a probability density of the form

$$p(x|\sigma) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right]. \quad (67)$$

The steps to be followed in deriving an expression for the probability distribution of the non-gaussian model are:

- 1) derive an expression for the probability density of the product $a(t)b(t)$ appearing in equation 65
- 2) using the density function from step 1, obtain an expression for the characteristic function of the first term of equation 64

- 3) find the characteristic function of the second term of equation 64
- 4) multiply the characteristic functions of steps 2 and 3 to obtain the characteristic function of the non-gaussian model
- 5) Fourier transform to obtain the density function of the non-gaussian model.

The first step is to find an expression for the probability density of the product $a(t)b(t)$. Since a and b are independent processes, it follows that the probability distribution function of their product is

$$P_{ab}(z|\sigma_a, \sigma_b) = \iint_{\xi} p_a(x|\sigma_a) p_b(y|\sigma_b) dx dy, \quad (68)$$

where ξ denotes that region of the $x-y$ plane in which the condition $xy \leq z$ is satisfied. The density functions p_a and p_b are both of the gaussian form presented in equation 67. Substitution of equation 67 into equation 68 followed by differentiation with respect to z and finally integration gives the following result for the density function of $a(t)b(t)$.

$$p_{ab}(z|\sigma_a, \sigma_b) = \frac{1}{\pi \sigma_a \sigma_b} K_0 \left(\frac{|z|}{\sigma_a \sigma_b} \right), \quad (69)$$

where K_0 is the modified Bessel function of the second type and order zero. Equation 69 completes the first step of the derivation.

Now note that $\sigma_a \sigma_b$ is the standard deviation of the product $a(t)b(t)$ and, according to the notation of figure 13, this product is equivalent to $d(t)$. Also, equation 56 states that the standard deviation of $d(t)$ is equal to σ_u by virtue of the choice of transfer functions of the model.

Thus $\sigma_a \sigma_b$ is equal to σ_u , and equation 69 can be written

$$p_{ab}(z|\sigma_u) = \frac{1}{\pi\sigma_u} K_0 \left(\frac{|z|}{\sigma_u} \right) . \quad (70)$$

The first term of equation 64 is, according to equation 65, the product of $a(t)b(t)$ scaled by the factor $(1+R^2)^{-1/2}$, so it follows immediately that

$$p_e(z|\sigma_u, R_u) = \frac{(1+R_u^2)^{1/2}}{\pi R_u \sigma_u} K_0 \left[\frac{|z|(1+R_u^2)^{1/2}}{\pi R_u \sigma_u} \right] . \quad (71)$$

The characteristic function of $e(t)$ can be found by Fourier transforming equation 71.

$$\phi_e(f|\sigma_u, R_u) = \int_{-\infty}^{\infty} p_e(z|\sigma_u, R_u) \exp(i 2\pi f z) dz \quad (72)$$

The particular form of the Fourier transform employed in equation 72 was chosen for two reasons. First, it will lead to some simplification of the expressions to be derived presently. Secondly, and perhaps more importantly, a computer program implementing this transformation will be used for numerical studies later in this report.

Substitution of equation 71 into equation 72 and evaluation of the resulting integral gives

$$\phi_e(f|\sigma_u, R_u) = \{1 + [2\pi R_u \sigma_u (1+R_u^2)^{-1/2} f]^2\}^{-1/2} . \quad (73)$$

Equation 73 completes the second step of the derivation.

Now consider the second term on the right hand side of equation 64. The process $h(t)$ is clearly gaussian because $e(t)$ is gaussian, and its

standard deviation is just that of $e(t)$ scaled by the factor $(1+R^2)^{-1/2}$. From equation 56 it is known that σ_e is identical to σ_u . Thus the probability density of $h(t)$ is

$$p_h(z|\sigma_u, R_u) = \frac{(1+R_u^2)^{1/2}}{\sigma_u(2\pi)^{1/2}} \exp\left[-\frac{z^2(1+R_u^2)}{2\sigma_u^2}\right]. \quad (74)$$

The characteristic function of h is found by Fourier transforming p_h in the same manner as equation 72.

$$\phi_h(f|\sigma_u, R_u) = \exp\{-2[\pi\sigma_u(1+R_u^2)^{-1/2}f]^2\} \quad (75)$$

This result completes the third step of the derivation.

Since $e(t)$ and $h(t)$ are independent random processes, it follows that the characteristic function of their sum is given by the product of their respective characteristic functions. Thus, for the case of the u -gust component of the non-gaussian model

$$\phi_g(f|\sigma_u, R_u) = \left[1 + \frac{(2\pi\sigma_u R_u f)^2}{1+R_u^2}\right]^{-1/2} \exp\left[-\frac{2(\pi\sigma_u f)^2}{1+R_u^2}\right]. \quad (76)$$

The characteristic functions of the vertical and lateral components of the model are obtained merely by replacing the subscript u of equation 76 with w and v respectively. This completes the fourth step of the derivation.

The probability density of the non-gaussian model can now be obtained by inverse Fourier transforming equation 76.

$$p_g(x|\sigma_u, R_u) = \int_{-\infty}^{\infty} \phi_g(f|\sigma_u, R_u) \exp(-i2\pi fx) dx \quad (77)$$

Equation 77 is easily evaluated numerically by means of the fast Fourier transform program *FFT* which will be found in the appendix of this report. Similar equations apply to each component of the model. The probability distribution function can be obtained directly from the density function of equation 77 by means of integration.

$$P_g(x|\sigma_u, R_u) = \int_{-\infty}^x p_g(y|\sigma_u, R_u) dy \quad (78)$$

Figure 15 presents the density and distribution functions of the non-gaussian model for several values of the parameter R . Tabulated values of these functions will be found in tables B1 and B2 of Appendix B.

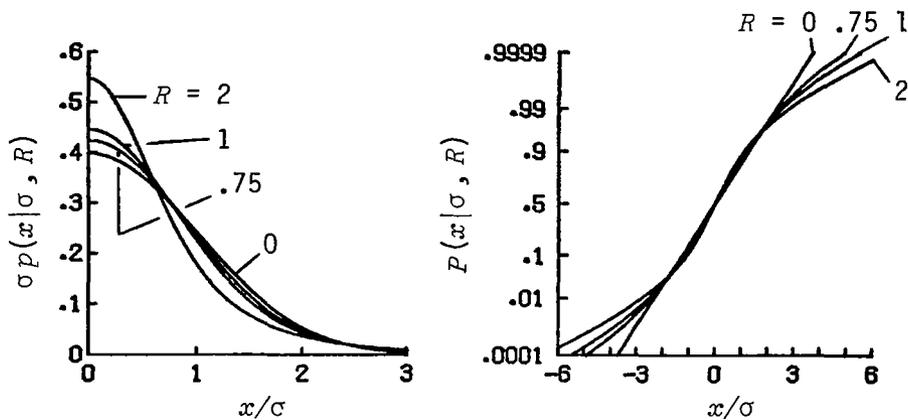


Figure 15.--Probability density and distribution functions of the non-gaussian turbulence model for various values of the parameter R .

In the next section of this report it will be of interest to compute the probability distributions which would be obtained by combining data from many samples of the non-gaussian model. It will be assumed that the parameter R is fixed, but σ will be allowed to vary randomly from sample to sample. If σ is distributed over the samples with the density function

p_σ , the probability density of the non-gaussian model based on all of the samples will be

$$p_g(x|R_u) = \int_0^\infty p_g(x|y, R_u) p_\sigma(y) dy, \quad (79)$$

and the cumulative gust velocity distribution of the model will be

$$P_g^c(x|R_u) = 1 - \int_{-\infty}^\infty p_g(y|R_u) dy. \quad (80)$$

Level Crossing Frequency

The level crossing frequency of the non-gaussian model will now be investigated. The basic relation used here will be the well known result of reference 36,

$$N_g(x_1|\sigma_g, \sigma_g^*) = \frac{1}{U} \int_0^\infty x_2 p_{g, \dot{g}}(x_1, x_2|\sigma_g, \sigma_g^*) dx_2. \quad (81)$$

N_g is the number of crossings with positive slope of the level x_1 per unit distance of flight. The function $p_{g, \dot{g}}(x_1, x_2)$ is the joint density of the turbulence time history and its first derivative.

Just as in the case of the gaussian model considered previously in this report, a problem arises in the application of this equation to the non-gaussian model because the first derivative of a stochastic process having one of the Dryden spectral densities does not exist. This difficulty can be overcome in a manner similar to that of the gaussian model by adding high frequency poles to the transfer functions of the non-gaussian model. These new poles can be looked upon as the effect of viscosity at very short wavelengths. It will be shown that, similar to the case of the gaussian model treated previously in this report, these poles have no

effect whatsoever on the probability distribution of the model, and act only as a scale factor of the level crossing frequency. Since vehicle responses virtually never involve such high frequencies, these poles need not be considered in response calculations.

The following discussion will at first be restricted to only the longitudinal gusts. The results will then be extended to the vertical and lateral components. The principal steps of the procedure are:

- 1) add high frequency poles to the transfer functions of the turbulence model
- 2) derive expressions for the standard deviation of the gust component and its first time derivative
- 3) examine the effect of these new poles on the spectral density of the model, showing that the spectra are still essentially the Dryden forms
- 4) use the results of step 2 to obtain an expression for the joint characteristic function of the gust component and its first derivative
- 5) inverse Fourier transform the joint characteristic function of step 4 to obtain the joint density of the gust time history and its first derivative
- 6) apply equation 81 to the joint density of step 5 to obtain the level crossing frequency of the non-gaussian model
- 7) review the results of steps 4 through 6 in order to determine the dependence of the level crossing frequency upon the poles added to the transfer functions in step 1 and the parameters σ_u , L_u ,

U , and R_u which appear in the result.

- 8) determine a set of universal curves from which the level crossing frequency for any set of parameters can be found without resorting to the complete process described above.

Level Crossing Frequency of the Longitudinal Component

Recall the three linear filters used to generate the longitudinal gusts (table 1, page 55). The spectrum produced by these filters is the Dryden u -gust form, equation 4. As was pointed out in the discussion of equation 13, this spectrum behaves like Ω^{-2} at high frequencies. Therefore the first derivative of the u -gust random process is not defined. In order to overcome this problem high frequency poles are added to the transfer functions as shown in equations 82, 83, and 84.

$$H_a(s) = \frac{4\sigma_u^2 k_d \frac{L_u}{U}}{\left(1 + \frac{2L_u s}{U}\right) \left(1 + \frac{\gamma_u s}{U}\right)} \quad (82)$$

$$H_b(s) = \frac{1}{\left(1 + \frac{2L_u s}{U}\right) \left(1 + \frac{\gamma_u s}{U}\right)} \quad (83)$$

$$H_c(s) = \frac{\sigma_u k_c \left(\frac{2L_u}{U}\right)^{1/2}}{\left(1 + \frac{L_u s}{U}\right) \left(1 + \frac{\gamma_u s}{U}\right)} \quad (84)$$

The constants k_c and k_d are scaling factors which will be chosen so as

to correct the effects of γ_u on the standard deviation of $c(t)$ and $d(t)$. The power spectral density of $d(t)$ can be found by substituting first equation 82 and then 83 into equation 47, multiplying the results to obtain the correlation function of $d(t)$, and finally Fourier transforming to obtain the power spectral density. The result is

$$\Phi_{dd}(\Omega) = \frac{(k_d \sigma_u)^2}{\left[1 - \left(\frac{\gamma_u}{2L_u}\right)^2\right]^2} \left\{ \frac{L_u}{\pi [1 + (L_u \Omega)^2]} - \frac{\left[\frac{2\gamma_u^2}{\pi(2L_u + \gamma_u)}\right]}{1 + \left(\frac{2\gamma_u L_u \Omega}{2L_u + \gamma_u}\right)^2} + \frac{\frac{\gamma_u^3}{8\pi L_u^2}}{1 + \left(\frac{\gamma_u \Omega}{2}\right)^2} \right\}. \quad (85)$$

The power spectral density of $c(t)$ is obtained by substituting equation 82 into equation 47 and Fourier transforming.

$$\Phi_{cc}(\Omega) = \frac{(k_c \sigma_u)^2 L_u}{\pi [1 + (L_u \Omega)^2] [1 + (\gamma_u \Omega)^2]} \quad (86)$$

The constants k_c and k_d are now chosen so that the standard deviations of both $c(t)$ and $d(t)$ are equal to σ_u .

$$k_c = \left[1 + \frac{\gamma_u}{L_u}\right]^{1/2} \quad (87)$$

$$k_d = 1 + \frac{\gamma_u}{2L_u} \quad (88)$$

$$\sigma_c = \sigma_d = \sigma_u \quad (89)$$

It can also be verified that, with these values of k_c and k_d , the standard deviations of $\hat{c}(t)$ and $\hat{d}(t)$ are equal to

$$\sigma_c^* = \sigma_d^* = U \sigma_u (\gamma_u L_u)^{-1/2}. \quad (90)$$

These results imply that the standard deviation of the non-gaussian model and its derivative are

$$\sigma_g = \sigma_u \quad (91)$$

$$\sigma_g^* = U \sigma_u (\gamma_u L_u)^{-1/2}. \quad (92)$$

Equations 89 through 92 complete the second step of the derivation.

The power spectral density of the model is

$$\Phi_{gg}(\Omega) = \Phi_{dd}(\Omega) R_u^2 (1 + R_u^2)^{-1} + \Phi_{cc}(\Omega) (1 + R_u^2)^{-1}. \quad (93)$$

Substitution of equations 85 and 86 into equation 93 gives an expression for Φ_{gg} which is far more complicated than the simple Dryden spectral density. However, on the assumption that γ_u is very much smaller than L_u , the power spectral density of the non-gaussian model becomes

$$\Phi_{gg}(\Omega) = \frac{\frac{\sigma_u^2 L_u}{\pi} \left(1 + \frac{\gamma_u}{L_u}\right)}{\left[1 + (L_u \Omega)^2\right] \left[1 + (\gamma_u \Omega)^2\right]} + O\left\{\left(\frac{\gamma_u}{L_u}\right)^2\right\}. \quad (94)$$

Equation 94 is, to second order in γ_u/L_u , the Dryden form of the u -gust power spectral density with an additional high frequency pole. This is precisely the result required in order to insure the existence of $\dot{g}(t)$, and equation 94 completes the third step of the level crossing derivation.

Note that the standard deviations of $c(t)$ and $d(t)$ have not been altered by the addition of γ_u . Thus the probability distributions discussed earlier in this section remain unchanged.

The level crossing frequency of the non-gaussian model will now be developed using the modified transfer functions, equations 82, 83, and 84. Consider the joint characteristic function of $g(t)$ and its first derivative. By definition this is

$$\phi_{g, \dot{g}}(f_1, f_2) = E\{\exp[i2\pi f_1 g(t) + i2\pi f_2 \dot{g}(t)]\} . \quad (95)$$

In terms of the processes $a(t)$, $b(t)$, and $c(t)$ used in the non-gaussian model, $g(t)$ and $\dot{g}(t)$ are

$$g(t) = a(t) b(t) R_u (1 + R_u^2)^{-1/2} + c(t) (1 + R_u^2)^{-1/2} \quad (96)$$

$$\dot{g}(t) = [\dot{a}(t) b(t) + a(t) \dot{b}(t)] R_u (1 + R_u^2)^{-1/2} + \dot{c}(t) (1 + R_u^2)^{-1/2} . \quad (97)$$

Substitution of these two expressions into equation 95 will give an expression for $\phi_{g, \dot{g}}$ in terms of a , \dot{a} , b , \dot{b} , c , and \dot{c} . If now the joint density of these random processes can be determined, the expected value appearing in equation 95 can be evaluated and the joint characteristic function found.

Note that the processes a , b , and c are zero mean, independent, and gaussian. Since the derivative of a gaussian process is always gaussian, it follows that \dot{a} , \dot{b} , and \dot{c} are zero mean, independent, and gaussian. Furthermore, since a random process and its first derivative are always uncorrelated, it follows from their gaussian nature that a , \dot{a} , b , \dot{b} , c , and \dot{c} are mutually independent. The joint density function of these processes can thus be written directly.

$$P_{a, \dot{a}, b, \dot{b}, c, \dot{c}}(x_1, x_2, x_3, x_4, x_5, x_6 | \sigma_a, \sigma_a^*, \sigma_b, \sigma_b^*, \sigma_c, \sigma_c^*) =$$

$$\frac{\exp\left\{-\frac{1}{2}\left[\left(\frac{x_1}{\sigma_a}\right)^2 + \left(\frac{x_2}{\sigma_a^*}\right)^2 + \left(\frac{x_3}{\sigma_b}\right)^2 + \left(\frac{x_4}{\sigma_b^*}\right)^2 + \left(\frac{x_5}{\sigma_c}\right)^2 + \left(\frac{x_6}{\sigma_c^*}\right)^2\right]\right\}}{(2\pi)^3 \sigma_a \sigma_a^* \sigma_b \sigma_b^* \sigma_c \sigma_c^*} \quad (98)$$

The standard deviations appearing in equation 98 can be found from the filter transfer functions, equations 82, 83, and 84.

$$\left. \begin{aligned} \sigma_a &= \frac{\sigma_u}{L_u} \left[\frac{U(2L_u + \gamma_u)}{2} \right]^{1/2} & \sigma_a^* &= \frac{U\sigma_u}{2L_u} \left[\frac{U(2L_u + \gamma_u)}{\gamma_u L_u} \right]^{1/2} \\ \sigma_b &= L_u \left[\frac{2}{U(2L_u + \gamma_u)} \right]^{1/2} & \sigma_b^* &= \left[\frac{UL_u}{\gamma_u(2L_u + \gamma_u)} \right]^{1/2} \\ \sigma_c &= \sigma_u & \sigma_c^* &= U\sigma_u (\gamma_u L_u)^{-1/2} \end{aligned} \right\} \quad (99)$$

Equations 96 through 99 can now be combined to evaluate the joint characteristic function of the non-gaussian model, equation 95.

$$\phi_{g, \dot{g}}(f_1, f_2 | \sigma_a, \sigma_a^*, \sigma_b, \sigma_b^*, \sigma_c, \sigma_c^*, R_u) =$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left\{i2\pi f_1 \left[\frac{R_u x_1 x_3 + x_5}{(1 + R_u^2)^{1/2}} \right] + i2\pi f_2 \left[\frac{R_u (x_1 x_4 + x_2 x_3) + x_6}{(1 + R_u^2)^{1/2}} \right] \right\} \cdot$$

$$P_{a, \dot{a}, b, \dot{b}, c, \dot{c}}(x_1, x_2, x_3, x_4, x_5, x_6 | \sigma_a, \sigma_a^*, \sigma_b, \sigma_b^*, \sigma_c, \sigma_c^*) dx_1 \dots dx_6$$

This completes the fourth step of the derivation.

(100)

Equation 100 can be integrated to give a closed form algebraic expression for the joint characteristic function of the non-gaussian u -gust component and its derivative. The joint density function required for the evaluation of the level crossing frequency can then be found, at least in principle, by means of the Fourier transformation

$$p_{g,\dot{g}}(x_1, x_2 | \sigma_a, \sigma_a^*, \sigma_b, \sigma_b^*, \sigma_c, \sigma_c^*, R_u) = \quad (101)$$

$$\iint_{-\infty}^{\infty} \phi_{g,\dot{g}}(f_1, f_2 | \sigma_a, \sigma_a^*, \sigma_b, \sigma_b^*, \sigma_c, \sigma_c^*, R_u) \exp(-i2\pi f_1 x_1 - 2\pi f_2 x_2) df_1 df_2.$$

Evaluation of this expression would complete the fifth step of the derivation. Finally, the level crossing frequency can be found by means of equation 79.

In reality, of course, the series of integrations required to pass from equation 100 to the evaluation of the level crossing frequency are extremely tedious if not overwhelmingly difficult. Fortunately, a far more convenient means of solution is available through use of a digital computer and the appropriate numerical methods. Program LEVXNG, listed in the appendix of this report, is one means of performing these calculations. This program was, in fact, used to compute the level crossing results to be presented in this section.

Program LEVXNG permits the rapid calculation of N_g for any set of parameters σ_u, L_u, U, R_u , and γ_u which might be of interest. However, merely calculating results for various cases does not yield much useful information concerning the dependence of the solution on its parameters. Furthermore, every time any of the parameters is changed, it becomes necessary to repeat the entire calculation in order to obtain the new

level crossing frequencies. Thus it would clearly be quite useful to have some idea as to how a change of parameters will affect the resulting level crossings of the model. With this in mind, a set of universal curves depending only upon the parameter R_u will now be derived. It will be shown that the level crossing frequency for any choice of the parameters σ_u, L_u, U , and γ_u can be found from these curves.

It is possible to integrate equation 100 with respect to x_5 and x_6 with a minimum of difficulty. The results of these integrations can be factored from the remaining integrals and identified as the joint characteristic function of e and \dot{e} . Equation 100 can then be rewritten in the form shown in equation 102. (For the sake of simplicity and clarity, the explicit parameter dependence notation will be temporarily suspended.)

$$\phi_{g, \dot{g}}(f_1, f_2) = \phi_{e, \dot{e}}(f_1, f_2) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (4) \int \frac{\exp(\bar{x}^T \tilde{A} \bar{x}) d\bar{x}}{(2\pi)^2 \sigma_a \sigma_a^* \sigma_b \sigma_b^*}$$

where

$$\phi_{g, \dot{g}}(f_1, f_2) = \exp \left[- \frac{2(\pi \sigma_e f_1)^2}{1 + R_u^2} - \frac{2(\pi \sigma_e^* f_2)^2}{1 + R_u^2} \right] \quad (102a)$$

$$\bar{x} = [x_1 \ x_2 \ x_3 \ x_4]^T$$

$$\tilde{A} = \left[\begin{array}{cccc} -(2\sigma_a^2)^{-1} & 0 & i2\pi f_1 R_u^* & i2\pi f_2 R_u^* \\ 0 & -(2\sigma_a^2)^{-1} & i2\pi f_2 R_u^* & 0 \\ i2\pi f_2 R_u^* & i2\pi f_2 R_u^* & -(2\sigma_b^2)^{-1} & 0 \\ i2\pi f_2 R_u^* & 0 & 0 & -(2\sigma_b^2)^{-1} \end{array} \right] \quad (102b)$$

$$R_u^* = R_u (1 + R_u^2)^{-1/2}$$

Now suppose that a special case of the joint characteristic function is computed. Let all of the parameters with the exception of R_u be set to unity.

$$\sigma_u = L_u = U = \gamma_u = 1.0 \quad (103)$$

Denote this case by $\hat{\phi}_{g,g}$. Note that $\hat{\phi}$ is a function only of f_1 , f_2 , and R_u .

Consider again equation 102 and introduce the following change of variables:

$$\left. \begin{array}{ll} x_1' = \frac{x_1}{\sigma_a} \left(\frac{3}{2} \right)^{1/2} & x_2' = \frac{x_2}{\sigma_a} \left(\frac{3}{4} \right)^{1/2} \\ x_3' = \frac{x_3}{\sigma_b} \left(\frac{2}{3} \right)^{1/2} & x_4' = \frac{x_4}{\sigma_b} \left(\frac{1}{3} \right)^{1/2} \end{array} \right\} \quad (104)$$

A comparison of this transformed expression with the special case $\hat{\phi}$, taking note of the equalities

$$\left. \begin{aligned} \sigma_a \sigma_b &= \sigma_c \\ \sigma_a^* \sigma_b^* 2^{1/2} &= \sigma_a^* \sigma_b^* 2^{1/2} = \sigma_c^* \end{aligned} \right\} (105)$$

of equation 99, leads to the conclusion

$$\phi_{g, \dot{g}}(f_1, f_2) = \hat{\phi}_{g, \dot{g}}(f_1 \sigma_c, f_2 \sigma_c^*) . \quad (106)$$

The joint density function of g and \dot{g} is obtained by means of equation 101. Substitution of equation 106 for $\phi_{g, \dot{g}}$ in equation 101 and introduction of the variables

$$\left. \begin{aligned} f_1' &= f_1 \sigma_c \\ f_2' &= f_2 \sigma_c^* \end{aligned} \right\} (107)$$

gives

$$p_{g, \dot{g}}(x_1, x_2) = \frac{1}{\sigma_c \sigma_c^*} \hat{p}_{g, \dot{g}}\left(\frac{x_1}{\sigma_c}, \frac{x_2}{\sigma_c^*}\right) . \quad (108)$$

The notation $\hat{p}_{g, \dot{g}}$ is used to signify the joint density corresponding to the joint characteristic function $\hat{\phi}_{g, \dot{g}}$.

The level crossing frequency of the non-gaussian model can now be found by substituting equation 108 into equation 81 and performing the change of variable $x_2' = x_2 / \sigma_c^*$. The result is

$$N_g(x) = \frac{\sigma_c^*}{U \sigma_c} \hat{N}_g\left(\frac{x}{\sigma_c}\right) \quad (109)$$

where \hat{N}_g is the level crossing frequency of the special case.

The explicit parameter notation can now be reinstated. Substitute for σ_c and σ_c from equation 99 and recall that the special case \hat{N}_g depends only on the parameter R_u . The final expression for N_g is

$$N_g(x|\sigma_u, L_u, R_u, \gamma_u) = (L_u \gamma_u)^{-1/2} \hat{N}_g\left(\frac{x}{\sigma_u} | R_u\right). \quad (110)$$

Note the similarity of equation 110 to the level crossing frequency of the gaussian model, equation 16. Just as in equation 16, the crossing frequency is inversely proportional to the square root of γ_u . Thus γ_u can be interpreted as a control on the zero crossing frequency of the non-gaussian model, and could be chosen so as to match equation 110 to some observed data. For the purposes of the present report, however, the value of γ_u can be allowed to remain arbitrary.

Equation 110 implies that once the level crossing frequency of the special case has been computed for a given value of R_u , the result for any choice of σ_u , L_u , and γ_u can be found directly. Alternatively, the equation states that all u -gust level crossing frequencies of the non-gaussian model can be reduced to a single function through the indicated scaling. In order to show that these results apply only to the u -gust component of the model, the subscript g will now be replaced by u .

The function $\hat{N}_u(x|R_u)$ has been computed using program LEVXNG which is described in Appendix A of this report. Figure 16 and table B3 (of Appendix B) present the results for several values of the parameter R_u .

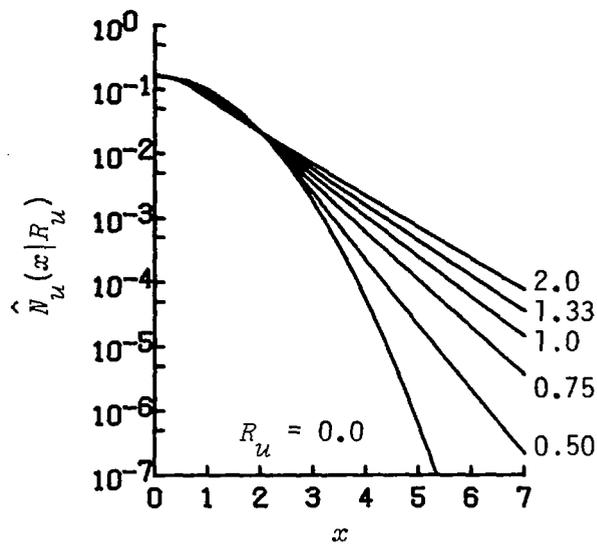


Figure 16.--Level crossing frequencies of the non-gaussian turbulence model longitudinal component for various values of the parameter R_u .

In the next section it will be of interest to evaluate the cumulative probability of exceedance which would result from combining data from a number of samples of the non-gaussian model. As in the case of the gaussian model described previously, it will be assumed that the parameters σ_u , L_u , and γ_u vary randomly from sample to sample. The standard deviation, σ_u , will be assumed independent of L_u and γ_u ; and the parameter R_u will be required to remain constant over all the samples.

Recall from the discussion of the gaussian model that the cumulative probability of exceedance is calculated by first finding the average level crossing frequency for all of the samples, then normalizing with respect to the average zero crossing frequency. In view of the above assumptions, the cumulative probability of exceedance of the u -gust component is

$$\frac{N_u(x|R_u)}{N_u(0|R_u)} = \frac{\int_0^\infty \int \int N_u(x|\sigma, L, R_u, \gamma) p_{\sigma_u}(\sigma) p_{L_u, \gamma_u}(L, \gamma) d\sigma dL d\gamma}{\int_0^\infty \int \int N_u(0|\sigma, L, R_u, \gamma) p_{\sigma_u}(\sigma) p_{L_u, \gamma_u}(L, \gamma) d\sigma dL d\gamma}, \quad (111)$$

where p_{σ_u} and p_{L_u, γ_u} are the probability density functions of σ_u, L_u , and γ_u . Equation 111 can be greatly simplified by substituting equation 110 for N_u . The result is

$$\frac{N_u(x|R_u)}{N_u(0|R_u)} = \hat{N}_u(0|R_u)^{-1} \int_0^\infty \hat{N}_u\left(\frac{x}{\sigma} | R_u\right) p_{\sigma_u}(\sigma) d\sigma. \quad (112)$$

Note that it is not necessary to know the distribution of either L_u or γ_u in order to evaluate equation 112, a result analogous to that for the gaussian model (Eq. 21).

This completes the analysis of the longitudinal gust level crossings. The methods applied above will now be extended to the vertical and lateral gust components.

Level Crossing Frequency of Lateral and Vertical Components

The level crossings of the lateral gust component will now be considered. Since the vertical and lateral components of the non-gaussian model use linear filters of the same form, all results for the lateral gusts will apply directly to the vertical gusts.

Recall from table 1 the transfer functions to be used in generating the lateral gust component. The power spectral density produced by these filters is the Dryden v -gust form, equation 5. Just as in the u -gust case considered above, the derivative of a process with the Dryden spectrum is not defined. In order to avoid this problem, the spectrum is modified

through the addition of high frequency poles to the transfer functions.

$$H_a(s) = \frac{\sigma_v (128)^{1/2} \left[\frac{L_v}{U} \right]^2 k_d}{\left(1 + \frac{2L_v s}{U} \right) \left(1 + \frac{\gamma_v s}{U} \right)} \quad (113)$$

$$H_b(s) = \frac{s k_d^2}{\left(1 + \frac{L_v s}{U} \right)^2 \left(1 + \frac{\gamma_v s}{U} \right)} \quad (114)$$

$$H_c(s) = \frac{\sigma_v \left[\frac{L_v}{U} \right]^{1/2} k_c \left(1 + 3^{1/2} \frac{L_v s}{U} \right)}{\left(1 + \frac{L_v s}{U} \right)^2 \left(1 + \frac{\nu_v s}{U} \right)} \quad (115)$$

Unlike the u -gust case, it is convenient to use two parameters, γ_v and ν_v , rather than only one. The constants k_c and k_d are scaling factors which will be chosen so as to correct the effects of γ_v and ν_v on the standard deviations of $e(t)$ and $d(t)$.

The power spectral densities of e and d can be derived using the above transfer functions. The condition

$$\sigma_e^2 = \sigma_d^2 = \sigma_v^2 \quad (116)$$

is satisfied by choosing

$$k_c = \frac{1 + \frac{\nu_v}{L_v}}{\left(1 + \frac{\nu_v}{2L_v} \right)^{1/2}} \quad (117)$$

and

$$k_d = \left(1 + \frac{\gamma_v}{2L_v}\right)^{1/2}. \quad (118)$$

The standard deviations of c and d can be made equal by choosing v_v to be a function of γ_v .

$$v_v = \frac{3\gamma_v L_v - 6L_v^2 + 6L_v^2 \left(1 + \frac{\gamma_v}{L_v} + \frac{7\gamma_v^2}{12L_v}\right)^{1/2}}{6L_v + \gamma_v} \quad (119)$$

The resulting standard deviations are

$$\sigma_c^2 = \sigma_d^2 = \sigma_v^2 \left(\frac{U}{2L_v}\right)^2 \left(\frac{6L_v + \gamma_v}{\gamma_v}\right). \quad (120)$$

With the above choices of parameters, and assuming the condition $\gamma_v \ll L_v$, the power spectral density of the lateral gust component becomes

$$\Phi_{vv}(\Omega) = \frac{\sigma_v^2 L_v \left(1 + \frac{3\gamma_v}{2L_v}\right) [1 + 3(L_v \Omega)^2]}{\pi [1 + (L_v \Omega)^2]^2 [1 + (\gamma_v \Omega)^2]} + O\left\{\left\{\frac{\gamma_v}{L_v}\right\}^2\right\}. \quad (121)$$

Now consider the level crossing frequency of the lateral component. Just as in the treatment of the longitudinal gust, an expression for the joint characteristic function of v and \dot{v} will be derived. This expression will be formally identical to equation 102, but in the present case the standard deviations are

$$\left. \begin{aligned}
 \sigma_a &= 2\sigma_v \left(\frac{2L_v}{U} \right)^{3/2} & \sigma_a^* &= 4L_v \sigma_v (U\gamma_v)^{-1/2} \\
 \sigma_b &= \frac{1}{2} \left(\frac{U}{2L_v} \right)^{3/2} & \sigma_b^* &= \left(\frac{U}{2L_v} \right)^{5/2} \left(\frac{L_v}{\gamma_v} \right)^{1/2} \left(1 + \frac{\gamma_v}{4L_v} \right)^{1/2} \\
 \sigma_c &= \sigma_v & \sigma_c^* &= \sigma_v \left(\frac{U}{2L_v} \right) \left(\frac{6L_v}{\gamma_v} \right)^{1/2} \left(1 + \frac{\gamma_v}{6L_v} \right)^{1/2} .
 \end{aligned} \right\} (122)$$

Using equation 122 and computer program LEVXNG listed in the appendix, it is possible to calculate exact results for the level crossing frequency. However, a more convenient solution of the problem can be found by introducing an approximation for σ_b^* and σ_c^* of equation 122 which will allow the derivation of universal curves similar to those for the longitudinal case (Fig. 16). On the assumption that L_v is much larger than γ_v , the expressions for σ_b^* and σ_c^* of equation 122 become

$$\left. \begin{aligned}
 \sigma_b^* &\doteq \left(\frac{U}{2L_v} \right)^{5/2} \left(\frac{L_v}{\gamma_v} \right)^{1/2} \\
 \sigma_c^* &\doteq \sigma_v \left(\frac{U}{2L_v} \right) \left(\frac{6L_v}{\gamma_v} \right)^{1/2} .
 \end{aligned} \right\} (123)$$

Employing these approximate results and following the procedure described above for the longitudinal gust component, derive the joint characteristic function of the special case

$$\sigma_v = L_v = U = \gamma_v = 1.0 , \quad (124)$$

and denote this result by $\hat{\phi}_{v,v}$. Return to equation 102 and introduce the change of variables

$$\begin{aligned}
 x_1' &= \frac{x_1}{\sigma_a} 2^{5/2} & x_2' &= \frac{x_2}{\sigma_a} 4 \\
 x_3' &= \frac{x_3}{\sigma_b} 2^{-5/2} & x_4' &= \frac{x_4}{\sigma_b} 2^{-5/2}.
 \end{aligned}
 \tag{125}$$

Comparison of this result with $\hat{\phi}_{v,v}$ gives

$$\phi_{v,v}(f_1, f_2) = \hat{\phi}_{v,v}(f_1 \sigma_c, f_2 \sigma_c). \tag{126}$$

Continuation of the same procedure used for the u -gust case leads to the final result

$$N_v(x|\sigma_v, L_v, R_v, \gamma_v) = \left[\frac{3}{2L_v \gamma_v} \right]^{1/2} \hat{N}_v \left[\frac{x}{\sigma_v} | R_v \right], \tag{127}$$

which is valid for $\gamma_v \ll L_v$. The level crossing frequency of the vertical gust component can be obtained by replacing the subscript v of equations 113 through 127 with the subcript w .

The special cases \hat{N}_v and \hat{N}_w have been computed by means of program LEVXNG. Figure 17 and table B4 (of Appendix B) present these results for several values of the parameter R .

A comparison of figures 16 and 17 reveals that the special cases appear to differ by only a constant of proportionality which depends weakly upon the parameter R . This suggests that a transformation exists such that the level crossing frequencies of all three components of the non-gaussian model can be expressed in terms of a single set of universal curves. This transformation is not derived here, but the possibility of its existence is mentioned as a point of interest.

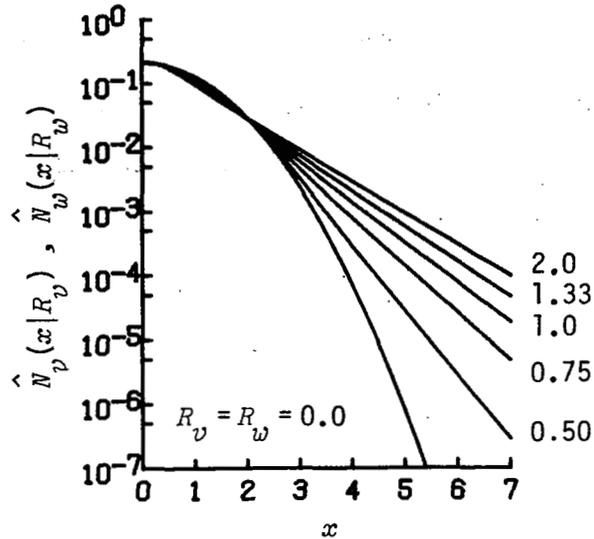


Figure 17.--Level crossing frequencies of the non-gaussian turbulence model lateral and vertical components for various values of the parameter R .

In the next section of this report it will be necessary to calculate cumulative probability of exceedance curves for the vertical and lateral gust components. Following the procedure described previously for the longitudinal components gives

$$\frac{N_v(x|R_v)}{N_v(0|R_v)} = \hat{N}_v(0|R_v)^{-1} \int_0^{\infty} \hat{N}_v\left(\frac{x}{\sigma} | R_v\right) p_{\sigma_v}(\sigma) d\sigma . \quad (128)$$

Just as in the u -gust case, the distribution of L_v and γ_v need not be known in order to evaluate this expression. The result for the vertical gusts can be obtained by replacing the subscript v of this equation with the subscript w . Equation 128 completes the analysis of the level crossing frequencies of the non-gaussian model.

Increment Distribution

The probability distribution of the non-gaussian model's velocity increments will now be derived. The following remarks apply to all three components of the model; however, only the u -gust component will be explicitly treated.

Consider the increment of the longitudinal gust component.

$$\Delta_u(t|\tau) = u(t) - u(t-\tau) \quad (129)$$

The parameter τ is a constant. According to figure 13, equation 129 can be rewritten as

$$\Delta_u(t|\tau) = \Delta_{ab}(t|\tau) + \Delta_c(t|\tau) \quad (130)$$

where

$$\Delta_{ab}(t|\tau) = [a(t)b(t) - a(t-\tau)b(t-\tau)] R_u (1 + R_u^2)^{-1/2} \quad (131)$$

$$\Delta_c(t|\tau) = [c(t) - c(t-\tau)] (1 + R_u^2)^{-1/2} . \quad (132)$$

The procedure to be followed in finding the probability distribution of Δ_u is

- 1) derive an expression for the characteristic function of Δ_c
- 2) derive an expression for the characteristic function of Δ_{ab}
- 3) multiply the results of steps 1 and 2 in order to obtain the characteristic function of Δ_u .
- 4) Fourier transform the result of step 3 to find the probability density of Δ_u .

Since Δ_c is a linear transformation of a gaussian process, it follows that it must also be gaussian. Its mean value is zero and its standard deviation is given by

$$\sigma_{\Delta_c}^2 = 2[\sigma_c^2 - C_{cc}(\tau)](1 + R_u^2)^{-1}, \quad (133)$$

where C_{cc} is the correlation function of $c(t)$, equation 55. The characteristic function of Δ_c is thus

$$\phi_{\Delta_c}(f | \sigma_u, L_u, U, R_u, \tau) = \quad (134)$$

$$\exp\{- (2\pi\sigma_u f)^2 (1 + R_u^2)^{-1} [1 - \exp(\frac{U}{L_u} |\tau|)]\}.$$

The characteristic function of Δ_{ab} is more difficult to evaluate. Since a and b are gaussian processes with zero means and correlation functions given by equations 51, 52, and 54, $\phi_{\Delta_{ab}}$ can be written in the form

$$\phi_{\Delta_{ab}}(f | \sigma_u, L_u, U, R_u, \tau) = \int_{-\infty}^{\infty} (4) \exp\left[\frac{i(\pi f R_u x_1 x_2 - x_2 x_4)}{(1 + R_u^2)^{1/2}} \right].$$

$$P_{a(t), a(t-\tau), b(t), b(t-\tau)}(x_1, x_2, x_3, x_4 | \sigma_u, L_u, U, \tau) dx_1 dx_2 dx_3 dx_4$$

where

$$P_{a(t), a(t-\tau), b(t), b(t-\tau)}(\bar{x} | \sigma_u, L_u, U, \tau) = \frac{\exp(-\frac{1}{2} \bar{x}^T \tilde{A}^{-1} \bar{x})}{(2\pi)^2 [\det(\tilde{A})]}$$

(135a)

$$\bar{x} = [x_1 \ x_2 \ x_3 \ x_4]^T$$

$$\tilde{A} = \begin{bmatrix} c_{aa}(0) & c_{aa}(\tau) & 0 & 0 \\ c_{aa}(\tau) & c_{aa}(0) & 0 & 0 \\ 0 & 0 & c_{bb}(0) & c_{bb}(\tau) \\ 0 & 0 & c_{bb}(\tau) & c_{bb}(0) \end{bmatrix} \quad (135b)$$

$$c_{aa}(\tau) = \sigma_u^2 8 \frac{L_u}{U} \exp\left(-\frac{U}{2L_u} |\tau|\right)$$

$$c_{bb}(\tau) = \frac{U}{8L_u} \exp\left(-\frac{U}{2L_u} |\tau|\right).$$

The characteristic function of Δ_u is given by

$$\phi_{\Delta_u}(f|\sigma_u, L_u, U, R_u, \tau) = \phi_{\Delta_{ab}}(f|\sigma_u, L_u, U, R_u, \tau) \phi_{\Delta_c}(f|\sigma_u, L_u, U, R_u, \tau) \quad (136)$$

The probability density of Δ_u can be found by Fourier transforming equation 136. A closed form solution of this problem has not been derived, however program INCPD listed in the appendix of this report can perform all required computations for a numerical solution of the problem.

Figure 18 presents density and distribution functions of the u -gust and its increment for a typical case. Comparison of the density functions of figure 18 with those of figure 9 on page 41 leads to the conclusion that the non-gaussian model does not properly model the velocity increment

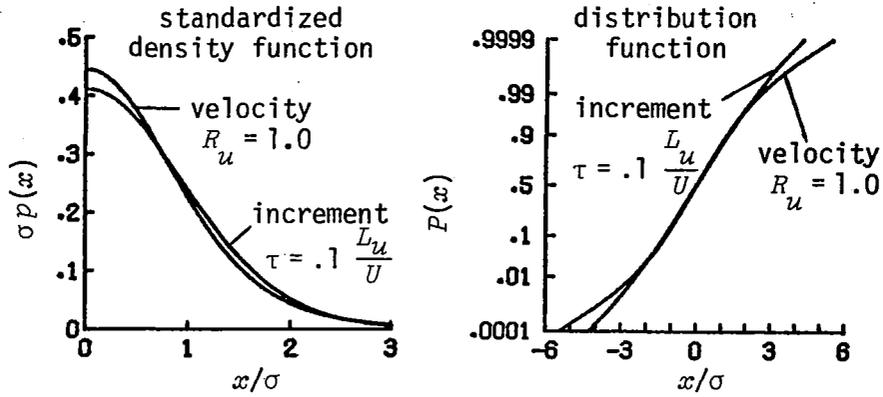


Figure 18.--Comparison of a typical non-gaussian model u -gust component probability distribution with that of the corresponding increment distribution.

distribution of atmospheric turbulence. Additional data in the form of probability distribution functions presented in references 22 and 25 confirm this conclusion. The increments of actual turbulence are sharply more non-gaussian than the velocity, while the increments of the model are less non-gaussian than the velocity. As a result, the non-gaussian model underestimates the occurrences of large increments.

The reason for this can be seen by considering the gaussian and non-gaussian portions of both the velocity and velocity increment. The velocity of the model is the sum of two independent parts; the product $a(t)b(t)$ which has a K_0 distribution (Fig. 12), and $c(t)$ which is gaussian. The relative intensity of these two parts can be shown to be [since $a(t)b(t)$ is equal to $d(t)$]

$$\frac{\sigma_{ab}}{\sigma_c} = R \left(\frac{C_{dd}(0)}{C_{cc}(0)} \right)^{1/2} = R . \quad (137)$$

The probability density of the gust velocity can be found by convolving the K_0 and gaussian densities to obtain a result which is, in general, non-gaussian. Similarly, the velocity increment of the model is the sum

of two independent parts; the increment Δ_e which is gaussian, and the increment Δ_{ab} which is non-gaussian. Just as above, the probability density functions of these two parts, and their relative intensities can be shown to be

$$\frac{\sigma_{\Delta_{ab}}}{\sigma_{\Delta_e}} = R \frac{[C_{dd}(0) - C_{dd}(\tau)]}{[C_{ee}(0) - C_{ee}(\tau)]} = R . \quad (138)$$

However, the probability distribution of Δ_{ab} is not sharply non-gaussian like the K_0 distribution. It is instead much smoother, tending to look like the exponential function, $\exp(-|x|)$. Because of this smoothness and because Δ_{ab} has the same intensity relative to Δ_e as has the product $a(t)b(t)$ relative to $c(t)$, it follows that when the density of Δ_{ab} is convolved with the gaussian density function, the result will be more nearly gaussian than the convolution of K_0 with the gaussian density. Thus, as shown in figure 18, the distribution of Δ_u is more nearly gaussian than the distribution of u itself.

There are two ways in which the increment distribution can be made more strongly non-gaussian than the velocity distribution; increase the intensity of Δ_{ab} relative to Δ_e while holding the ratio σ_{ab}/σ_e constant, or make the distribution of Δ_{ab} more non-gaussian than the K_0 distribution. The second of these does not appear to be possible. The first, however, could be achieved by modifying C_{ee} and C_{dd} of equation 138 so that the ratio $\sigma_{\Delta_{ab}}/\sigma_{\Delta_e}$ is greater than R . This would require changing the transfer functions of the model, and would invalidate many of the results obtained so far. For this reason, the possibility of modifying the model so as to more accurately model the increment distribution of atmospheric turbulence will not be discussed in this report. Instead, this subject

will be suggested as a topic for further research.

Because the model in its present form does not properly reproduce the increment distributions of actual turbulence, its use should be restricted to those applications which require only the accurate modeling of the gust velocity distribution and level crossing frequency. These applications will typically involve responses of the rigid body modes of the aircraft which are not as excited by large increments with short time lags as are the structural modes.

Summary

This completes the statistical analysis of the proposed non-gaussian turbulence model. Before proceeding to the next section, a brief summary of this section is in order.

- 1) Equations 38 through 63 showed that the linear filters of the model could be chosen so as to produce the Dryden spectral densities. The results are contained in table 1 which summarizes the transfer functions for each of the three gust components.
- 2) Equations 64 through 80 were concerned with the probability distribution of the non-gaussian model. Figure 15 presents density and distribution functions attainable with the model.
- 3) Equations 81 through 128 derived the level crossing frequency of the model. The results of this derivation are presented in figures 16 and 17, which show sets of universal curves from which the level crossing frequency of the model can be derived for any set of parameters.

- 4) The last part of this section, equations 129 through 138 has discussed the increment distribution of the non-gaussian model. The results, presented in figure 18 for a typical case of the longitudinal gust component, show that the non-gaussian model in its present form does not properly model the increment distribution of atmospheric turbulence. The reason for this behavior has been discussed and a possible solution to this problem has been suggested as a topic for further research.

VALIDITY OF THE NON-GAUSSIAN MODEL

The results presented in the previous section have indicated that the probability distribution and level crossing frequency of the non-gaussian model exhibit characteristics very similar to those of single samples of atmospheric turbulence. In this section it will be shown that the model also fits the cumulative probability distributions and cumulative probability of exceedance distributions measured in atmospheric turbulence. In addition, it will be shown that the non-gaussian model fits these data better than the gaussian model.

Experimental Data

The experimental data used here will be a portion of those obtained during the LO-LOCAT project sponsored by the United States Air Force (Ref. 11). Although these data are exclusively from low altitude flight, they were selected for use in this report for two reasons.

- 1) The large body of data collected during the LO-LOCAT program can be divided into categories on the basis of flight altitude, atmospheric stability, and terrain roughness while retaining a large number of samples in each category. This will permit comparison of the models with turbulence data obtained under quite restricted conditions without the necessity of using small data samples.
- 2) These data are readily available to anyone wishing to either verify the results presented in this report or extend them to a wider variety of cases.

The disadvantage associated with using only this low altitude data is that any conclusions which might be drawn from them cannot be

generalized to higher altitudes. Further work will be required in order to show the validity of the model at high altitudes.

All of the results presented in this section are based on vertical gusts. Computations for longitudinal and lateral gust components have indicated that the results presented here are representative of all three components.

The data categories used in this section are described in table 2. These particular categories were selected because they represent a wide range of terrain, altitude, and atmospheric stability conditions.

Goodness-of-Fit-Criteria

The object of this section is to compare both the gaussian and non-gaussian models with experimentally measured probability distributions and level crossing frequencies. Before any comparison can be made, however, it will be necessary to define some goodness-of-fit criterion. That is, some objective test which can be used as an indication of how well the theoretical curves of the turbulence models fit the measured data.

A problem arises in the application of standard statistical methods such as the chi-squared or Kolmogorov-Smirnov tests because they require that the experimental data be based on independent samples. Unfortunately, the data of reference 11 which are to be used here are not based entirely on independent samples. Hence these tests are not applicable.

There appears to be no easy solution to this problem. However, for the purposes of this report, it will be possible to avoid this difficulty by assuming that the data can be treated as essentially exact. Any possible differences between the true statistics and the measured

Table 2.--Description of data categories used in this report.

| Category | Description |
|----------|--|
| 111000 | Vertical gust data based on 5,700 kilometers (3,536 statute miles) of flight at an altitude of 76 meters (250 feet) above the surface. All data collected during 109 flights over high mountains in very stable atmospheric conditions. |
| 112000 | Vertical gust data based on 7,350 kilometers (4,577 statute miles) of flight at an altitude of 76 meters (250 feet) above the surface. All data collected during 140 flights over high mountains in stable atmospheric conditions. |
| 113000 | Vertical gust data based on 6,800 kilometers (4,226 statute miles) of flight at an altitude of 76 meters (250 feet) above the surface. All data collected during 129 flights over high mountains in neutral atmospheric conditions. |
| 121000 | Vertical gust data based on 5,800 kilometers (3,620 statute miles) of flight at an altitude of 228 meters (750 feet) above the surface. All data collected during 112 flights over high mountains in very stable atmospheric conditions. |
| 122000 | Vertical gust data based on 7,800 kilometers (4,840 statute miles) of flight at an altitude of 228 meters (750 feet) above the surface. All data collected during 147 flights over high mountains in stable atmospheric conditions. |
| 123000 | Vertical gust data based on 5,000 kilometers (3,122 statute miles) of flight at an altitude of 228 meters (750 feet) above the surface. All data collected during 95 flights over high mountains in neutral atmospheric conditions. |
| 413000 | Vertical gust data based on 2,900 kilometers (1,811 statute miles) of flight at an altitude of 76 meters (250 feet) above the surface. All data collected during 55 flights over plains in neutral atmospheric conditions. |
| 414000 | Vertical gust data based on 2,300 kilometers (1,446 statute miles) of flight at an altitude of 76 meters (250 feet) above the surface. All data collected during 44 flights over plains in unstable atmospheric conditions. |

statistics will be ignored. This is a common approach to the analysis of turbulence data (e.g., Ref. 23). This assumption is acceptable in the present work for two reasons.

- 1) The purpose of this section is only to indicate to the reader that the non-gaussian model produces a better fit of the experimental data than the gaussian model. The results to be presented will show that in every case the gaussian model underestimates the occurrences of the high velocity gusts. Even though this error may or may not be judged statistically significant by one of the standard tests, it is clearly significant for the purposes of aircraft design if it occurs in every case tested. Thus it is contended that a rigorous statistical test of significance is not required for the purposes of this report.
- 2) As indicated in reference 11, the LO-LOCAT data used here (which were selected for the reasons presented previously in this section) contain some nonstationary effects. Run tests of both the mean and mean square indicated that approximately 30% of the LO-LOCAT turbulence samples could not be accepted as stationary at the 0.02 level of significance. Unfortunately, the models used in this report assume turbulence to be a stationary process. Thus the presence of nonstationary effects in the experimental data can be expected to have some effect on the results to be presented here, and the magnitude of this effect is unknown.

For these reasons, a more careful analysis of the data is not warranted at this time.

Since the experimental data are assumed to represent the true statistics of atmospheric turbulence, the problem reduces to one of simple curve fitting. Three goodness-of-fit criteria have been investigated in the research reported here. The first of these is the integral of the squared error,

$$\epsilon_{ise}(R) = \int_0^{x_{max}} |F_{data}(x) - F_{model}(x|R)|^2 dx \quad (139)$$

where x_{max} is the highest gust velocity measured. The functions F_{data} and F_{model} denote either the cumulative probability of exceedance or the cumulative probability distribution of the data and the model respectively. Note that since the model distribution depends upon the parameter R , the error criterion also depends upon R . The best fit of the experimental data is chosen to be the turbulence model with the value of R which minimizes ϵ_{ise} . Note that if setting R to zero minimizes ϵ_{ise} then the gaussian model is the best fit.

The second error criterion investigated was the maximum difference between the experimental data and the model.

$$\epsilon_{max}(R) = \max_{0 \leq x \leq x_{max}} |F_{data}(x) - F_{model}(x|R)| \quad (140)$$

The quantities used in this equation are the same as those in equation 139. Again, the best fit of the data is chosen to be the model with that value of R which minimizes ϵ_{max} .

Application of both ϵ_{ise} and ϵ_{max} to a number of data samples revealed that both criteria produced essentially identical results. For this reason, and because it is a more difficult test to apply, ϵ_{ise} will not be used in the numerical calculations of this report.

The reader should note that both ϵ_{ise} and ϵ_{max} apply primarily at low gust velocities, where F is large, rather than at high gust velocities, where F is very small. Thus both criteria tend to ignore errors in modeling the occurrences of very high gust velocities and are therefore not ideally suited for determining the best fit of the data for the purposes of aircraft design.

A third criterion, which is more sensitive to the occurrences of high gust velocities, has been studied. This is the maximum absolute difference of logarithms.

$$\epsilon_{\log(R)} = \max_{0 \leq x \leq x_{max}} \left| \log_{10}[F_{data}(x)] - \log_{10}[F_{model}(x|R)] \right|. \quad (141)$$

This criterion seems to be quite sensitive to small errors at the high gust velocities and may therefore ignore more serious errors at lower gust velocities. It does however lead to a very good fit of the LO-LOCAT data, especially when plotted on logarithmic scales as is the usual practice.

The above discussion makes it apparent that the two error criteria, ϵ_{max} and ϵ_{\log} , emphasize different aspects of the curve fitting problem. For this reason, both criteria will be used in the following tests.

Generation of Model Data

The cumulative probability of exceedance and cumulative probability distribution of the non-gaussian model vertical component were generated according to equations 80 and 128 derived in the previous section of this report. The assumptions involved in these equations will be found in that section. The probability density of σ_{ω} for each case was obtained from distribution functions presented in reference 11.

It was also assumed that the parameter R of the model could be considered constant for each data category. Since these categories are defined by altitude, atmospheric stability, and terrain characteristics, this assumption is equivalent to requiring R to be in some sense determined by these factors. As R is merely a parameter which arises from the manner in which the non-gaussian model is physically interpreted (Fig. 13), there is no difficulty in assigning to it this functional dependence. The explicit form of the dependence is, of course, unknown.

Results were computed numerically for R values of 0.0, 0.5, 0.75, 1.0, 1.33, and 2.0. The first of these ($R = 0.0$) corresponds to the gaussian turbulence model. The five other cases correspond to the non-gaussian model with varying degrees of non-gaussian behavior.

Results

Table 3 presents the computed error criteria ϵ_{max} and ϵ_{log} for the various data categories and values of R . Error values for the cumulative probability of exceedance and cumulative probability distribution have been summed because the same value of R does not necessarily minimize a given error criterion for both functions simultaneously. In every case the non-gaussian model provides a better fit of the experimental data

Table 3.--Goodness-of-fit criteria for various LO-LOCAT data categories.

| Category | Test | Turbulence Model Parameter | | | | | |
|----------|------------------|----------------------------|-----------|------------|-----------|------------|-----------|
| | | $R = 0.0$ Gaussian | $R = 0.5$ | $R = 0.76$ | $R = 1.0$ | $R = 1.33$ | $R = 2.0$ |
| 111000 | ϵ_{max} | 2.5E-1 | 2.3E-1 | 2.0E-1 | 1.6E-1 | 1.1E-1 | 4.8E-2 |
| | ϵ_{log} | 8.9E 0 | 4.5E 0 | 2.7E 0 | 1.6E 0 | 9.3E-1 | 3.9E-1 |
| 112000 | ϵ_{max} | 1.7E-1 | 1.6E-1 | 1.3E-1 | 1.0E-1 | 6.0E-2 | 1.0E-1 |
| | ϵ_{log} | 5.1E 0 | 2.5E 0 | 1.2E 0 | 4.3E-1 | 2.8E-1 | 5.8E-1 |
| 113000 | ϵ_{max} | 1.2E-1 | 1.0E-1 | 6.9E-2 | 2.5E-2 | 4.4E-2 | 1.4E-1 |
| | ϵ_{log} | 3.0E 0 | 1.4E 0 | 3.1E-1 | 5.0E-1 | 9.8E-1 | 1.4E 0 |
| 121000 | ϵ_{max} | 2.4E-1 | 2.2E-1 | 1.9E-1 | 1.5E-1 | 9.3E-2 | 3.1E-2 |
| | ϵ_{log} | 8.0E 0 | 4.5E 0 | 2.8E 0 | 2.0E 0 | 1.3E 0 | 8.2E-1 |
| 122000 | ϵ_{max} | 1.7E-1 | 1.5E-1 | 1.2E-1 | 8.0E-2 | 4.0E-2 | 8.4E-2 |
| | ϵ_{log} | 3.8E 0 | 2.3E 0 | 1.2E 0 | 6.2E-1 | 1.8E-1 | 4.0E-1 |
| 123000 | ϵ_{max} | 8.2E-2 | 6.5E-2 | 3.6E-2 | 4.1E-2 | 8.1E-2 | 1.7E-1 |
| | ϵ_{log} | 4.0E 0 | 2.1E 0 | 8.2E-1 | 2.4E-1 | 4.9E-1 | 8.9E-1 |
| 413000 | ϵ_{max} | 1.1E-1 | 9.2E-2 | 6.0E-2 | 6.6E-2 | 1.1E-1 | 1.9E-1 |
| | ϵ_{log} | 4.0E 0 | 1.5E 0 | 1.8E-1 | 7.3E-1 | 1.3E 0 | 1.8E 0 |
| 414000 | ϵ_{max} | 4.7E-2 | 3.2E-2 | 2.3E-2 | 6.1E-2 | 1.2E-1 | 2.2E-1 |
| | ϵ_{log} | 5.5E 0 | 2.9E 0 | 1.5E 0 | 7.2E-1 | 5.9E-1 | 8.4E-1 |

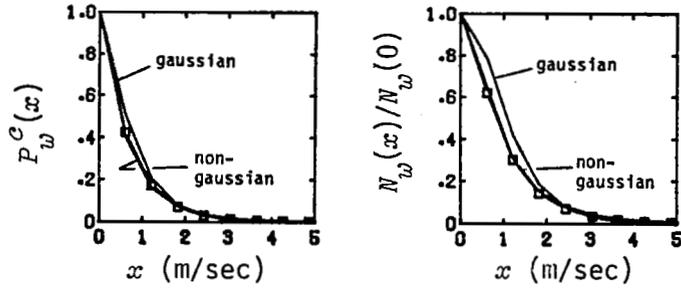
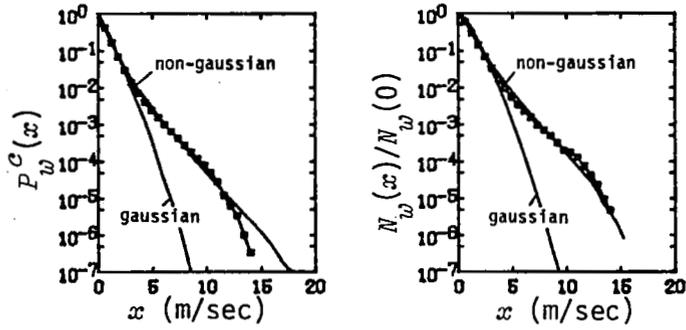
than does the gaussian model, although the best value of R for a given data category depends upon the error criterion used. This is especially true for those data collected over plains. Note, however, (Table 3) that in every case it is possible to choose an R value which simultaneously reduces both criteria below their values for the gaussian model.

Figures 19 and 20 presented on the next several pages compare the experimental data with both the indicated best fit models of table 3 and the gaussian model. It will be noted that in every case the gaussian model underestimates the occurrences of high gust velocities. In general, these results and others not presented here indicate that the two criteria agree much better for the high mountain data than they do for the plains data.

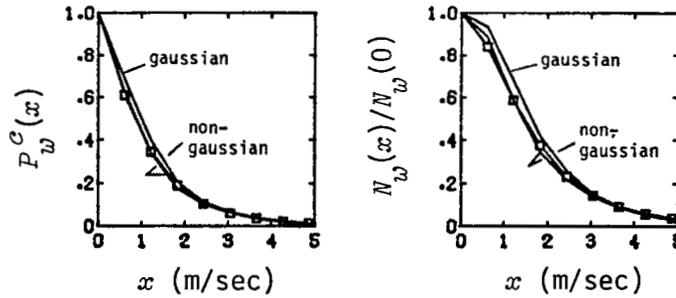
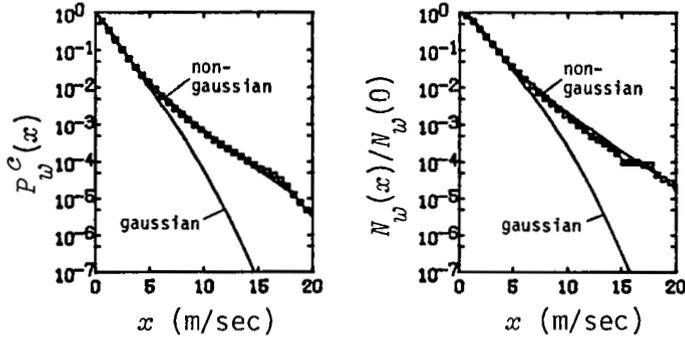
Table 4 summarizes the values of R which yield the best fit of the data for each of the error criteria. These results seem to indicate that the parameter R of the turbulence model may be related to the terrain and stability parameters of the LO-LOCAT data categories. It appears that R increases with atmospheric stability and terrain roughness, but is relatively unaffected by altitude. This result, however, cannot be verified without a much more careful investigation using more data and applying a regression analysis in order to objectively analyze the dependence of R upon the data parameters.

Summary

This section has presented a comparison of the gaussian and non-gaussian models with experimentally measured cumulative probability of exceedance and cumulative probability distributions of low altitude turbulence. Two simple error criteria have been used to select the best

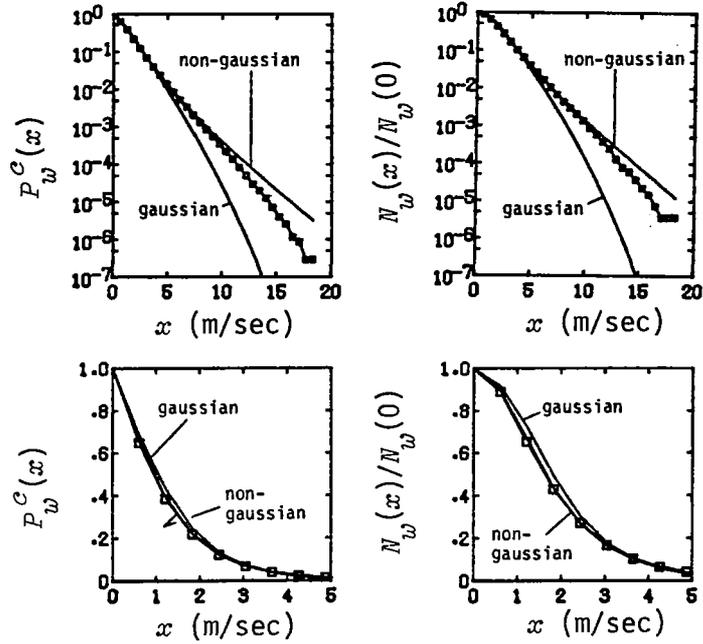


LO-LOCAT data category 111000

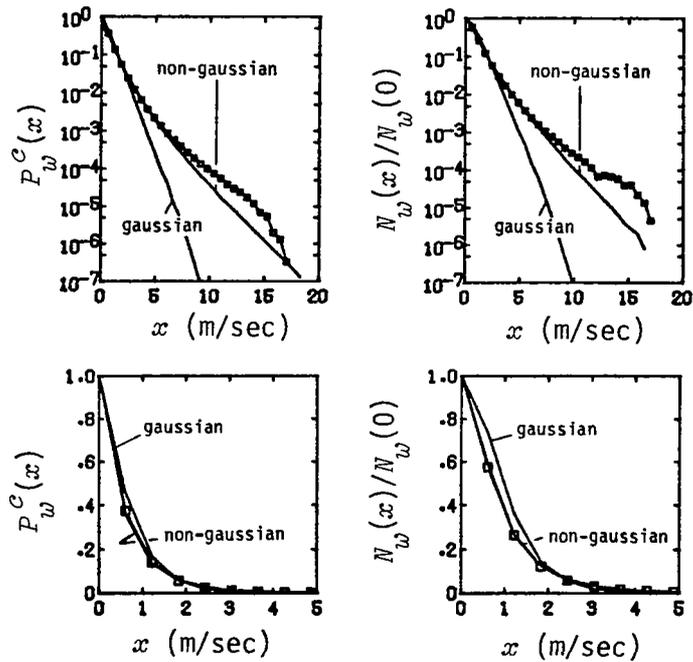


LO-LOCAT data category 112000

Figure 19.--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the ϵ_{\max} best fit criterion.



LO-LOCAT data category 113000



LO-LOCAT data category 121000

Figure 19 (cont.)--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the ϵ_{\max} best fit criterion.

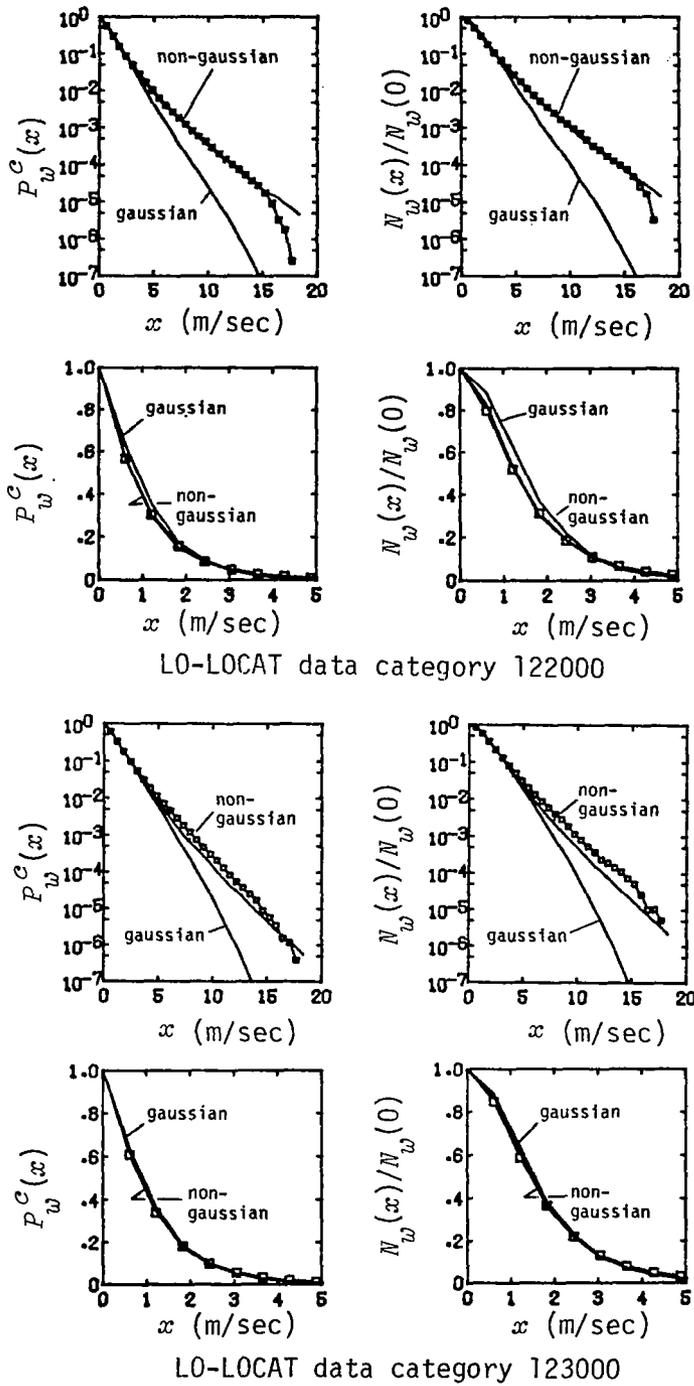
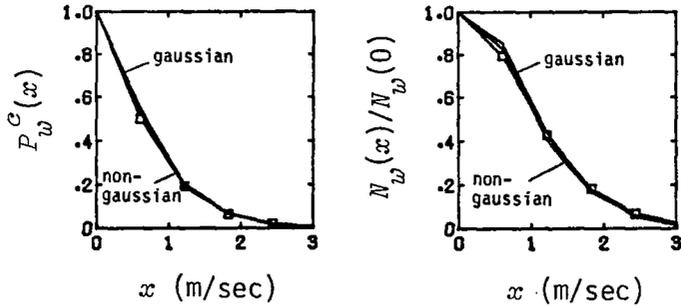
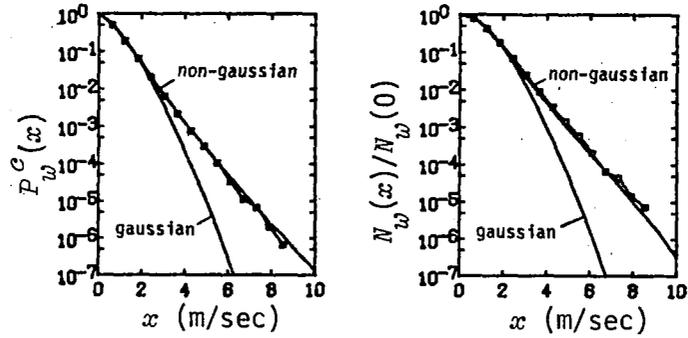
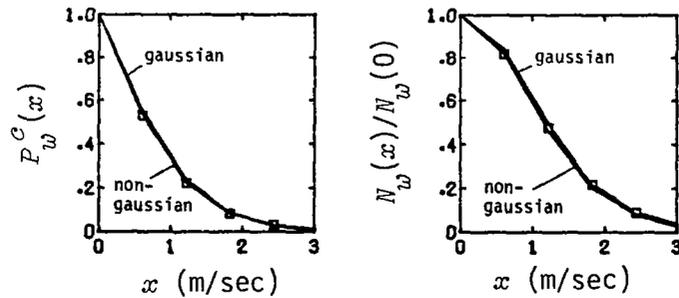
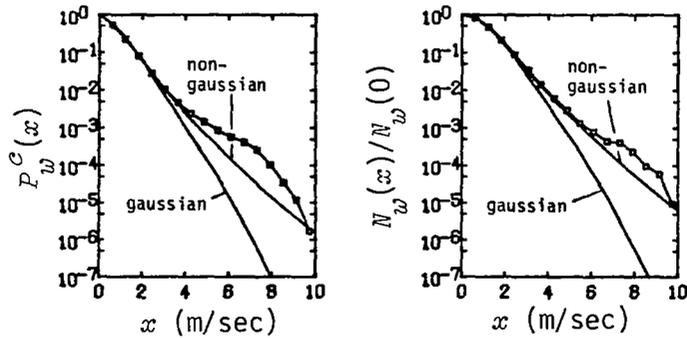


Figure 19 (cont.)--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the ϵ_{\max} best fit criterion.

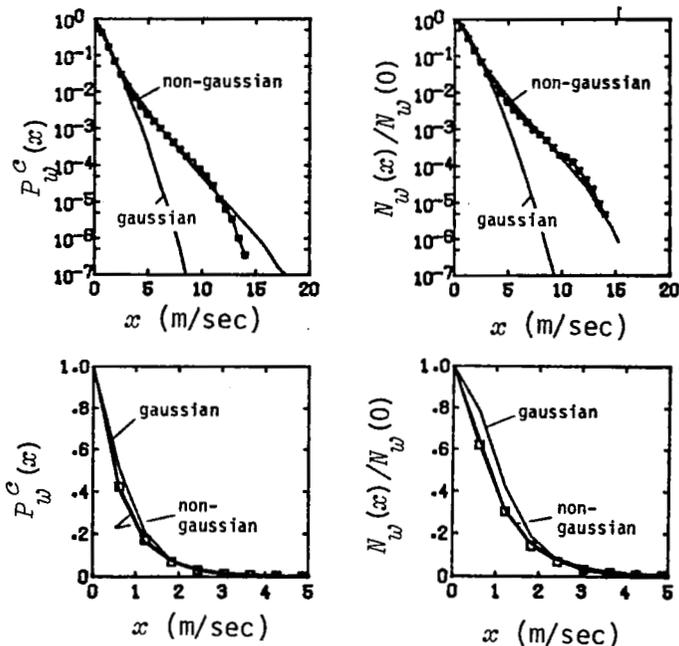


LO-LOCAT data category 413000

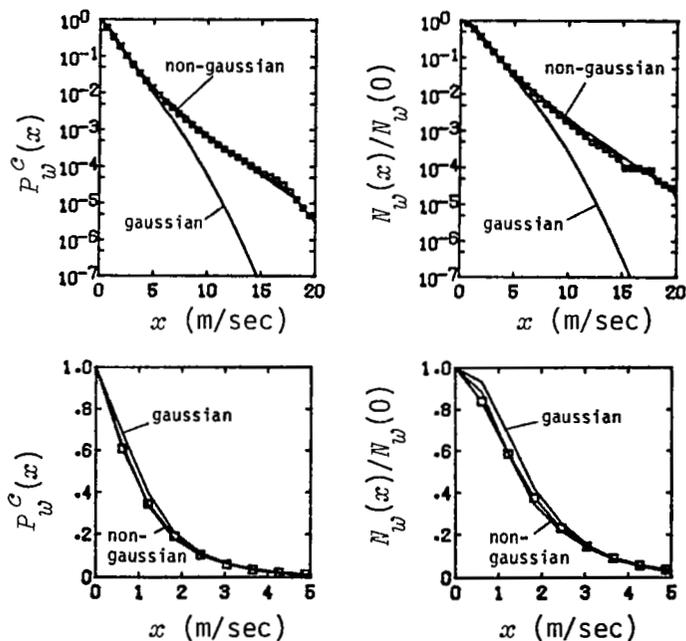


LO-LOCAT data category 414000

Figure 19 (cont.)--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the ϵ_{\max} best fit criterion.



LO-LOCAT data category 111000



LO-LOCAT data category 112000

Figure 20.--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the $\epsilon_{1\log}$ best fit criterion.

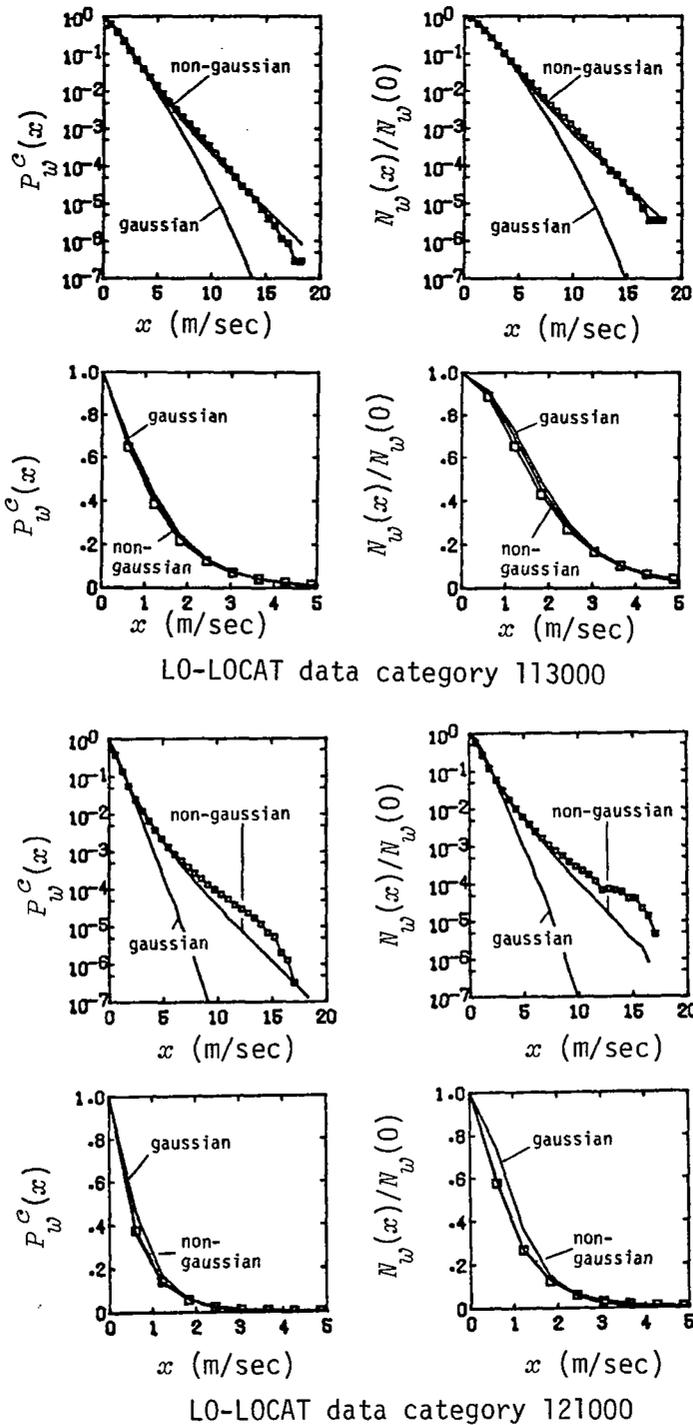


Figure 20 (cont.)--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the ϵ_{\log} best fit criterion.

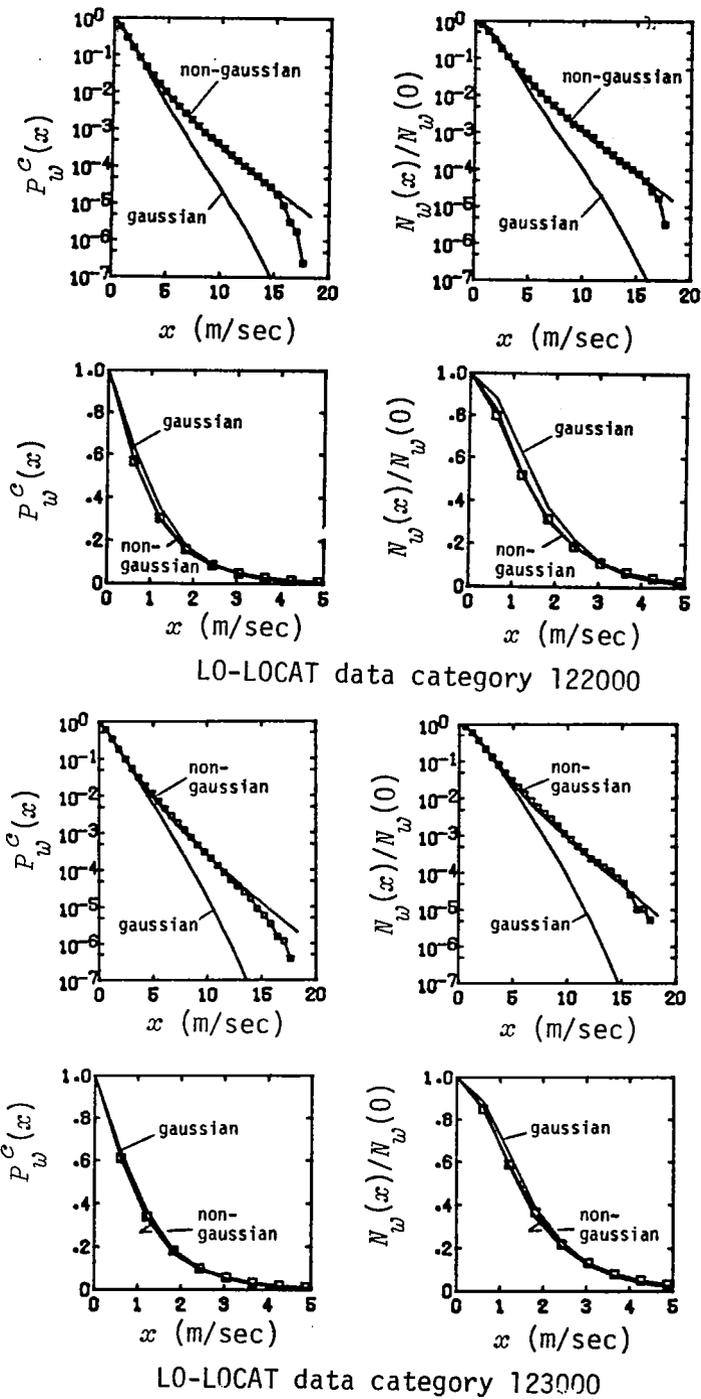
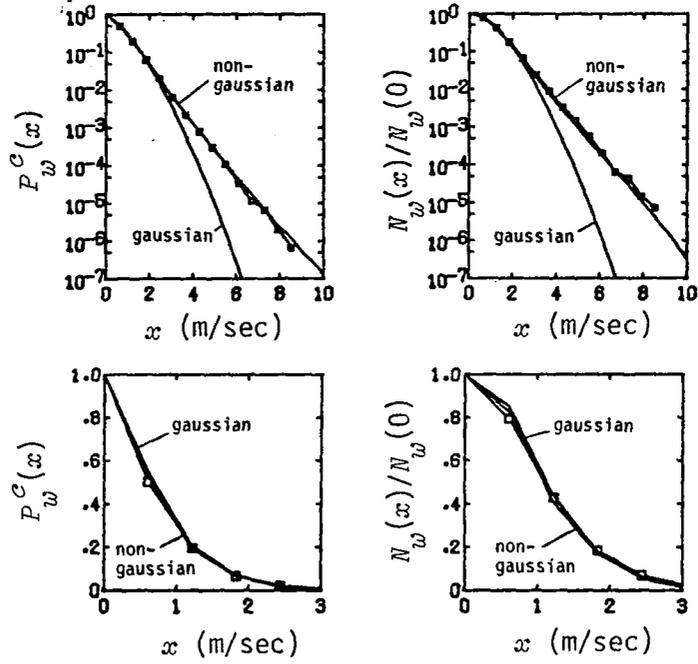
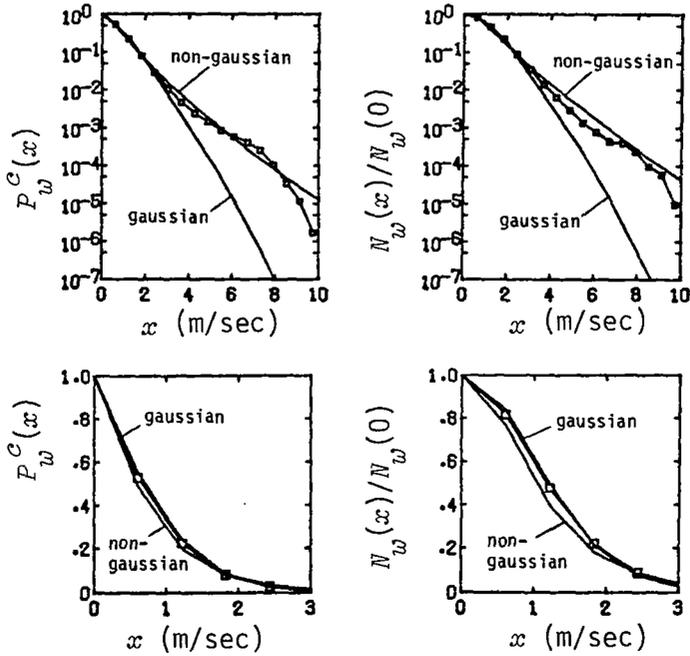


Figure 20 (cont.)--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the $\epsilon_{1\log}$ best fit criterion.



LO-LOCAT data category 413000



LO-LOCAT data category 414000

Figure 20 (cont.)--Various LO-LOCAT data plotted on both logarithmic and linear scales compared with the gaussian model and the non-gaussian model selected according to the ϵ_{log} best fit criterion.

values of the parameter R which fit the experimental data. Figures 19 and 20 present the resulting curves. Table 4 lists the values of R which minimize the error for each data category. These R values seem to exhibit a systematic increase with increasing atmospheric stability and terrain roughness. However, this result cannot be verified without a more careful analysis.

The results of this section indicate that the non-gaussian model is a better fit of experimental data than the gaussian model. The conclusion drawn is that the non-gaussian model is a better representation of atmospheric turbulence, at least as far as the modeling of distribution functions and level crossing frequencies for the purposes of aircraft design is concerned.

Table 4.--Values of the turbulence model parameter R which produce best fits of the experimental data.

| Data Category | R which Minimizes ϵ_{max} | R which Minimizes ϵ_{log} |
|---------------|--------------------------------------|--------------------------------------|
| 111000 | 2.0 | 2.0 |
| 112000 | 1.33 | 1.33 |
| 113000 | 1.0 | 0.75 |
| 121000 | 2.0 | 2.0 |
| 122000 | 1.33 | 1.33 |
| 123000 | 0.75 | 1.0 |
| 413000 | 0.75 | 0.75 |
| 414000 | 0.75 | 1.33 |

CALCULATION OF NON-GAUSSIAN RESPONSE STATISTICS

The preceding sections of this report have discussed the non-gaussian model in detail and shown that its statistics are consistent with those of experimentally measured turbulence data. The proposed model is of little value, however, if it cannot be used to study vehicle responses to turbulence. The purpose of this section is to investigate some methods by which response statistics of linear vehicles can be found using the non-gaussian model. The statistics of particular interest here are:

- 1) power spectral density
- 2) probability distribution
- 3) level crossing frequency.

The assumptions made regarding the nature of the turbulence are the same as those used in defining the non-gaussian model (page 47). The vehicles considered in this report will be required to satisfy the following conditions:

- 1) they must be stable linear systems
- 2) their transfer functions must have at least two more poles than zeros
- 3) their transfer functions must be rational
- 4) their transfer functions must have no multiple poles.

Only the first and second of these conditions is absolutely necessary. The third and fourth conditions are required because they permit simplification of the computer programs which will be used in the next section of this report. These two conditions could therefore be removed by writing more general programs.

The following discussion will first center on some standard techniques by which the above mentioned statistics might be calculated for responses to single components of the non-gaussian model. It will be shown that these methods are not entirely suitable for finding probability distributions or level crossing frequencies. Attention will then be turned to an approximate method of computing these statistics from the eigenvalues and eigenfunctions of a certain unsymmetric kernel. The problem of responses caused by two or all three components of the non-gaussian model will then be briefly discussed.

Standard Solution Techniques

Figure 21 shows the combined turbulence model - vehicle system which is to be analyzed. Note that this system is linear throughout with the exception of the single multiplication. It is this single nonlinearity which creates difficulties in response calculations. Now consider some standard techniques by which the three response statistics listed above might be computed.

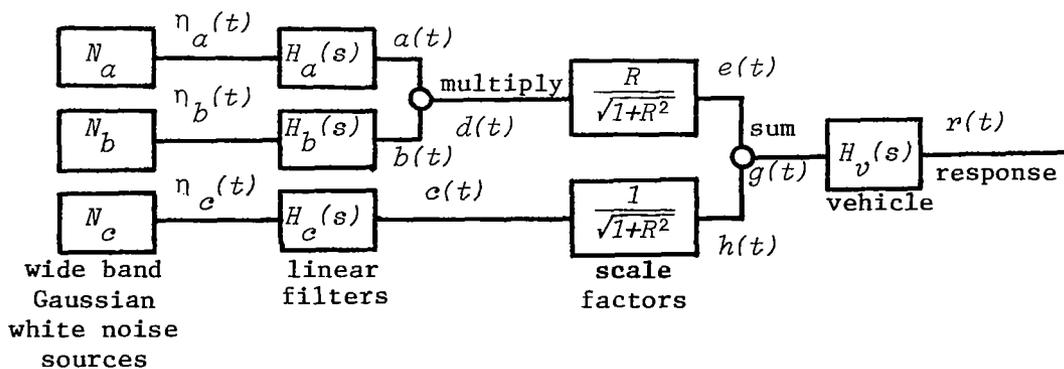


Figure 21.--Block diagram of the turbulence model-vehicle system which is to be analyzed in order to determine vehicle response statistics.

Power Spectral Density

Since the vehicle is assumed to be a linear system, and since the power spectral density of the turbulence time history is assumed known (Eq. 4, 5, or 6), the power spectral density of the vehicle response can be found from the well known input-output spectral relationship for linear systems (Ref. 38).

$$\phi_{rr}(\Omega) = |H(i U \Omega)|^2 \phi_{gg}(\Omega) , \quad (142)$$

where $H(s)$ is the transfer function relating the input $g(t)$ to the output $r(t)$. The calculation of response power spectral densities will therefore not present any difficulty, and will not be discussed further in this section.

Response Distribution and Level Crossing Frequency

The calculation of response distribution functions and level crossing frequencies provides somewhat more of a challenge than did the power spectral density discussed above. Two commonly used techniques are mentioned here as practical methods which may have application in some cases.

The first method discussed will be simulation. This is the direct approach to the problem of calculating response statistics, and may be the most convenient method of dealing with the non-gaussian model for many applications. Just as the name of this method suggests, the entire turbulence model-vehicle system is programmed on an analog or digital computer and the vehicle is "flown" through many miles of turbulence while its responses are recorded. After a sufficient amount of data have been collected, estimates are made of the response statistics.

The simulation method has both advantages and disadvantages. Among its advantages is the fact that it can be used to study both linear and nonlinear vehicles. It can also be applied to nonstationary problems such as the evaluation of control requirements during landing approaches. The disadvantage of this method is that it does not readily yield results concerning rare events such as encounters with very high velocity gusts. The computer time required to estimate these occurrences may be prohibitive.

The simulation approach to response calculations may thus be a very useful technique in some cases, but it is not a convenient method of estimating the occurrences of rare events. Thus it will probably not be satisfactory when information on the tails of the probability distribution or level crossing frequency is sought.

The second method of computing response statistics is the Gram-Charlier expansion. This is a technique for expanding probability density functions in an orthogonal series of Hermite polynomials. The method can be applied to both one- and two-dimensional density functions, and so could be used to obtain the density function of the response, as well as the joint density function of the response and its first derivative. The first of these is equivalent to the response distribution function, and the second can be used (through Eq. 81) to determine the response level crossing frequency. This method could, at least in principle, be used to obtain the response statistics which are of interest here.

Unfortunately, the Gram-Charlier technique cannot be applied in all cases because the coefficients of the expansion become very difficult to calculate if more than the first two or three terms are required.

Consequently, this method is only useful if a very minimal number of terms is needed. Since the first term of the expansion turns out to be the gaussian density function, it follows that the Gram-Charlier technique is useful only when the density function being expanded is very nearly gaussian. The method also becomes impractical if the tails of the distribution to be expanded do not decay as rapidly as those of the gaussian density function.

On the basis of preliminary analysis it appears that in many cases the non-gaussian turbulence model will produce response density functions which require an unreasonable number of terms to converge. Table 5 shows the number of terms required to expand the density function of the model itself for various values of the parameter R . The shape of these density functions can be inferred from figure 15 on page 61. These results make it clear that any distribution which differs from gaussian by more than a slight degree will not be suitable for analysis by the Gram-Charlier technique.

Table 5.--Number of non-zero terms of Gram-Charlier expansion required to represent density functions of the non-gaussian model to various accuracies over the range of zero to six standard deviations.

| ACCURACY | MODEL PARAMETER R | | | |
|----------|---------------------|-----|----|-----|
| | .1 | .15 | .2 | .25 |
| ± 10% | 1 | 2 | 4 | 5 |
| ± 20% | 1 | 1 | 3 | 5 |
| ± 50% | 1 | 1 | 2 | 3 |

In the following parts of this section, an expansion method will be derived which works best in the case of a strongly non-gaussian response. This new method will thus compliment the Gram-Charlier expansion as a suitable technique for finding response statistics. Furthermore, unlike the Gram-Charlier technique, it can be shown that this expansion has the characteristic of approximating the tails of the distribution very well, even when only a few terms are used.

The Gram-Charlier method will not be discussed further in this report. However, references 24 and 46 provide good descriptions of the technique and its use in computing one- and two-dimensional density functions.

Decomposition of Response

The above remarks have considered two well known methods of dealing with system responses to non-gaussian inputs, and pointed to some of the shortcomings of each. A method will now be developed which specifically treats the response of linear systems to the non-gaussian turbulence model. Before beginning the derivation of this new method, however, some introductory remarks are in order.

Consider again the turbulence model-vehicle system of figure 21. Since the vehicle is assumed to be linear, the system can be redrawn as shown in figure 22. Note that the definition of $g(t)$ in this figure differs from that used previously. The process g is now gaussian because a gaussian process remains gaussian when passed through a linear filter. The process $k(t)$ on the other hand is the result of passing $d(t)$, which has a K_0 probability density (Fig. 12), through a linear filter. In general, the distribution of $k(t)$ is unknown. The total vehicle response,

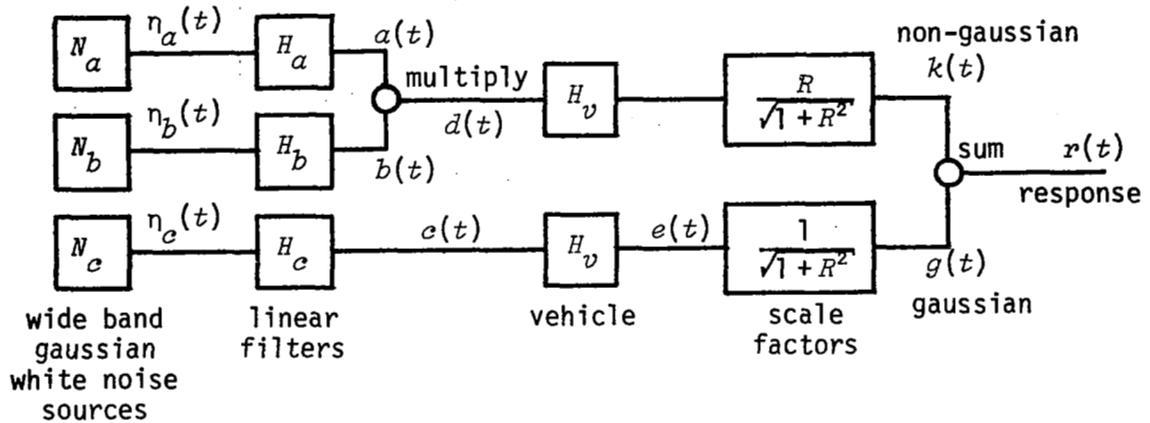


Figure 22.--Turbulence model - vehicle system with the vehicle response decomposed into gaussian and non-gaussian parts.

$r(t)$, is the sum of $g(t)$ and $k(t)$. Figure 22 has thus shown how the response of the vehicle can be decomposed into gaussian and non-gaussian parts.

Approximate Solution for Response Probability

Distribution Function

Figure 22 has shown that the vehicle response can be written as a sum of independent gaussian and non-gaussian parts. This immediately suggests the possibility of finding the distribution function of the response from its characteristic function. The steps in this procedure are:

- 1) find the characteristic function of the gaussian portion of the response, ϕ_g
- 2) find the characteristic function of the non-gaussian portion of the response, ϕ_k
- 3) multiply ϕ_g and ϕ_k to obtain the characteristic function of the total response, ϕ_r

- 4) inverse Fourier transform ϕ_x to obtain the density function of the response, p_x .

The explicit parameter dependence notation used in previous discussions of characteristic functions and probability distributions will not be used in this section in order to simplify notation. The reader should be aware, however, that the results obtained here depend upon the parameters of the turbulence model as well as the dynamic characteristics of the vehicle.

As the first step in determining the response distribution function, the characteristic function of the gaussian portion of the response will be found. Recall the definition of the characteristic function used in this report (Eq. 72). For the gaussian portion of the response, the result is

$$\phi_g(f) = \exp[-2(\pi \sigma_e f)^2(1+R^2)^{-1}] . \quad (143)$$

The standard deviation appearing in this expression, σ_e , can be found from the power spectral density of the response, equation 142. This completes the first step of the derivation.

Now consider the characteristic function of $k(t)$. Two very important assumptions will be made concerning this process. It is almost certain that neither of these assumptions is necessary in order to obtain the results presented here; however, their use will greatly simplify the derivation of these results.

- 1) It is assumed that negligible error will occur in the response statistics if the bandwidth of the white noise sources in figure 22 is fixed at some very high but finite limit as

shown in figure 23. By virtue of the second assumption regarding the vehicle transfer functions, only vehicles which act as low pass or band pass filters will be considered in this report. Furthermore, the filters used in the turbulence model itself (Table 1) are either low pass or band pass. Consequently, even when the nonlinear nature of the system is taken into account, it is clear that it will always be possible to select a value of Ω_c (e.g., 10^{10} or 10^{100}) such that the variance of the response and its first derivative will not be significantly affected by further increase of the noise bandwidth. In view of this, it is reasonable to expect that the response probability distribution and level crossing frequency will also be unaffected by increases in bandwidth.

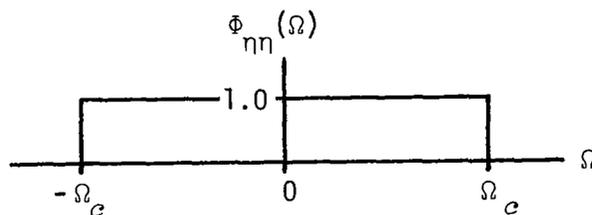


Figure 23.--Power spectral density of band limited gaussian white noise.

- 2) It is also assumed that the impulse response functions of the filters appearing in figure 22 can be truncated at some very large value of their argument, as shown in figure 24, without causing significant error in the vehicle response. This seems quite reasonable in view of the fact that the impulse response functions

of the vehicle and the turbulence model filters all decay exponentially. If this assumption were not true, it would follow that the response of the vehicle depended to a significant degree upon the infinite past. Thus the response could never reach a state of statistical equilibrium, and the assumption of stationarity which has been made throughout this report would not be valid.

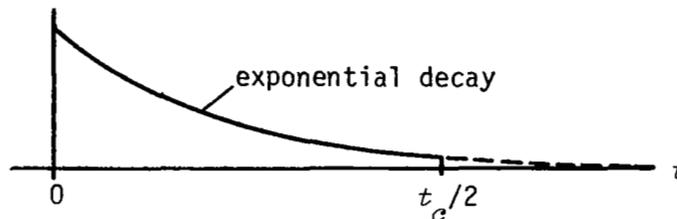


Figure 24.--Truncated impulse response function.

Once these two assumptions have been made, the eigenvalue expansion of the response distribution function follows quickly from a certain theorem due to Schmidt which will be stated shortly.

Consider again figure 22. From linear system theory it follows that the response time history $k(t)$ can be written as the iterated integral

$$k(t) = \frac{R}{(1+R^2)^{1/2}} \iiint_{-\infty}^{\infty} h_v(\delta) h_a(\alpha') h_b(\beta') \eta_a(t-\delta-\alpha') \eta_b(t-\delta-\beta') d\alpha' d\beta' d\delta . \quad (144)$$

Now, because of the first assumption above limiting the bandwidth of

η_a and η_b , it follows (Ref. 47, page 170) that these processes are continuous with probability one. The integrand of equation 144 is therefore integrable and, by Fubini's theorem (Ref. 48), the order of integration can be freely interchanged. Substitute the change of variables

$$\left. \begin{aligned} \alpha &= t - \alpha' \\ \beta &= t - \beta' \end{aligned} \right\} \quad (145)$$

and integrate with respect to δ . The resulting expression for $k(t)$ is

$$k(t) = R(1+R^2)^{-1/2} \int_{-\infty}^{\infty} \eta_a(t-\alpha) \left\{ \int_{-\infty}^{\infty} \eta_b(t-\beta) h(\alpha, \beta) d\beta \right\} d\alpha, \quad (146)$$

where the integration with respect to β is arbitrarily chosen to be performed first, and the kernel $h(\alpha, \beta)$ is given by

$$h(\alpha, \beta) = \int_{-\infty}^{\infty} h_v(\delta) h_a(\alpha - \delta) h_b(\beta - \delta) d\delta. \quad (147)$$

Note that for the vertical and lateral components of the turbulence model, the impulse response functions h_a and h_b are not identical, thus $h(\alpha, \beta)$ is not generally symmetric.

As a consequence of the second assumption above concerning truncation of the impulse response functions, $h(\alpha, \beta)$ is non-zero only if α and β satisfy the condition

$$0 \leq \alpha, \beta \leq t_e, \quad (148)$$

where $t_e/2$ is the truncation time defined in figure 24. Because $h(\alpha, \beta)$ is zero whenever α or β does not satisfy equation 148, it follows that finite limits of integration can be used in equation 146.

Now consider Schmidt's theorem concerning integral equations with unsymmetric kernels (Ref. 49, 50). This theorem states that every function q having one of the two forms

$$q(\alpha) = \int_{\xi} h(\alpha, \beta) \eta(\beta) d\beta \quad (149)$$

$$q(\beta) = \int_{\xi} h(\alpha, \beta) \eta(\alpha) d\alpha \quad (150)$$

where ξ is a finite interval, is the sum of its uniformly and absolutely convergent Fourier series with respect to the orthonormal system $\psi_n(\alpha)$ in the first case and with respect to the orthonormal system $\chi_n(\beta)$ in the second case. The functions ψ_n and χ_n are defined by the eigenfunction relationships

$$\lambda_n^2 \psi_n(\alpha) = \int_{\xi} \left\{ \int_{\xi} h(\alpha, \delta) h(\beta, \delta) d\delta \right\} \psi_n(\beta) d\beta \quad n = 1, 2, \dots \quad (151)$$

$$\lambda_n^2 \chi_n(\beta) = \int_{\xi} \left\{ \int_{\xi} h(\delta, \alpha) h(\delta, \beta) d\delta \right\} \chi_n(\alpha) d\alpha \quad n = 1, 2, \dots; \quad (152)$$

and are related to each other by the equations

$$\lambda_n \psi_n(\alpha) = \int_{\xi} h(\alpha, \beta) \chi_n(\beta) d\beta \quad n = 1, 2, \dots \quad (153)$$

$$\lambda_n \chi_n(\alpha) = \int_{\xi} h(\alpha, \beta) \psi_n(\beta) d\beta \quad n = 1, 2, \dots \quad (154)$$

The two sets of functions $\psi_n(\alpha)$ and $\chi_n(\beta)$ are known as the adjoint eigenfunctions of the kernel $h(\alpha, \beta)$, and the constants λ_n are said to be the eigenvalues of the kernel.

Schmidt's theorem holds if the following conditions are satisfied:

- 1) the functions $h(\alpha, \beta)$ and $h(\alpha, \beta)^2$ are integrable
- 2) the integrals $\int_{\xi} h(\alpha, \beta)^2 d\alpha$ and $\int_{\xi} h(\alpha, \beta)^2 d\beta$ are bounded
- 3) the functions η and η^2 are integrable.

All three of these conditions are satisfied for the inner integral of equation 146. The first and second conditions will always be satisfied by the well behaved linear systems of this report. The third condition is satisfied because of the bandlimited nature of the white noise functions (Ref. 47, page 170).

The set of eigenvalues λ_n is assumed to be ordered according to decreasing magnitude. It is also necessary that they satisfy Bessel's inequality,

$$\sum_{n=1}^{\infty} \lambda_n^2 \leq \iint_{\xi} h(\alpha, \beta)^2 d\alpha d\beta, \quad (155)$$

Schmidt's theorem will be applied to the inner integral of equation 146. Before doing so, however, it is useful to make the following observation. Note that equation 146 expresses k as a function of time. Since k is a stationary process however, its statistical properties cannot depend upon the time at which they are evaluated. Thus, for the purposes of calculating the probability distribution and level crossing frequency of $k(t)$, it must be permissible to fix t at any desired value. The value chosen in this report is zero. Therefore, upon taking equation 148 into account, equation 146 can be written

$$k(0) = R(1+R^2)^{-1/2} \int_0^t \eta_\alpha(-\alpha) \left\{ \int_0^t \eta_\beta(-\beta) h(\alpha, \beta) d\beta \right\} d\alpha. \quad (156)$$

This is the equation to which Schmidt's theorem will be applied.

According to the theorem, the bracketed integral of equation 156 can be expressed as an absolutely and uniformly convergent series.

$$\int_0^t \eta_\beta(-\beta) h(\alpha, \beta) d\beta = \sum_{n=1}^{\infty} \psi_n(\alpha) \iint_0^t h(\delta, \beta) \eta_\beta(-\beta) \psi_n(\delta) d\beta d\delta. \quad (157)$$

All functions are continuous on $[0, t]$, so the order of integration can be interchanged and integration with respect to δ carried out first.

By equation 154, the result is

$$\int_0^t \eta_\beta(-\beta) h(\alpha, \beta) d\beta = \sum_{n=1}^{\infty} \lambda_n \psi_n(\alpha) \int_0^t \eta_\beta(-\beta) \chi_n(\beta) d\beta. \quad (158)$$

Equation 158 can be substituted into equation 156 and, because the series is absolutely and uniformly convergent, integration with respect to α can be carried out term by term. The result is

$$k(0) = R(1+R^2)^{-1/2} \sum_{n=1}^{\infty} \lambda_n A_n B_n, \quad (159)$$

where

$$A_n = \int_0^t \eta_\alpha(-\alpha) \psi_n(\alpha) d\alpha \quad (160)$$

$$B_n = \int_0^t \eta_\beta(-\beta) \chi_n(\beta) d\beta. \quad (161)$$

A_n and B_n are random variables with the following properties.

- 1) A_n and B_n are normally distributed because they are linear transformations of gaussian processes.
- 2) The mean value of each A_n and B_n is zero.
- 3) All A_n are independent of all B_n because η_a is independent of η_b .
- 4) For all n and m the correlation of A_n with A_m and B_n with B_m is given by

$$E\{A_n A_m\} = \int_0^t \int_0^t \psi_n(\alpha) \psi_m(\beta) C_{\eta_a \eta_a}(\alpha - \beta) d\alpha d\beta \quad (162)$$

$$E\{B_n B_m\} = \int_0^t \int_0^t \chi_n(\alpha) \chi_m(\beta) C_{\eta_b \eta_b}(\alpha - \beta) d\alpha d\beta \quad (163)$$

where $C_{\eta_a \eta_a}$ and $C_{\eta_b \eta_b}$ are the correlation functions of η_a and η_b respectively.

At this point a further approximation is introduced. Equation 159 expresses the non-gaussian portion of the vehicle response at an arbitrary instant ($t = 0$) as an infinite summation of random variables. In order to apply this equation to the practical evaluation of the response density function, it is going to be necessary to evaluate the eigenvalues, λ_n . Clearly it is unreasonable to expect to be able to evaluate more than the first few of these. Consequently, if equation 159 is to be of any practical use, it will be necessary that only the first few terms of the series be significant. Results to be derived shortly will show that the variance of the random variables A_n and B_n is approximately unity,

and therefore the terms of equation 159 will decrease rapidly only if the eigenvalues decrease very rapidly. It will be assumed that this is the case.

Should this assumption prove invalid for some particular vehicle, it would follow that k could be represented by a summation of random variables with variances of comparable magnitude. These variables would be independent; and, though not identically distributed, it would not be unreasonable to expect that their sum would be nearly gaussian. In such a case the vehicle response would be almost gaussian, and the Graham-Charlier expansion mentioned earlier in this section would be a promising method of approach.

Under the assumption that only a few terms of the expansion are required to adequately represent k , equation 159 becomes

$$k(0) = R(1+R^2)^{-1/2} \sum_{n=1}^N \lambda_n A_n B_n. \quad (164)$$

This assumption will also permit it to be shown that the random variables A_n and B_n can be considered independent and $(0, 1)$ normal.

Consider the correlation of A_n with A_m , equation 162. The correlation function of η_a can be written as the Fourier transform of its power spectral density. Noting the form of $\Phi_{\eta_a \eta_a}$ shown in figure 22, and using the transformation presented in equation 38, this relationship becomes

$$C_{\eta_a \eta_a}(\tau) = \int_{-\Omega_c}^{\Omega_c} \exp(i U \Omega \tau) d\Omega \quad (165)$$

Substitution of this result into equation 162 and interchange of the order

of integration gives

$$E\{A_n A_m\} = \int_{-\Omega_c}^{\Omega_c} \left\{ \int_0^t \psi_n(\alpha) \exp(i U \Omega \alpha) d\alpha \right\} \left\{ \int_0^t \psi_m(\beta) \exp(-i U \Omega \beta) d\beta \right\} d\Omega \quad (166)$$

where it is assumed that ψ_n and ψ_m are zero outside of the interval $[0, t_c]$. The bracketed terms of equation 166 can be identified as the Fourier transform of ψ_n and ψ_m .

Inspection of the kernel defined in equation 147 will now show that $h(\alpha, \beta)$ is zero if either α or β is zero, but $h(t_c, \beta)$ and $h(\alpha, t_c)$ are not necessarily zero. It follows therefore from equation 151 that $\psi_n(0)$ is equal to zero, but $\psi_n(t_c)$ is not necessarily zero. Thus $\psi_n(\alpha)$ and $\psi_m(\beta)$ in equation 166 are continuous everywhere except possibly at the points α and β equal to t_c . At these points, ψ_n and ψ_m may exhibit a simple step discontinuity. The theory of Fourier transforms (Ref. 51) then requires that the absolute value of the Fourier transforms appearing in equation 166 must decrease at least as rapidly as Ω^{-1} for large Ω . The product of transforms must therefore decrease at least as rapidly as Ω^{-2} . Thus, for all possible combinations of the indices n and m bounded by N , it must be possible to choose a single, finite value of Ω_c such that $E\{A_n A_m\}$ differs arbitrarily little from equation 166 with Ω_c replaced by infinity. Equation 166 therefore reduces to

$$E\{A_n A_m\} \doteq \int_0^t \psi_n(\alpha) \psi_m(\alpha) d\alpha \quad (167)$$

Since the functions ψ_n and ψ_m are orthonormal, it follows that

$$E\{A_n A_m\} \triangleq \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases} \quad (168)$$

A completely analogous result holds for equation 163.

Equation 168, along with the other properties of A_n and B_n listed previously, implies that the A_n and B_n or equation 164 can be assumed to be independent normal random variables with zero mean value and unit variance. It has been shown previously in this report that the product of independent gaussian random variables has a K_o probability distribution (Eq. 70). Hence, each term of equation 164 is a random variable with a density function of the form

$$p_n(x) = \frac{1}{\pi\sigma_n} K_o\left(\frac{|x|}{\sigma_n}\right) \quad n = 1, 2, \dots, N, \quad (169)$$

where

$$\sigma_n^2 = E\{(R\lambda_n A_n B_n)^2 (1+R^2)^{-1}\} = (R\lambda_n)^2 (1+R^2)^{-1}. \quad (170)$$

The reader will now recall that the object of all this is to obtain an approximate expression for the characteristic function of the non-gaussian portion of the vehicle response. It has just been shown that this response can be represented as a finite summation of independent random variables; therefore, the characteristic function in question must be given by a product of characteristic functions, one for each term of equation 164. It follows from equations 73, 169, and 170 that each term of this product is of the form

$$\phi_n(f) = [1 + (2\pi R \lambda_n f)^2 (1 + R^2)^{-1}]^{-1/2}, \quad (171)$$

and the characteristic function of k is

$$\phi_k(f) = \prod_{n=1}^N [1 + (2\pi R \lambda_n f)^2 (1 + R^2)^{-1}]^{-1/2}. \quad (172)$$

This completes the second step of the response density function derivation.

The characteristic function of the total vehicle response is now obtained by multiplying equations 143 and 172.

$$\phi_x(f) = \exp[-2(\pi \sigma_e f)^2 (1 + R^2)^{-1}] \prod_{n=1}^N [1 + (2\pi R \lambda_n f)^2 (1 + R^2)^{-1}]^{-1/2} \quad (173)$$

This completes the third step of the derivation. The density function of x can now be found by numerically Fourier transforming equation 173. Program PDIST presented in Appendix A of this report performs this calculation.

Computer programs EIGU and EIGVW, which will also be found in Appendix A can be used to obtain the eigenfunctions and eigenvalues of the vehicle response to the longitudinal gust component and to the lateral and vertical gust components respectively. Note from equations 151 and 152 that, since the function $h(\alpha, \beta)$ is symmetric for the longitudinal gust case, the orthonormal sets ψ_n and χ_n will be identical. Thus program EIGU computes only one set of eigenfunctions. The eigenvalues obtained from either EIGU or EIGVW can then be used in program PDIST to obtain the probability density and distribution function of the vehicle

response. The next section of this report will present a numerical example showing the application of these programs.

Approximate Solution for Response Level

Crossing Frequency

Recall from equation 81 that the level crossing frequency of a stochastic process can be found from a knowledge of the joint distribution of the process and its derivative,

$$N_p(x) = \int_0^{\infty} \dot{x} p_{x, \dot{x}}(x, \dot{x}) d\dot{x} \quad (174)$$

The reader should note that the explicit parameter dependence notation used in earlier sections of this report is not used here in order to simplify notation. The results presented here will, of course, depend upon the parameters of the turbulence model as well as the dynamic characteristics of the vehicle.

Previously in this section it was shown that the distribution function of the response could be approximated by means of a characteristic function approach. It will now be shown that the joint distribution of the response and its derivative can be approximated by an analogous procedure. The steps to be followed are:

- 1) obtain the joint characteristic function of the gaussian portion of the response and its derivative, $\phi_{g, \dot{g}}$
- 2) obtain the joint characteristic function of the non-gaussian portion of the response and its derivative, $\phi_{k, \dot{k}}$

- 3) multiply these results to obtain the joint characteristic function of the complete response and its derivative, $\phi_{r, \dot{r}}$
- 4) inverse Fourier transform to obtain the joint density of the response and its derivative, $p_{r, \dot{r}}$
- 5) apply equation 174 to obtain the level crossing frequency of the response, $N_r(x)$.

Refer to figure 22 and note that the derivative of the response can be obtained by merely replacing the vehicle transfer function $H_v(s)$ with $sH_v(s)$.

The joint characteristic function of the gaussian portion of the response and its derivative can be found immediately.

$$\phi_{g, \dot{g}}(f_1, f_2) = \exp\{-2(1+R^2)^{-1}[(\pi \sigma_e f_1)^2 + (\pi \sigma_{\dot{e}} f_2)^2]\} \quad (175)$$

where the standard deviations can be obtained from the power spectral density of $e(t)$. This completes the first step of the derivation.

The joint characteristic function of k and \dot{k} will now be approximated by extending the results obtained above for the characteristic function of k alone. By equation 164, k can be represented as a finite summation of random variables.

$$k(0) = R(1+R^2)^{-1/2} \sum_{n=1}^N \lambda_n A_n B_n \quad (176)$$

It will now be assumed that a similar approximation applies to \dot{k} ,

$$\dot{k}(0) = R(1+R^2)^{-1/2} \sum_{m=1}^M \Lambda_m C_m D_m, \quad (177)$$

where Λ_m are the eigenvalues of the kernel analogous to $h(\alpha, \beta)$ of equation 147; and C_m, D_m are $(0, 1)$ normal, independent random variables. The adjoint eigenfunctions of the \dot{k} kernel are assumed to be $\psi_m(\alpha)$ and $\chi_m(\beta)$. It is further assumed that the noise bandwidths, Ω_c of figure 23, and the impulse response truncation times, $t_c/2$ of figures 24, are identical for both k and \dot{k} .

Now even though all of the random variables A_n and B_n are mutually independent, and all of the random variables C_m and D_m are mutually independent, it is readily verified that the variables A_n are correlated with the variables C_m and likewise for the variables B_n and D_m . These correlations can be shown to be

$$E\{A_n C_m\} = \int_0^{t_c} \psi_n(\alpha) \psi_m(\alpha) d\alpha \quad (178)$$

$$E\{B_n D_m\} = \int_0^{t_c} \chi_n(\beta) \chi_m(\beta) d\beta. \quad (179)$$

In order to derive an approximate expression for the joint characteristic function of k and \dot{k} it is convenient to introduce the following notation.

$$\bar{A} = [A_1 A_2 \dots A_N C_1 C_2 \dots C_M]^T \quad (180)$$

$$\bar{B} = [B_1 B_2 \dots B_N D_1 D_2 \dots D_M]^T \quad (181)$$

$$\tilde{A} = E\{\bar{A}\bar{A}^T\} \quad (182)$$

$$\tilde{B} = E\{\bar{B}\bar{B}^T\} \quad (183)$$

Since the elements of \bar{A} are independent of those in \bar{B} , it follows that the joint density of all the random variables in both \bar{A} and \bar{B} is given by

$$p_{\bar{A}, \bar{B}}(\bar{x}, \bar{y}) = p_{\bar{A}}(\bar{x}) p_{\bar{B}}(\bar{y}), \quad (184)$$

where

$$p_{\bar{A}}(\bar{x}) = [(2\pi)^{\frac{M+N}{2}} \det(\tilde{A})]^{-1} \exp[-\bar{x}^T (\frac{1}{2} \tilde{A}^{-1}) \bar{x}], \quad (185)$$

and a similar expression applies to $p_{\bar{B}}$. The joint characteristic function of k and \dot{k} is defined to be

$$\phi_{k, \dot{k}}(f_1, f_2) = E\{\exp(i2\pi k f_1 + i2\pi \dot{k} f_2)\}. \quad (186)$$

Substitution for k and \dot{k} from equations 176 and 177 gives

$$\phi_{k, \dot{k}}(f_1, f_2) = E\{\exp(\bar{A}^T \tilde{C} \bar{B})\} \quad (187)$$

where

$$\tilde{C} = i2\pi R(1+R^2)^{-1/2} [\text{diag}(f_1 \lambda_1, \dots, f_1 \lambda_N, f_2 \Lambda_1, \dots, f_2 \Lambda_M)] . \quad (188)$$

The joint characteristic function $\phi_{k, \dot{k}}$ can now be written in the integral form

$$\phi_{k, \dot{k}}(f_1, f_2) = \int_{-\infty}^{\infty} [2(M+N)] \int_{-\infty}^{\infty} \exp(\bar{x}^T \bar{C} \bar{y}) p_{\bar{A}}(\bar{x}) p_{\bar{B}}(\bar{y}) d\bar{x} d\bar{y}, \quad (189)$$

This expression can be integrated numerically to obtain the value of $\phi_{k, \dot{k}}$ for any choice of f_1 and f_2 . This completes the second step of the derivation.

The third, fourth, and fifth steps of the derivation are carried out by multiplying equations 175 and 189, applying a two-dimensional Fourier transform, and then integrating as indicated in equation 174. Program RLEVX, presented in Appendix A of this report performs all of the computations described here including the evaluation of the covariance matrices, equations 182 and 183. Use of this program in connection with the eigensolution programs EIGU and EIGVW as shown in the next section of this report thus provides an automated method of computing the level crossing frequency of a linear response to the non-gaussian turbulence model. This completes the discussion of the level crossing problem.

Response to Multiple Inputs

The above remarks have all dealt with the case of a single component of the turbulence model. Because the three components of the model are independent processes, however, these results can be easily extended to include the case of a vehicle disturbed simultaneously by two or all three gust components.

Recall that the three quantities of interest are the response power spectral density, distribution function, and level crossing frequency. The power spectral density of a linear response to several independent inputs is merely the sum of the spectral densities due to each input

alone. Equation 142 can therefore be used to calculate a power spectrum for the response to each gust component and the total response power spectrum found by summing.

The distribution function of the response to multiple inputs can be found through its characteristic function. Since the response to each gust component is an independent process, the characteristic function of the total response will merely be the product of the characteristic functions corresponding to each component. Thus equation 173 could be used to find the characteristic function of the response to each component of the turbulence model, and the characteristic function of the total response found by multiplying. This product could then be Fourier transformed to yield the required distribution function.

A similar argument applies to the determination of response level crossings. In this case it is the joint characteristic function of the total response and its derivative which are found by multiplying the joint characteristic functions corresponding to each gust component alone. The joint density of the response and its derivative are then obtained by Fourier transforming, and equation 174 is used to calculate the level crossing frequency.

Summary

This section has investigated the response of a linear vehicle to the proposed non-gaussian turbulence model. An expression for the power spectral density of the response has been presented (Eq. 142), and approximate numerical procedures have been suggested for computing the response distribution function (Eq. 143 through 173) and level crossing frequency (Eq. 174 through 189). Finally, the problem of calculating

response statistics induced by simultaneous gust components has been briefly discussed. In the next section of this report, the techniques developed above will be used to calculate the responses of an aircraft-autopilot system to the non-gaussian model.

NUMERICAL EXAMPLE

The previous section of this report has derived techniques which can be used to calculate the power spectral density, probability distribution, and level crossing frequency of a linear response to the proposed non-gaussian turbulence model. The purpose of this section is to illustrate the application of these techniques to a simple problem, and to compare the results with those obtained using the gaussian model. The computer programs mentioned will be found in Appendix A of this report.

Problem Statement

The aircraft to be studied is a STOL vehicle in cruising flight. Altitude is controlled by a simple autopilot system using altitude error, its integral, and its derivative in a feedback loop driving the elevator so as to return the aircraft to the commanded altitude.

It is supposed that this aircraft - autopilot system is perturbed by the vertical component of the non-gaussian model, and it is desired to investigate the deviations from the commanded cruise altitude which the system experiences. The power spectral density, probability distribution, and level crossing frequency of the altitude error are to be calculated.

Model of the Vehicle

As mentioned above, the aircraft considered here is a STOL vehicle in cruising flight. The commanded altitude is 305 meters (1,000 feet) above sea level, and the true airspeed in equilibrium flight is 76 meters per second (249.7 feet per second). The transfer function of the altitude error due to vertical gust disturbances for this particular

aircraft - autopilot system is

$$\frac{z}{w}(s) = \frac{40.92 [s] [s + (.0363 \pm .2083i)] [s + 2.912]}{[s + (.0443 \pm .0131i)] [s + (.3035 \pm .2908i)] [s + (2.112 \pm 2.404i)]} \quad (190)$$

where the notation $[s + (a \pm ib)]$ has been used to indicate complex conjugate pairs of poles or zeros. Note that this transfer function satisfies all of the conditions stated at the beginning of the previous section (page 106). The derivative of the altitude error is

$$\frac{\dot{z}}{w}(s) = s \frac{z}{w}(s) \quad (191)$$

In order to apply the computer programs presented in the appendix of this report, it will be necessary to express these two transfer functions in partial fraction form.

$$\begin{aligned} \frac{z}{w}(s) = & \frac{[-1.946 \pm 5.538i]}{[s + (.0443 \pm .0131i)]} + \frac{[-4.016 \pm 8.026i]}{[s + (.3035 \pm .2908i)]} \\ & + \frac{[5.964 \mp 4.816i]}{[s + (2.112 \pm 2.404i)]} \end{aligned} \quad (192)$$

$$\begin{aligned} \frac{\dot{z}}{w}(s) = & \frac{[.158 \mp .220i]}{[s + (.0443 \pm .0131i)]} + \frac{[3.552 \mp 1.268i]}{[s + (.3035 \pm .2908i)]} \\ & + \frac{[24.18 \mp 4.164i]}{[s + (2.112 \pm 2.404i)]} \end{aligned} \quad (193)$$

Turbulence Model Parameters

The turbulence model used here is the vertical component of the non-gaussian model. The parameters of the model are chosen to be

$$\left. \begin{aligned} L_w &= 142 \text{ m (465 ft)} \\ \sigma_w &= 0.305 \text{ m/sec (1 ft/sec)} \\ R_w &= 1.0 \end{aligned} \right\} (194)$$

These parameters are typical of those which would be encountered at an altitude of 228 meters (750 feet) over plains in unstable atmospheric conditions. With this choice of σ_w and L_w , the power spectral density of the turbulence becomes

$$\phi_{ww}(\Omega) = 2.10 \frac{(1 + 60,500 \Omega^2)}{(1 + 20,200 \Omega^2)^2} \quad (195)$$

The probability density and distribution functions of the gust velocity will be found in figure 15 on page 61, and the normalized level crossing frequency is given in figure 17 on page 80.

Response Power Spectral Density

The power spectral density of the altitude error is obtained by substituting equations 190 and 195 into equation 142. Figure 25 presents the resulting spectrum for positive values of Ω . The complete spectrum is, of course, symmetric about the origin. The variance of the altitude error can be calculated by integrating the power spectral density as indicated in equation 7. The result is

$$\sigma_r^2 = 13.26 \text{ (m)}^2 [142.8 \text{ (ft)}^2] \quad (196)$$

Note that the power spectral density and variance are independent of the turbulence model parameter R .

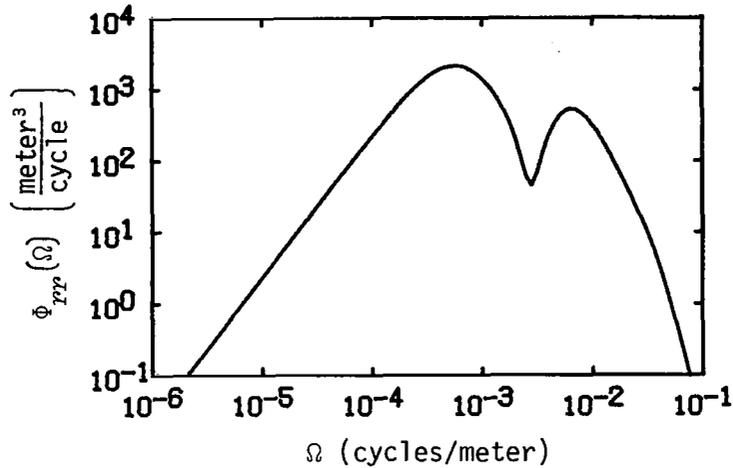


Figure 25.--Power spectral density of the altitude error.

Response Distribution Function

The probability distribution of the altitude error will now be calculated. This computation is in two parts,

- 1) use program EIGVW to compute the significant eigenvalues of the response
- 2) use these eigenvalues as input to program PDIST to obtain the response probability distribution function.

The first step of the procedure is the calculation of the altitude error eigensolutions. The transfer function of the system (Eq. 192) and the turbulence model parameters (Eq. 194), when used as input to program

EIGVW, result in the eigenvalues and adjoint eigenfunctions presented in figure 26. Note that program EIGVW cannot compute the eigenvalues without also computing the corresponding adjoint eigenfunctions.

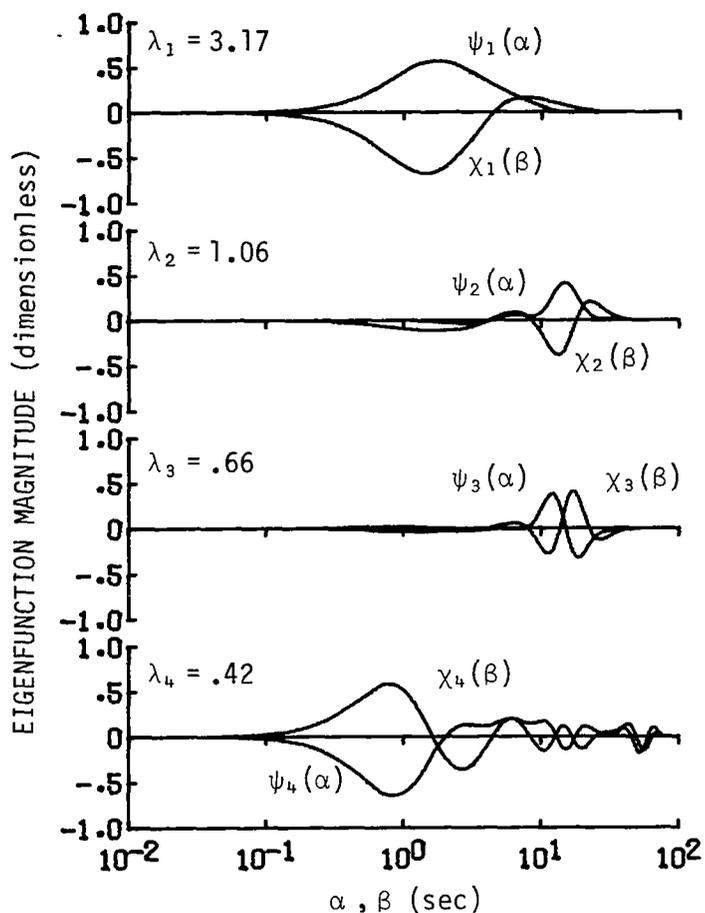


Figure 26.--Adjoint eigenfunctions and eigenvalues of the altitude error.

The second step in computing the response distribution is the use of the eigenvalues of figure 26 as input to program PDIST. This program has computed the probability density and distribution functions presented in figure 27. (Although used as input to PDIST, the eigenvalues of the response were not actually used in computing the gaussian distribution.

In fact, the gaussian result is merely the well known normal distribution which could be evaluated without the use of a computer

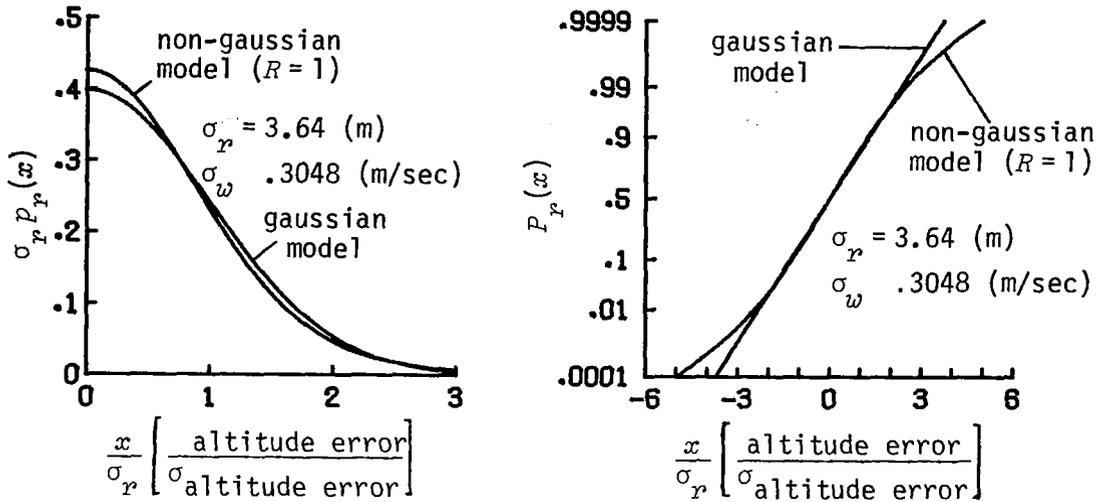


Figure 27.--Probability density and distribution functions of the altitude error in response to both gaussian and non-gaussian turbulence models.

Note that the non-gaussian model predicts that the occurrence of large altitude errors will occupy a much greater percentage of time than indicated by the gaussian model. For example, the computed values of the gaussian and non-gaussian distribution functions for a response magnitude of $3.5\sigma_r$ are 0.99977 and 0.99860 respectively. These numbers imply that the absolute value of the altitude error predicted by the gaussian model will exceed $3.5\sigma_r$ about 0.05% of the total flight time while the error predicted by the non-gaussian model will exceed this level about 0.28% of the time. These predictions differ by more than a factor of five.

Response Level Crossing Frequency

Consider now the level crossing frequency of the response. This calculation is in three parts,

- 1) compute the significant eigenvalues and eigenfunctions of the altitude error by means of program EIGVW
- 2) compute the significant eigenvalues and eigenfunctions of the altitude error derivative by means of program EIGVW
- 3) use the results of the first two steps as input to program RLEVX and obtain the level crossing frequency of the altitude error.

The eigensolutions of the altitude error have already been calculated, and are presented in figure 26. The eigensolutions of the altitude error derivative can be calculated using equations 193 and 194 as input data to program EIGVW. The results are presented in figure 28.

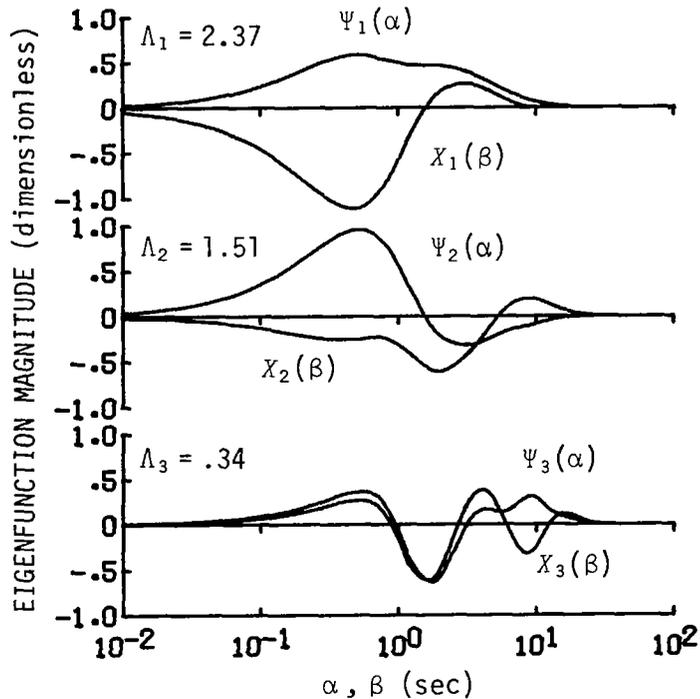


Figure 28.--Adjoint eigenfunctions and eigenvalues of the altitude error time derivative.

All of these eigenvalues and eigenfunctions (Figs. 26 and 28) are now used with program RLEVX to compute the level crossing frequency curves presented in figure 29. It should be noted that the gaussian result

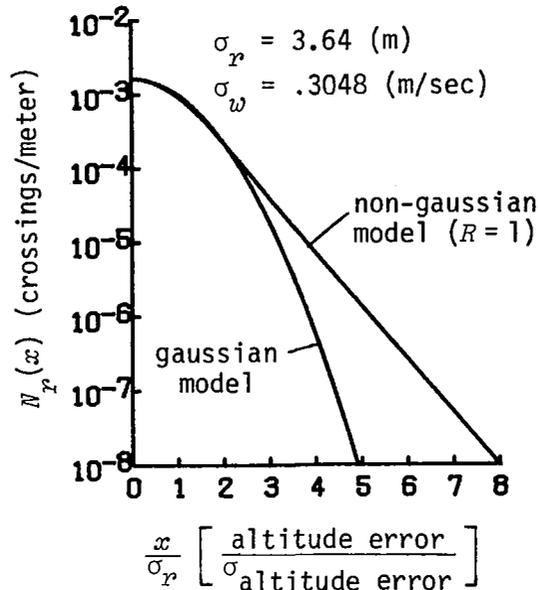


Figure 29.--Level crossing frequency of the altitude error in response to both gaussian and non-gaussian turbulence models.

follows directly from Rice's equation (Eq. 12), and could have been obtained without use of a computer.

Note that the non-gaussian model predicts that large altitude errors will occur much more frequently than indicated by the gaussian model. For example, the crossing frequencies of the $4\sigma_r$ response level predicted by the gaussian and non-gaussian models are 4.274×10^{-7} and 7.116×10^{-6} crossings per meter respectively. The gaussian value implies that the $4\sigma_r$ level will be exceeded once every 2,340 kilometers, while the non-gaussian value implies that this level will be exceeded once every 141 kilometers. These results differ by a factor of almost 17. Furthermore, the ratio of the two results increases rapidly with increasing response magnitude. At the $5\sigma_r$ level they differ by a factor of well over 100.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The previous sections of this report have developed a new model of atmospheric turbulence which is proposed for use in aircraft design work. In this section, the principal conclusions are summarized and some areas requiring further investigation are discussed.

Principal Conclusions

This report has reviewed the problem of modeling continuous atmospheric turbulence for the purposes of aircraft design. The model now in wide use; which assumes turbulence to be a homogeneous, stationary gaussian process with a specified power spectral density; has been discussed and its properties compared with experimentally measured characteristics of atmospheric turbulence. This comparison has shown that the gaussian model does not properly reproduce the number of high velocity gusts which are encountered in the atmosphere. Neither can it properly model the observed patchy character of turbulence or the experimentally measured distributions of velocity increments.

A modified form of the gaussian model, the gaussian patch model, has also been considered. This model is similar to the gaussian model, but assumes large regions of turbulence to be composed of independent patches which are homogeneous, stationary, and gaussian. The intensity of the turbulence varies randomly from patch to patch, but is assumed constant within each patch. The patch size is assumed to be sufficiently great that transient effects between patches can be neglected. It has been shown that this model cannot reproduce the distributions of velocity increments measured in turbulence. Furthermore, experimental measurement of patch sizes implied by this model has shown that the most intense

patches are so short (on the order of 2-3 km) that in many cases the neglect of transient effects between patches cannot be justified.

A non-gaussian model of turbulence has been proposed. This model has been formulated from the idea of representing the patchy character of turbulence by a product of independent gaussian processes. The new model can be viewed as an extended or generalized form of the gaussian model, and the gaussian model is included as a special case.

The statistical properties of the proposed model have been derived and compared with some experimentally measured properties of low altitude atmospheric turbulence. The results have shown that the power spectral densities, probability distributions, and level crossing frequencies observed in the atmosphere can be modeled quite well. However, it has also been shown that this model, like the gaussian and gaussian patch models, does not reproduce the velocity increment distributions of turbulence. The reason for this difficulty has been discussed and a modification of the model has been suggested which may correct the problem. Until a solution is found, it is suggested that the proposed model (as well as the gaussian and gaussian patch models discussed above) is not a good representation of the high frequency components of turbulence. Hence, these models are probably not suitable for studies of high frequency vehicle responses such as those involving structural modes.

Methods by which the non-gaussian model can be used to calculate vehicle response statistics have been investigated and techniques for computing the power spectral density, probability distribution, and level crossing frequency of linear responses to the model have been developed. These results have shown that the response power spectral density can be obtained by the usual methods of linear system theory, and therefore

calculated without difficulty. Evaluation of response probability distributions and level crossing frequencies, on the other hand, has required calculation of the eigenfunctions and eigenvalues of a certain kernel function which is, in general, unsymmetric. Computer programs presented in Appendix A of this report have been developed as a means of automating these calculations.

The altitude response statistics of a simple, linear, STOL aircraft-autopilot system subjected to the vertical component of the non-gaussian model have been calculated. The results have shown that, compared to the gaussian model, the non-gaussian model may predict more than a hundredfold increase in the occurrences of large response magnitudes.

Suggestions for Further Research

The turbulence model and application methods developed in this report can be considered as only the first step in producing a practical tool for use in aircraft design work. Additional research in several areas will be required if this goal is ever to be attained. The following is a list of several areas which the author believes to be topics of useful research regarding this turbulence model.

Improved Data Fitting Procedures

It has been shown that the LO-LOCAT data presented in this report can apparently be fit better by the non-gaussian turbulence model than by the currently used gaussian model. The results of Table 4, however, indicates that it is important to choose the correct goodness-of-fit criterion if satisfactory results are to be obtained. Thus some rational

procedure for obtaining the best over-all fit of the data must be found. This procedure would have application not only to the non-gaussian turbulence model described here, but also to any other turbulence models which might be proposed in the future.

Relationship of the Non-gaussian Model to Meteorological Parameters

The proposed non-gaussian turbulence model contains a parameter which controls its probability distribution and level crossing frequency. Results presented previously in this report (Table 4) indicate that the value of this parameter which leads to the best fit of experimental turbulence data depends, at least partially, on variables such as atmospheric stability, surface roughness, and height above the surface. It should be possible to carry out a regression analysis to determine the relationship between these variables and the parameter. In order to be of practical value, this analysis will have to consider much more data than used in the present report.

Improved Mathematical Development

The results derived in this report have relied very heavily upon heuristic assumptions such as the use of band limited white noise and truncated impulse response functions. These assumptions can surely be relaxed in a rigorous treatment. Although increased rigor would not be expected to alter any of the results which have been presented, it would almost certainly lead to a better understanding of the non-gaussian model and a greater appreciation of its limitations.

An especially useful result of such an investigation might be a rapid procedure for obtaining approximate solutions for response statistics.

It is intuitively obvious, and easy to show from the development of equations 144 through 189, that the vehicle response statistics become identical to the statistics of the turbulence as the bandwidth of the vehicle transfer function becomes infinitely wide with respect to the bandwidth of the turbulence. On the other hand, as the vehicle bandwidth becomes indefinitely narrow with respect to that of the turbulence, the Gerschgorin Circle Theorem concerning upper bounds of eigenvalues (Ref. 49) requires that the non-gaussian portion of the vehicle response become the sum of an infinite number of independent, identically distributed random variables. Hence, the Central Limit Theorem (Ref. 37) requires that this response become gaussian, and therefore the total response of the vehicle will become gaussian. It would be extremely useful to develop a method for estimating (or at least bounding) response statistics for the case of intermediate bandwidth vehicles.

Extensions of the Model

Several possible improvements of the turbulence model proposed in this report merit further investigation.

- 1) Reference 45 has suggested the possibility of defining a patchiness parameter for the non-gaussian model. It would be interesting to see if this parameter could be related to meteorological variables of turbulence, and if this parameter has any significant effect upon response statistics.
- 2) Nothing in the mathematical development presented here has prohibited the use of linear filters with irrational transfer functions in the turbulence model. This

suggests the possibility of using such filters to obtain the von Karman power spectral densities rather than the Dryden spectra assumed in the present report.

- 3) The present results are concerned only with a "point" model of turbulence, that is, the turbulence field is represented by three orthogonal gust components which are assumed to act at the vehicle center of gravity. Turbulence, however, is a distributed phenomenon; and for large aircraft it is important to take into account the distribution of the gust field over the surface of the vehicle. It appears that the spatial distribution representation used in reference 34 can, in principal, be incorporated into the non-gaussian model of this report without difficulty. This could be accomplished by adding an independent rolling gust component to the present model and using the vertical and lateral gust components of the present model in conjunction with an appropriate time delay to represent gusts occurring at the vehicle tail. Unfortunately, the computational procedures required to calculate vehicle responses to this model would become very unwieldy because it would be necessary to consider correlated inputs to the system rather than independent inputs as assumed in this report. Thus, either a different approach to the spatial distribution problem will have to be found, or the computational procedures will have to be greatly speeded.

- 4) It has been shown that the velocity increment distribution of the non-gaussian model do not match those observed in the atmosphere. As discussed previously in this report (page 84), it appears that this problem might be corrected by modifying the transfer functions used in the model.

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APPENDIX A
COMPUTER PROGRAMS

The purpose of this appendix is to present a number of computer programs which are of use in working with the non-gaussian model, and to give examples of their use. Programs to be presented are:

1. PDIST - a program to compute probability distributions of either the non-gaussian model or vehicle responses to the model
2. LEVXNG - a program to compute universal level crossing frequency curves for the non-gaussian model
3. INCPD - a program to compute probability distributions of the non-gaussian model velocity increments
4. EIGU - a program to compute eigenvalues and eigenfunctions of vehicle responses to the longitudinal component of the non-gaussian model
5. EIGVW - a program to compute eigenvalues and adjoint eigenfunctions of vehicle responses to the vertical or lateral components of the non-gaussian model
6. RLEVX - a program to compute level crossing frequency curves of vehicle responses to the non-gaussian model.

Listings of all programs and subroutines will be found at the end of this appendix. All coding is in FORTRAN language, version 2.3 for Control Data 6000 Series Computer Systems. Modifications required for compatibility with other systems should be slight.

The sample programs to be presented here were run on the University of Washington CDC-6400 computer using the U of W version of the Control

Data RUN compiler, and the SCOPE 3.4 operating system. Storage requirements and execution times reflect this operating environment.

Numerous comment cards have been incorporated into each of the card decks presented here as an aid in understanding their operation. The input data required by each program is also described by comment cards.

Program PDIST

PDIST is a program which computes probability distributions for both the non-gaussian turbulence model and vehicle responses to the model. The example presented here calculates the probability distribution of the model for an R parameter value of 1.0.

Card Decks

The following card decks were required to produce the sample calculation presented here:

PDIST

FFT

FFTRS

Input Data

The following five data cards were used in producing this example. The meaning and format of these cards are explained by comments in the listing of program PDIST.

| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 |
|--|------------------|----|----|----|----|----|----|
| TEST OF PDIST, PROBABILITY DISTRIBUTION OF NON-GAUSSIAN MODEL FOR R PARAM. = 1.0 | | | | | | | |
| 1 | 1.0000000000E+00 | | | | | | |
| 1.0 | 1.0000000000E+00 | | | | | | |

Results

The above named card decks were compiled, loaded, and executed on the University of Washington CDC-6400 computer system. Storage requirement including I/O buffers and all system programs was 16,406₈ words. Execution, including compilation and loading, required 3₁₀ seconds of central processor time. The following output was generated.

```

TEST OF PDIST, PROBABILITY DISTRIBUTION OF NON-GAUSSIAN MODEL FOR R PARAM. = 1.0
VARIANCE = 1.00000
NUMBER OF NON-GAUSSIAN EIGENVALUES = 1
EIGENVALUES ARE:
    1.00000000E+00

SIGMA RATIO OF TURBULENCE MODEL = 1.00000

SUM OF EIGENVALUES SQUARED = 1.0000000E+00

FOR VARIANCE OF 1.0000000E+00
NON-GAUSSIAN VARIANCE = 5.0000000E-01
GAUSSIAN VARIANCE INCLUDING CORRECTION FOR
NEGLECTED EIGENVALUES = 5.0000000E-01

FOR TOTAL VARIANCE OF UNITY
NON-GAUSSIAN VARIANCE = 5.0000000E-01
GAUSSIAN VARIANCE INCLUDING CORRECTION FOR
NEGLECTED EIGENVALUES = 5.0000000E-01

SQUARED EIGENVALUES SCALED TO GIVE CORRECT NON-GAUSSIAN
CONTRIBUTION TO UNIT VARIANCE:
    5.0000000E-01

```

TEST OF PDIST, PROBABILITY DISTRIBUTION OF NON-GAUSSIAN MODEL FOR R PARAM. = 1.0

| NORMALIZED VARIABLE X/SIGMA X | STOIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTDIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 0.000 | 4.455E-01 | 5.60000E-01 | 0. | 4.455E-01 |
| .050 | 4.447E-01 | 5.22262E-01 | 5.000E-02 | 4.447E-01 |
| .100 | 4.423E-01 | 5.44445E-01 | 1.000E-01 | 4.423E-01 |
| .150 | 4.384E-01 | 5.66469E-01 | 1.500E-01 | 4.384E-01 |
| .200 | 4.330E-01 | 5.88260E-01 | 2.000E-01 | 4.330E-01 |
| .250 | 4.261E-01 | 6.09741E-01 | 2.500E-01 | 4.261E-01 |
| .300 | 4.178E-01 | 6.30844E-01 | 3.000E-01 | 4.178E-01 |
| .350 | 4.083E-01 | 6.51503E-01 | 3.500E-01 | 4.083E-01 |
| .400 | 3.976E-01 | 6.71656E-01 | 4.000E-01 | 3.976E-01 |
| .450 | 3.859E-01 | 6.91249E-01 | 4.500E-01 | 3.859E-01 |
| .500 | 3.733E-01 | 7.10231E-01 | 5.000E-01 | 3.733E-01 |
| .550 | 3.598E-01 | 7.28561E-01 | 5.500E-01 | 3.598E-01 |
| .600 | 3.457E-01 | 7.46202E-01 | 6.000E-01 | 3.457E-01 |
| .650 | 3.311E-01 | 7.63125E-01 | 6.500E-01 | 3.311E-01 |
| .700 | 3.161E-01 | 7.79306E-01 | 7.000E-01 | 3.161E-01 |
| .750 | 3.008E-01 | 7.94730E-01 | 7.500E-01 | 3.008E-01 |
| .800 | 2.854E-01 | 8.09387E-01 | 8.000E-01 | 2.854E-01 |
| .850 | 2.700E-01 | 8.23274E-01 | 8.500E-01 | 2.700E-01 |
| .900 | 2.547E-01 | 8.36392E-01 | 9.000E-01 | 2.547E-01 |
| .950 | 2.396E-01 | 8.48749E-01 | 9.500E-01 | 2.396E-01 |
| 1.000 | 2.248E-01 | 8.60357E-01 | 1.000E+00 | 2.248E-01 |
| 1.050 | 2.103E-01 | 8.71234E-01 | 1.050E+00 | 2.103E-01 |
| 1.100 | 1.963E-01 | 8.81399E-01 | 1.100E+00 | 1.963E-01 |
| 1.150 | 1.828E-01 | 8.90876E-01 | 1.150E+00 | 1.828E-01 |
| 1.200 | 1.699E-01 | 8.99691E-01 | 1.200E+00 | 1.699E-01 |
| 1.250 | 1.575E-01 | 9.07872E-01 | 1.250E+00 | 1.575E-01 |
| 1.300 | 1.457E-01 | 9.15450E-01 | 1.300E+00 | 1.457E-01 |
| 1.350 | 1.346E-01 | 9.22455E-01 | 1.350E+00 | 1.346E-01 |
| 1.400 | 1.241E-01 | 9.28919E-01 | 1.400E+00 | 1.241E-01 |
| 1.450 | 1.142E-01 | 9.34874E-01 | 1.450E+00 | 1.142E-01 |
| 1.500 | 1.050E-01 | 9.40352E-01 | 1.500E+00 | 1.050E-01 |
| 1.550 | 9.638E-02 | 9.45384E-01 | 1.550E+00 | 9.638E-02 |
| 1.600 | 8.838E-02 | 9.50001E-01 | 1.600E+00 | 8.838E-02 |
| 1.650 | 8.096E-02 | 9.54232E-01 | 1.650E+00 | 8.096E-02 |
| 1.700 | 7.410E-02 | 9.58106E-01 | 1.700E+00 | 7.410E-02 |
| 1.750 | 6.777E-02 | 9.61650E-01 | 1.750E+00 | 6.777E-02 |
| 1.800 | 6.194E-02 | 9.64891E-01 | 1.800E+00 | 6.194E-02 |
| 1.850 | 5.659E-02 | 9.67852E-01 | 1.850E+00 | 5.659E-02 |
| 1.900 | 5.168E-02 | 9.70557E-01 | 1.900E+00 | 5.168E-02 |
| 1.950 | 4.719E-02 | 9.73027E-01 | 1.950E+00 | 4.719E-02 |
| 2.000 | 4.309E-02 | 9.75283E-01 | 2.000E+00 | 4.309E-02 |
| 2.050 | 3.934E-02 | 9.77342E-01 | 2.050E+00 | 3.934E-02 |
| 2.100 | 3.592E-02 | 9.79223E-01 | 2.100E+00 | 3.592E-02 |
| 2.150 | 3.281E-02 | 9.80940E-01 | 2.150E+00 | 3.281E-02 |
| 2.200 | 2.997E-02 | 9.82508E-01 | 2.200E+00 | 2.997E-02 |
| 2.250 | 2.739E-02 | 9.83941E-01 | 2.250E+00 | 2.739E-02 |
| 2.300 | 2.503E-02 | 9.85250E-01 | 2.300E+00 | 2.503E-02 |
| 2.350 | 2.289E-02 | 9.86448E-01 | 2.350E+00 | 2.289E-02 |
| 2.400 | 2.094E-02 | 9.87543E-01 | 2.400E+00 | 2.094E-02 |
| 2.450 | 1.917E-02 | 9.88545E-01 | 2.450E+00 | 1.917E-02 |
| 2.500 | 1.756E-02 | 9.89463E-01 | 2.500E+00 | 1.756E-02 |

TEST OF PDIST, PROBABILITY DISTRIBUTION OF NON-GAUSSIAN MODEL FOR R PARAM. = 1.0

| NORMALIZED VARIABLE X/SIGMA X | STOIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTOIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 2.500 | 1.756E-02 | 9.89463E-01 | 2.500E+00 | 1.756E-02 |
| 2.550 | 1.608E-02 | 9.90303E-01 | 2.550E+00 | 1.608E-02 |
| 2.600 | 1.474E-02 | 9.91073E-01 | 2.600E+00 | 1.474E-02 |
| 2.650 | 1.352E-02 | 9.91779E-01 | 2.650E+00 | 1.352E-02 |
| 2.700 | 1.241E-02 | 9.92427E-01 | 2.700E+00 | 1.241E-02 |
| 2.750 | 1.139E-02 | 9.93022E-01 | 2.750E+00 | 1.139E-02 |
| 2.800 | 1.046E-02 | 9.93568E-01 | 2.800E+00 | 1.046E-02 |
| 2.850 | 9.612E-03 | 9.94069E-01 | 2.850E+00 | 9.612E-03 |
| 2.900 | 8.836E-03 | 9.94530E-01 | 2.900E+00 | 8.836E-03 |
| 2.950 | 8.126E-03 | 9.94954E-01 | 2.950E+00 | 8.126E-03 |
| 3.000 | 7.476E-03 | 9.95344E-01 | 3.000E+00 | 7.476E-03 |
| 3.050 | 6.881E-03 | 9.95702E-01 | 3.050E+00 | 6.881E-03 |
| 3.100 | 6.335E-03 | 9.96033E-01 | 3.100E+00 | 6.335E-03 |
| 3.150 | 5.835E-03 | 9.96337E-01 | 3.150E+00 | 5.835E-03 |
| 3.200 | 5.375E-03 | 9.96617E-01 | 3.200E+00 | 5.375E-03 |
| 3.250 | 4.954E-03 | 9.96875E-01 | 3.250E+00 | 4.954E-03 |
| 3.300 | 4.567E-03 | 9.97113E-01 | 3.300E+00 | 4.567E-03 |
| 3.350 | 4.211E-03 | 9.97332E-01 | 3.350E+00 | 4.211E-03 |
| 3.400 | 3.884E-03 | 9.97534E-01 | 3.400E+00 | 3.884E-03 |
| 3.450 | 3.583E-03 | 9.97721E-01 | 3.450E+00 | 3.583E-03 |
| 3.500 | 3.306E-03 | 9.97893E-01 | 3.500E+00 | 3.306E-03 |
| 3.550 | 3.052E-03 | 9.98052E-01 | 3.550E+00 | 3.052E-03 |
| 3.600 | 2.817E-03 | 9.98199E-01 | 3.600E+00 | 2.817E-03 |
| 3.650 | 2.601E-03 | 9.98334E-01 | 3.650E+00 | 2.601E-03 |
| 3.700 | 2.402E-03 | 9.98459E-01 | 3.700E+00 | 2.402E-03 |
| 3.750 | 2.219E-03 | 9.98574E-01 | 3.750E+00 | 2.219E-03 |
| 3.800 | 2.050E-03 | 9.98681E-01 | 3.800E+00 | 2.050E-03 |
| 3.850 | 1.894E-03 | 9.98780E-01 | 3.850E+00 | 1.894E-03 |
| 3.900 | 1.750E-03 | 9.98871E-01 | 3.900E+00 | 1.750E-03 |
| 3.950 | 1.618E-03 | 9.98955E-01 | 3.950E+00 | 1.618E-03 |
| 4.000 | 1.495E-03 | 9.99033E-01 | 4.000E+00 | 1.495E-03 |
| 4.050 | 1.383E-03 | 9.99104E-01 | 4.050E+00 | 1.383E-03 |
| 4.100 | 1.278E-03 | 9.99171E-01 | 4.100E+00 | 1.278E-03 |
| 4.150 | 1.182E-03 | 9.99232E-01 | 4.150E+00 | 1.182E-03 |
| 4.200 | 1.093E-03 | 9.99289E-01 | 4.200E+00 | 1.093E-03 |
| 4.250 | 1.011E-03 | 9.99342E-01 | 4.250E+00 | 1.011E-03 |
| 4.300 | 9.356E-04 | 9.99391E-01 | 4.300E+00 | 9.356E-04 |
| 4.350 | 8.657E-04 | 9.99436E-01 | 4.350E+00 | 8.657E-04 |
| 4.400 | 8.010E-04 | 9.99477E-01 | 4.400E+00 | 8.010E-04 |
| 4.450 | 7.412E-04 | 9.99516E-01 | 4.450E+00 | 7.412E-04 |
| 4.500 | 6.860E-04 | 9.99551E-01 | 4.500E+00 | 6.860E-04 |
| 4.550 | 6.349E-04 | 9.99584E-01 | 4.550E+00 | 6.349E-04 |
| 4.600 | 5.877E-04 | 9.99615E-01 | 4.600E+00 | 5.877E-04 |
| 4.650 | 5.441E-04 | 9.99643E-01 | 4.650E+00 | 5.441E-04 |
| 4.700 | 5.037E-04 | 9.99669E-01 | 4.700E+00 | 5.037E-04 |
| 4.750 | 4.664E-04 | 9.99694E-01 | 4.750E+00 | 4.664E-04 |
| 4.800 | 4.318E-04 | 9.99716E-01 | 4.800E+00 | 4.318E-04 |
| 4.850 | 3.999E-04 | 9.99737E-01 | 4.850E+00 | 3.999E-04 |
| 4.900 | 3.704E-04 | 9.99756E-01 | 4.900E+00 | 3.704E-04 |
| 4.950 | 3.430E-04 | 9.99774E-01 | 4.950E+00 | 3.430E-04 |
| 5.000 | 3.177E-04 | 9.99791E-01 | 5.000E+00 | 3.177E-04 |

TEST OF POIST, PROBABILITY DISTRIBUTION OF NON-GAUSSIAN MODEL FOR R PARAM. = 1.0

| NORMALIZED VARIABLE X/SIGMA X | STOIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTOIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 5.000 | 3.177E-04 | 9.99791E-01 | 5.000E+00 | 3.177E-04 |
| 5.050 | 2.943E-04 | 9.99806E-01 | 5.050E+00 | 2.943E-04 |
| 5.100 | 2.727E-04 | 9.99820E-01 | 5.100E+00 | 2.727E-04 |
| 5.150 | 2.526E-04 | 9.99833E-01 | 5.150E+00 | 2.526E-04 |
| 5.200 | 2.340E-04 | 9.99845E-01 | 5.200E+00 | 2.340E-04 |
| 5.250 | 2.169E-04 | 9.99857E-01 | 5.250E+00 | 2.169E-04 |
| 5.300 | 2.009E-04 | 9.99867E-01 | 5.300E+00 | 2.009E-04 |
| 5.350 | 1.862E-04 | 9.99877E-01 | 5.350E+00 | 1.862E-04 |
| 5.400 | 1.726E-04 | 9.99886E-01 | 5.400E+00 | 1.726E-04 |
| 5.450 | 1.599E-04 | 9.99894E-01 | 5.450E+00 | 1.599E-04 |
| 5.500 | 1.482E-04 | 9.99902E-01 | 5.500E+00 | 1.482E-04 |
| 5.550 | 1.374E-04 | 9.99909E-01 | 5.550E+00 | 1.374E-04 |
| 5.600 | 1.274E-04 | 9.99915E-01 | 5.600E+00 | 1.274E-04 |
| 5.650 | 1.181E-04 | 9.99921E-01 | 5.650E+00 | 1.181E-04 |
| 5.700 | 1.095E-04 | 9.99927E-01 | 5.700E+00 | 1.095E-04 |
| 5.750 | 1.015E-04 | 9.99932E-01 | 5.750E+00 | 1.015E-04 |
| 5.800 | 9.409E-05 | 9.99937E-01 | 5.800E+00 | 9.409E-05 |
| 5.850 | 8.724E-05 | 9.99942E-01 | 5.850E+00 | 8.724E-05 |
| 5.900 | 8.090E-05 | 9.99946E-01 | 5.900E+00 | 8.090E-05 |
| 5.950 | 7.501E-05 | 9.99950E-01 | 5.950E+00 | 7.501E-05 |
| 6.000 | 6.956E-05 | 9.99954E-01 | 6.000E+00 | 6.956E-05 |
| 6.050 | 6.451E-05 | 9.99957E-01 | 6.050E+00 | 6.451E-05 |
| 6.100 | 5.983E-05 | 9.99960E-01 | 6.100E+00 | 5.983E-05 |
| 6.150 | 5.549E-05 | 9.99963E-01 | 6.150E+00 | 5.549E-05 |
| 6.200 | 5.147E-05 | 9.99966E-01 | 6.200E+00 | 5.147E-05 |
| 6.250 | 4.774E-05 | 9.99968E-01 | 6.250E+00 | 4.774E-05 |
| 6.300 | 4.428E-05 | 9.99970E-01 | 6.300E+00 | 4.428E-05 |
| 6.350 | 4.107E-05 | 9.99973E-01 | 6.350E+00 | 4.107E-05 |
| 6.400 | 3.810E-05 | 9.99974E-01 | 6.400E+00 | 3.810E-05 |
| 6.450 | 3.535E-05 | 9.99976E-01 | 6.450E+00 | 3.535E-05 |
| 6.500 | 3.279E-05 | 9.99978E-01 | 6.500E+00 | 3.279E-05 |
| 6.550 | 3.042E-05 | 9.99980E-01 | 6.550E+00 | 3.042E-05 |
| 6.600 | 2.823E-05 | 9.99981E-01 | 6.600E+00 | 2.823E-05 |
| 6.650 | 2.619E-05 | 9.99982E-01 | 6.650E+00 | 2.619E-05 |
| 6.700 | 2.430E-05 | 9.99984E-01 | 6.700E+00 | 2.430E-05 |
| 6.750 | 2.255E-05 | 9.99985E-01 | 6.750E+00 | 2.255E-05 |
| 6.800 | 2.092E-05 | 9.99986E-01 | 6.800E+00 | 2.092E-05 |
| 6.850 | 1.942E-05 | 9.99987E-01 | 6.850E+00 | 1.942E-05 |
| 6.900 | 1.802E-05 | 9.99988E-01 | 6.900E+00 | 1.802E-05 |
| 6.950 | 1.672E-05 | 9.99989E-01 | 6.950E+00 | 1.672E-05 |
| 7.000 | 1.552E-05 | 9.99990E-01 | 7.000E+00 | 1.552E-05 |
| 7.050 | 1.440E-05 | 9.99990E-01 | 7.050E+00 | 1.440E-05 |
| 7.100 | 1.337E-05 | 9.99991E-01 | 7.100E+00 | 1.337E-05 |
| 7.150 | 1.241E-05 | 9.99992E-01 | 7.150E+00 | 1.241E-05 |
| 7.200 | 1.151E-05 | 9.99992E-01 | 7.200E+00 | 1.151E-05 |
| 7.250 | 1.069E-05 | 9.99993E-01 | 7.250E+00 | 1.069E-05 |
| 7.300 | 9.920E-06 | 9.99993E-01 | 7.300E+00 | 9.920E-06 |
| 7.350 | 9.208E-06 | 9.99994E-01 | 7.350E+00 | 9.208E-06 |
| 7.400 | 8.548E-06 | 9.99994E-01 | 7.400E+00 | 8.548E-06 |
| 7.450 | 7.935E-06 | 9.99995E-01 | 7.450E+00 | 7.935E-06 |
| 7.500 | 7.366E-06 | 9.99995E-01 | 7.500E+00 | 7.366E-06 |

TEST OF PDIST, PROBABILITY DISTRIBUTION OF NON-GAUSSIAN MODEL FOR R PARAM. = 1.0

| NORMALIZED VARIABLE X/SIGMA X | STOIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTOIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 7.500 | 7.366E-06 | 9.99995E-01 | 7.500E+00 | 7.366E-06 |
| 7.550 | 6.838E-06 | 9.99995E-01 | 7.550E+00 | 6.838E-06 |
| 7.600 | 6.348E-06 | 9.99996E-01 | 7.600E+00 | 6.348E-06 |
| 7.650 | 5.894E-06 | 9.99996E-01 | 7.650E+00 | 5.894E-06 |
| 7.700 | 5.472E-06 | 9.99996E-01 | 7.700E+00 | 5.472E-06 |
| 7.750 | 5.080E-06 | 9.99997E-01 | 7.750E+00 | 5.080E-06 |
| 7.800 | 4.717E-06 | 9.99997E-01 | 7.800E+00 | 4.717E-06 |
| 7.850 | 4.380E-06 | 9.99997E-01 | 7.850E+00 | 4.380E-06 |
| 7.900 | 4.067E-06 | 9.99997E-01 | 7.900E+00 | 4.067E-06 |
| 7.950 | 3.776E-06 | 9.99997E-01 | 7.950E+00 | 3.776E-06 |
| 8.000 | 3.506E-06 | 9.99998E-01 | 8.000E+00 | 3.506E-06 |
| 8.050 | 3.256E-06 | 9.99998E-01 | 8.050E+00 | 3.256E-06 |
| 8.100 | 3.023E-06 | 9.99998E-01 | 8.100E+00 | 3.023E-06 |
| 8.150 | 2.808E-06 | 9.99998E-01 | 8.150E+00 | 2.808E-06 |
| 8.200 | 2.607E-06 | 9.99998E-01 | 8.200E+00 | 2.607E-06 |
| 8.250 | 2.421E-06 | 9.99998E-01 | 8.250E+00 | 2.421E-06 |
| 8.300 | 2.249E-06 | 9.99998E-01 | 8.300E+00 | 2.249E-06 |
| 8.350 | 2.088E-06 | 9.99999E-01 | 8.350E+00 | 2.088E-06 |
| 8.400 | 1.939E-06 | 9.99999E-01 | 8.400E+00 | 1.939E-06 |
| 8.450 | 1.801E-06 | 9.99999E-01 | 8.450E+00 | 1.801E-06 |
| 8.500 | 1.673E-06 | 9.99999E-01 | 8.500E+00 | 1.673E-06 |
| 8.550 | 1.554E-06 | 9.99999E-01 | 8.550E+00 | 1.554E-06 |
| 8.600 | 1.443E-06 | 9.99999E-01 | 8.600E+00 | 1.443E-06 |
| 8.650 | 1.340E-06 | 9.99999E-01 | 8.650E+00 | 1.340E-06 |
| 8.700 | 1.245E-06 | 9.99999E-01 | 8.700E+00 | 1.245E-06 |
| 8.750 | 1.156E-06 | 9.99999E-01 | 8.750E+00 | 1.156E-06 |
| 8.800 | 1.074E-06 | 9.99999E-01 | 8.800E+00 | 1.074E-06 |
| 8.850 | 9.978E-07 | 9.99999E-01 | 8.850E+00 | 9.978E-07 |
| 8.900 | 9.269E-07 | 9.99999E-01 | 8.900E+00 | 9.269E-07 |
| 8.950 | 8.610E-07 | 9.99999E-01 | 8.950E+00 | 8.610E-07 |
| 9.000 | 7.999E-07 | 9.99999E-01 | 9.000E+00 | 7.999E-07 |
| 9.050 | 7.430E-07 | 9.99999E-01 | 9.050E+00 | 7.430E-07 |
| 9.100 | 6.903E-07 | 1.00000E+00 | 9.100E+00 | 6.903E-07 |
| 9.150 | 6.413E-07 | 1.00000E+00 | 9.150E+00 | 6.413E-07 |
| 9.200 | 5.957E-07 | 1.00000E+00 | 9.200E+00 | 5.957E-07 |
| 9.250 | 5.535E-07 | 1.00000E+00 | 9.250E+00 | 5.535E-07 |
| 9.300 | 5.142E-07 | 1.00000E+00 | 9.300E+00 | 5.142E-07 |
| 9.350 | 4.777E-07 | 1.00000E+00 | 9.350E+00 | 4.777E-07 |
| 9.400 | 4.438E-07 | 1.00000E+00 | 9.400E+00 | 4.438E-07 |
| 9.450 | 4.124E-07 | 1.00000E+00 | 9.450E+00 | 4.124E-07 |
| 9.500 | 3.831E-07 | 1.00000E+00 | 9.500E+00 | 3.831E-07 |
| 9.550 | 3.560E-07 | 1.00000E+00 | 9.550E+00 | 3.560E-07 |
| 9.600 | 3.308E-07 | 1.00000E+00 | 9.600E+00 | 3.308E-07 |
| 9.650 | 3.073E-07 | 1.00000E+00 | 9.650E+00 | 3.073E-07 |
| 9.700 | 2.856E-07 | 1.00000E+00 | 9.700E+00 | 2.856E-07 |
| 9.750 | 2.653E-07 | 1.00000E+00 | 9.750E+00 | 2.653E-07 |
| 9.800 | 2.466E-07 | 1.00000E+00 | 9.800E+00 | 2.466E-07 |
| 9.850 | 2.291E-07 | 1.00000E+00 | 9.850E+00 | 2.291E-07 |
| 9.900 | 2.129E-07 | 1.00000E+00 | 9.900E+00 | 2.129E-07 |
| 9.950 | 1.978E-07 | 1.00000E+00 | 9.950E+00 | 1.978E-07 |
| 10.000 | 1.838E-07 | 1.00000E+00 | 1.000E+01 | 1.838E-07 |

Program LEVXNG

LEVXNG is a program which computes universal level crossing frequency curves for the non-gaussian turbulence model. The example presented here shows the calculation of the u -gust curve for an R parameter value of 1.0.

Card Decks

The following card decks were required to produce the sample calculation presented here:

LEVXNG

CF2

INVR

FFT

Input Data

Program LEVXNG is written so as to require no input data from punched cards. The functioning of the program is determined by DATA statements as described by comment cards in the deck listing.

Results

The above named card decks were compiled, loaded, and executed on the University of Washington CDC-6400 computer system. Storage requirement including I/O buffers and all system programs was 40,500₈ words. Execution, including compilation and loading, required 19₁₀ seconds of central processor time. The following output was generated.

COVARIANCE MATRICIES OF FIRST AND SECOND
GAUSSIAN VECTORS ARE

7.071E-01 0.
0. 3.536E-01
7.071E-01 0.
0. 3.536E-01

FUNCTIONAL RELATIONSHIP MATRICIES FOR FIRST
AND SECOND TRANSFORM VARIABLES ARE

1.000E+00 0.
0. 0.
0. 1.000E+00
1.000E+00 0.

R PARAMETER = 1.000

DETERMINANTS OF COVARIANCE MATRICIES

DETA = 2.50000E-01
DETB = 2.50000E-01

X INCREMENTS: DX(1) = 3.0000000E-01 DX(2) = 3.0000000E-01
F INCREMENTS: DF(1) = 5.2083333E-02 DF(2) = 5.2083333E-02

FIRST ROW, FIRST COLUMN, AND DIAGONAL OF
JOINT CHARACTERISTIC FUNCTION

| | | | |
|--------------|--------------|--------------|--------------|
| 1.000E+00 0. | 9.485E-01 0. | 8.154E-01 0. | 6.456E-01 0. |
| 4.782E-01 0. | 3.348E-01 0. | 2.229E-01 0. | 1.415E-01 0. |
| 8.566E-02 0. | 4.949E-02 0. | 2.727E-02 0. | 1.433E-02 0. |
| 7.172E-03 0. | 3.419E-03 0. | 1.552E-03 0. | 6.700E-04 0. |
| 2.752E-04 0. | 1.875E-04 0. | 3.990E-05 0. | 1.408E-05 0. |
| 4.718E-06 0. | 1.502E-06 0. | 4.543E-07 0. | 1.305E-07 0. |
| 3.557E-08 0. | 9.209E-09 0. | 2.263E-09 0. | 5.277E-10 0. |
| 1.168E-10 0. | 2.454E-11 0. | 4.896E-12 0. | 9.534E-13 0. |
| 3.320E-13 0. | 9.534E-13 0. | 4.896E-12 0. | 2.454E-11 0. |
| 1.168E-10 0. | 5.277E-10 0. | 2.263E-09 0. | 9.209E-09 0. |
| 3.557E-08 0. | 1.305E-07 0. | 4.543E-07 0. | 1.502E-06 0. |
| 4.718E-06 0. | 1.408E-05 0. | 3.990E-05 0. | 1.075E-04 0. |
| 2.752E-04 0. | 6.700E-04 0. | 1.552E-03 0. | 3.419E-03 0. |
| 7.172E-03 0. | 1.433E-02 0. | 2.727E-02 0. | 4.949E-02 0. |
| 8.566E-02 0. | 1.415E-01 0. | 2.229E-01 0. | 3.348E-01 0. |
| 4.782E-01 0. | 6.456E-01 0. | 8.154E-01 0. | 9.485E-01 0. |
| | | | |
| 1.000E+00 0. | 9.482E-01 0. | 8.115E-01 0. | 6.333E-01 0. |
| 4.562E-01 0. | 3.067E-01 0. | 1.942E-01 0. | 1.165E-01 0. |
| 6.642E-02 0. | 3.608E-02 0. | 1.870E-02 0. | 9.242E-03 0. |
| 4.360E-03 0. | 1.562E-03 0. | 8.420E-04 0. | 3.446E-04 0. |
| 1.344E-04 0. | 4.992E-05 0. | 1.766E-05 0. | 5.951E-06 0. |
| 1.908E-06 0. | 5.219E-07 0. | 1.689E-07 0. | 4.659E-08 0. |
| 1.222E-08 0. | 3.849E-09 0. | 7.226E-10 0. | 1.627E-10 0. |
| 3.493E-11 0. | 7.080E-12 0. | 1.368E-12 0. | 2.579E-13 0. |
| 8.731E-14 0. | 2.579E-13 0. | 1.368E-12 0. | 7.080E-12 0. |
| 3.483E-11 0. | 1.627E-10 0. | 7.226E-10 0. | 3.849E-09 0. |
| 1.222E-08 0. | 4.659E-08 0. | 1.689E-07 0. | 5.819E-07 0. |
| 1.908E-06 0. | 5.951E-06 0. | 1.766E-05 0. | 4.992E-05 0. |
| 1.344E-04 0. | 3.446E-04 0. | 8.420E-04 0. | 1.962E-03 0. |
| 4.360E-03 0. | 9.242E-03 0. | 1.870E-02 0. | 3.608E-02 0. |
| 6.642E-02 0. | 1.165E-01 0. | 1.942E-01 0. | 3.067E-01 0. |
| 4.562E-01 0. | 6.333E-01 0. | 8.115E-01 0. | 9.482E-01 0. |
| | | | |
| 1.000E+00 0. | 9.006E-01 0. | 6.727E-01 0. | 4.343E-01 0. |
| 2.494E-01 0. | 1.291E-01 0. | 6.049E-02 0. | 2.569E-02 0. |
| 9.890E-03 0. | 3.448E-03 0. | 1.088E-03 0. | 3.105E-04 0. |
| 8.010E-05 0. | 1.867E-05 0. | 3.932E-06 0. | 7.476E-07 0. |
| 1.283E-07 0. | 1.986E-08 0. | 2.773E-09 0. | 3.491E-10 0. |
| 3.963E-11 0. | 4.055E-12 0. | 3.739E-13 0. | 3.106E-14 0. |
| 2.325E-15 0. | 1.567E-16 0. | 9.511E-18 0. | 5.199E-19 0. |
| 2.558E-20 0. | 1.133E-21 0. | 4.528E-23 0. | 1.723E-24 0. |
| 2.096E-25 0. | 1.723E-24 0. | 4.528E-23 0. | 1.133E-21 0. |
| 2.558E-20 0. | 5.199E-19 0. | 9.511E-18 0. | 1.567E-16 0. |
| 2.325E-15 0. | 3.106E-14 0. | 3.739E-13 0. | 4.055E-12 0. |
| 3.963E-11 0. | 3.491E-10 0. | 2.773E-09 0. | 1.986E-08 0. |
| 1.283E-07 0. | 7.476E-07 0. | 3.932E-06 0. | 1.867E-05 0. |
| 8.010E-05 0. | 3.105E-04 0. | 1.088E-03 0. | 3.448E-03 0. |
| 9.890E-03 0. | 2.569E-02 0. | 6.049E-02 0. | 1.291E-01 0. |
| 2.494E-01 0. | 4.343E-01 0. | 6.727E-01 0. | 9.006E-01 0. |

FIRST ROW, FIRST COLUMN, AND DIAGONAL OF
JOINT PROBABILITY DENSITY

| | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 2.010E-01 0. | 1.876E-01 | 1.585E-14 | 1.530E-01 | 5.968E-14 | 1.098E-01 | 2.787E-14 |
| 7.043E-02-1.121E-14 | 4.124E-02-8.940E-14 | 2.272E-02-7.865E-15 | 1.219E-02 | 5.151E-14 | 6.565E-03-2.791E-15 | 3.615E-03 4.223E-14 |
| 6.565E-03-2.791E-15 | 3.615E-03 4.223E-14 | 2.044E-03 8.691E-15 | 1.182E-03-4.116E-16 | 6.955E-04-5.759E-16 | 4.142E-04-8.974E-14 | 2.490E-04-6.401E-14 |
| 6.955E-04-5.759E-16 | 4.142E-04-8.974E-14 | 2.490E-04-6.401E-14 | 1.508E-04-8.972E-15 | 9.190E-05-1.665E-16 | 5.629E-05-2.205E-14 | 3.462E-05-1.613E-13 |
| 9.190E-05-1.665E-16 | 5.629E-05-2.205E-14 | 3.462E-05-1.613E-13 | 2.138E-05-8.330E-14 | 1.324E-05-3.469E-17 | 8.230E-06 4.884E-14 | 5.127E-06 2.303E-14 |
| 1.324E-05-3.469E-17 | 8.230E-06 4.884E-14 | 5.127E-06 2.303E-14 | 3.202E-06-4.956E-15 | 2.005E-06-1.549E-17 | 1.258E-06 1.657E-14 | 7.924E-07 1.142E-14 |
| 2.005E-06-1.549E-17 | 1.258E-06 1.657E-14 | 7.924E-07 1.142E-14 | 1.448E-07-2.652E-14 | 3.209E-07-1.041E-17 | 2.102E-07 2.647E-14 | 1.448E-07-2.652E-14 |
| 3.209E-07-1.041E-17 | 2.102E-07 2.647E-14 | 1.448E-07-2.652E-14 | 1.106E-07-9.477E-15 | 9.992E-08 0. | 1.106E-07-9.477E-15 | 1.448E-07-2.652E-14 |
| 9.992E-08 0. | 1.106E-07-9.477E-15 | 1.448E-07-2.652E-14 | 7.924E-07-1.142E-14 | 3.209E-07 1.041E-17 | 5.016E-07 3.228E-14 | 7.924E-07-1.142E-14 |
| 3.209E-07 1.041E-17 | 5.016E-07 3.228E-14 | 7.924E-07-1.142E-14 | 1.258E-06-1.657E-14 | 2.005E-06 1.549E-17 | 3.202E-06-4.956E-15 | 5.127E-06-2.303E-14 |
| 2.005E-06 1.549E-17 | 3.202E-06-4.956E-15 | 5.127E-06-2.303E-14 | 8.230E-06-4.884E-14 | 1.324E-05 3.469E-17 | 2.138E-05 8.330E-14 | 3.462E-05 1.613E-13 |
| 1.324E-05 3.469E-17 | 2.138E-05 8.330E-14 | 3.462E-05 1.613E-13 | 5.629E-05-2.205E-14 | 9.190E-05 1.665E-16 | 1.508E-04 8.972E-15 | 2.490E-04 6.401E-14 |
| 9.190E-05 1.665E-16 | 1.508E-04 8.972E-15 | 2.490E-04 6.401E-14 | 4.142E-04-8.974E-14 | 6.955E-04 5.759E-16 | 1.182E-03 4.116E-16 | 2.044E-03-8.691E-15 |
| 6.955E-04 5.759E-16 | 1.182E-03 4.116E-16 | 2.044E-03-8.691E-15 | 3.615E-03-4.223E-14 | 7.043E-02 1.121E-14 | 1.219E-02-7.865E-15 | 1.530E-01-5.968E-14 |
| 7.043E-02 1.121E-14 | 1.219E-02-7.865E-15 | 1.530E-01-5.968E-14 | 1.098E-01 2.787E-14 | 2.010E-01 0. | 1.876E-01 1.585E-14 | 1.530E-01 5.968E-14 |

| | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 2.010E-01 0. | 1.889E-01 | 1.310E-14 | 1.573E-01 | 6.195E-14 | 1.166E-01 | 3.267E-14 |
| 7.766E-02-1.188E-14 | 4.709E-02-8.896E-14 | 2.643E-02-9.276E-15 | 1.399E-02 | 4.908E-14 | 7.106E-03-3.124E-15 | 3.523E-03 3.880E-14 |
| 7.106E-03-3.124E-15 | 3.523E-03 3.880E-14 | 1.725E-03 1.035E-14 | 8.404E-04 6.580E-15 | 4.097E-04-4.996E-16 | 2.004E-04-7.192E-14 | 9.849E-05-6.581E-14 |
| 4.097E-04-4.996E-16 | 2.004E-04-7.192E-14 | 9.849E-05-6.581E-14 | 4.872E-05-1.049E-14 | 2.427E-05 0. | 1.217E-05-2.148E-14 | 6.144E-06-1.660E-13 |
| 2.427E-05 0. | 1.217E-05-2.148E-14 | 6.144E-06-1.660E-13 | 3.121E-06-8.840E-14 | 1.595E-06 5.551E-17 | 8.192E-07 5.382E-14 | 4.227E-07 2.695E-14 |
| 1.595E-06 5.551E-17 | 8.192E-07 5.382E-14 | 4.227E-07 2.695E-14 | 2.190E-07 4.252E-15 | 1.138E-07-1.578E-17 | 5.937E-08 1.720E-14 | 3.106E-08 1.298E-14 |
| 1.138E-07-1.578E-17 | 5.937E-08 1.720E-14 | 3.106E-08 1.298E-14 | 1.630E-08-3.294E-14 | 8.601E-09 0. | 4.599E-09 2.872E-14 | 2.559E-09-2.726E-14 |
| 8.601E-09 0. | 4.599E-09 2.872E-14 | 2.559E-09-2.726E-14 | 1.605E-09-8.551E-15 | 1.326E-09 0. | 1.605E-09-8.551E-15 | 2.559E-09-2.726E-14 |
| 1.326E-09 0. | 1.605E-09-8.551E-15 | 2.559E-09-2.726E-14 | 4.599E-09-2.872E-14 | 8.601E-09 0. | 1.630E-08 3.294E-14 | 3.106E-08-1.298E-14 |
| 8.601E-09 0. | 1.630E-08 3.294E-14 | 3.106E-08-1.298E-14 | 5.937E-08-1.720E-14 | 1.138E-07 1.578E-17 | 2.190E-07-4.252E-15 | 4.227E-07-2.695E-14 |
| 1.138E-07 1.578E-17 | 2.190E-07-4.252E-15 | 4.227E-07-2.695E-14 | 8.192E-07 5.382E-14 | 1.595E-06-5.551E-17 | 3.121E-06 8.840E-14 | 6.144E-06 1.660E-13 |
| 1.595E-06-5.551E-17 | 3.121E-06 8.840E-14 | 6.144E-06 1.660E-13 | 1.217E-05 2.148E-14 | 2.427E-05 0. | 4.872E-05 1.049E-14 | 9.849E-05 6.581E-14 |
| 2.427E-05 0. | 4.872E-05 1.049E-14 | 9.849E-05 6.581E-14 | 2.004E-04 7.192E-14 | 4.097E-04 4.996E-16 | 8.404E-04-6.580E-15 | 1.725E-03-1.035E-14 |
| 4.097E-04 4.996E-16 | 8.404E-04-6.580E-15 | 1.725E-03-1.035E-14 | 3.523E-03-3.880E-14 | 7.106E-03 3.124E-15 | 1.399E-02-4.908E-14 | 2.643E-02 9.276E-15 |
| 7.106E-03 3.124E-15 | 1.399E-02-4.908E-14 | 2.643E-02 9.276E-15 | 1.166E-01-3.267E-14 | 7.766E-02 1.188E-14 | 1.166E-01-3.267E-14 | 1.573E-01-6.195E-14 |
| 7.766E-02 1.188E-14 | 1.166E-01-3.267E-14 | 1.573E-01-6.195E-14 | 1.889E-01 1.310E-14 | 2.010E-01 0. | 1.765E-01 2.591E-14 | 1.206E-01 8.954E-14 |

| | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 2.010E-01 0. | 1.765E-01 | 2.591E-14 | 1.206E-01 | 8.954E-14 | 6.625E-02 | 2.759E-14 |
| 3.097E-02-2.301E-14 | 1.335E-02-5.181E-14 | 5.719E-03-1.307E-14 | 2.533E-03-3.073E-15 | 1.163E-03-5.733E-15 | 5.471E-04-3.705E-15 | 2.618E-04-1.515E-15 |
| 1.163E-03-5.733E-15 | 5.471E-04-3.705E-15 | 2.618E-04-1.515E-15 | 1.268E-04-5.384E-16 | 6.193E-05-9.759E-16 | 3.047E-05-9.785E-16 | 1.508E-05-5.539E-16 |
| 6.193E-05-9.759E-16 | 3.047E-05-9.785E-16 | 1.508E-05-5.539E-16 | 7.501E-06-2.129E-16 | 3.747E-06-2.488E-16 | 1.878E-06-9.795E-17 | 9.444E-07-4.088E-17 |
| 3.747E-06-2.488E-16 | 1.878E-06-9.795E-17 | 9.444E-07-4.088E-17 | 6.198E-08 2.876E-17 | 2.408E-07-4.987E-17 | 1.220E-07-3.266E-17 | 6.198E-08 2.876E-17 |
| 2.408E-07-4.987E-17 | 1.220E-07-3.266E-17 | 6.198E-08 2.876E-17 | 3.156E-08-1.610E-17 | 1.612E-08 7.875E-18 | 8.275E-09 2.700E-17 | 4.284E-09 4.009E-18 |
| 1.612E-08 7.875E-18 | 8.275E-09 2.700E-17 | 4.284E-09 4.009E-18 | 2.256E-09-3.290E-18 | 1.227E-09-7.660E-18 | 7.088E-10-3.888E-17 | 4.537E-10-1.575E-17 |
| 1.227E-09-7.660E-18 | 7.088E-10-3.888E-17 | 4.537E-10-1.575E-17 | 3.380E-10 1.063E-17 | 3.056E-10 0. | 3.380E-10 1.063E-17 | 4.537E-10 1.575E-17 |
| 3.056E-10 0. | 3.380E-10 1.063E-17 | 4.537E-10 1.575E-17 | 8.275E-09-2.700E-17 | 1.227E-09 7.660E-18 | 2.256E-09-3.290E-18 | 4.284E-09-4.009E-18 |
| 1.227E-09 7.660E-18 | 2.256E-09-3.290E-18 | 4.284E-09-4.009E-18 | 6.198E-08-2.876E-17 | 1.612E-08 7.875E-18 | 3.156E-08-1.610E-17 | 6.198E-08-2.876E-17 |
| 1.612E-08 7.875E-18 | 3.156E-08-1.610E-17 | 6.198E-08-2.876E-17 | 1.878E-06-9.795E-17 | 2.408E-07 4.987E-17 | 4.763E-07 9.191E-17 | 3.156E-08 1.610E-17 |
| 2.408E-07 4.987E-17 | 4.763E-07 9.191E-17 | 3.156E-08 1.610E-17 | 1.508E-05-5.539E-16 | 3.747E-06 2.488E-16 | 7.501E-06 2.129E-16 | 1.508E-05-5.539E-16 |
| 3.747E-06 2.488E-16 | 7.501E-06 2.129E-16 | 1.508E-05-5.539E-16 | 3.047E-05-9.785E-16 | 6.193E-05 9.759E-16 | 1.268E-04 5.384E-16 | 2.618E-04 1.515E-15 |
| 6.193E-05 9.759E-16 | 1.268E-04 5.384E-16 | 2.618E-04 1.515E-15 | 5.471E-04-3.705E-15 | 1.163E-03 5.733E-15 | 2.533E-03 3.073E-15 | 5.719E-03 1.307E-14 |
| 1.163E-03 5.733E-15 | 2.533E-03 3.073E-15 | 5.719E-03 1.307E-14 | 1.765E-01-2.591E-14 | 3.097E-02 2.301E-14 | 6.625E-02-2.759E-14 | 1.206E-01-8.954E-14 |
| 3.097E-02 2.301E-14 | 6.625E-02-2.759E-14 | 1.206E-01-8.954E-14 | 1.765E-01 2.591E-14 | 2.010E-01 0. | 1.765E-01 2.591E-14 | 1.206E-01 8.954E-14 |

COMPUTED VALUES OF X AND N(X)

| | |
|---------------|---------------|
| 0. | 1.6183602E-01 |
| 3.0000000E-01 | 1.5262845E-01 |
| 6.0000000E-01 | 1.2847151E-01 |
| 9.0000000E-01 | 9.7523343E-02 |
| 1.2000000E+00 | 6.7960316E-02 |
| 1.5000000E+00 | 4.4515143E-02 |
| 1.8000000E+00 | 2.8145278E-02 |
| 2.1000000E+00 | 1.7584277E-02 |
| 2.4000000E+00 | 1.1020905E-02 |
| 2.7000000E+00 | 6.9694671E-03 |
| 3.0000000E+00 | 4.4460260E-03 |
| 3.3000000E+00 | 2.8545952E-03 |
| 3.6000000E+00 | 1.8407513E-03 |
| 3.9000000E+00 | 1.1904340E-03 |
| 4.2000000E+00 | 7.7143429E-04 |
| 4.5000000E+00 | 5.0066131E-04 |
| 4.8000000E+00 | 3.2530378E-04 |
| 5.1000000E+00 | 2.1155798E-04 |
| 5.4000000E+00 | 1.3768585E-04 |
| 5.7000000E+00 | 8.9662966E-05 |
| 6.0000000E+00 | 5.8419782E-05 |
| 6.3000000E+00 | 3.8980485E-05 |
| 6.6000000E+00 | 2.4833030E-05 |
| 6.9000000E+00 | 1.6201600E-05 |
| 7.2000000E+00 | 1.0576907E-05 |
| 7.5000000E+00 | 6.9123730E-06 |
| 7.8000000E+00 | 4.5273138E-06 |
| 8.1000000E+00 | 2.9793597E-06 |
| 8.4000000E+00 | 1.9817650E-06 |
| 8.7000000E+00 | 1.3499307E-06 |
| 9.0000000E+00 | 9.6704274E-07 |
| 9.3000000E+00 | 7.6236168E-07 |
| 9.6000000E+00 | 6.9808990E-07 |
| 9.9000000E+00 | 7.6236168E-07 |
| 1.0200000E+01 | 9.6704274E-07 |

LEVEL CROSSINGS
LEVEL CROSSING FREQUENCY OF THE NON-GAUSSIAN MODEL, R= 1.000

| DIMENSIONAL LEVEL X | NON-DIMENSIONAL LEVEL X/SIGMA X | CROSSINGS PER UNIT TIME | CROSSINGS PER ZERO CROSSING |
|---------------------------|---------------------------------------|----------------------------|--------------------------------|
| 0. | 0. | 1.618E-01 | 1.000E+00 |
| 2.000E-01 | 2.000E-01 | 1.576E-01 | 9.741E-01 |
| 4.000E-01 | 4.000E-01 | 1.459E-01 | 9.014E-01 |
| 6.000E-01 | 6.000E-01 | 1.285E-01 | 7.938E-01 |
| 8.000E-01 | 8.000E-01 | 1.081E-01 | 6.677E-01 |
| 1.000E+00 | 1.000E+00 | 8.721E-02 | 5.389E-01 |
| 1.200E+00 | 1.200E+00 | 6.796E-02 | 4.199E-01 |
| 1.400E+00 | 1.400E+00 | 5.153E-02 | 3.184E-01 |
| 1.600E+00 | 1.600E+00 | 3.831E-02 | 2.367E-01 |
| 1.800E+00 | 1.800E+00 | 2.815E-02 | 1.739E-01 |
| 2.000E+00 | 2.000E+00 | 2.057E-02 | 1.271E-01 |
| 2.200E+00 | 2.200E+00 | 1.503E-02 | 9.290E-02 |
| 2.400E+00 | 2.400E+00 | 1.102E-02 | 6.810E-02 |
| 2.600E+00 | 2.600E+00 | 8.111E-03 | 5.012E-02 |
| 2.800E+00 | 2.800E+00 | 5.995E-03 | 3.704E-02 |
| 3.000E+00 | 3.000E+00 | 4.446E-03 | 2.747E-02 |
| 3.200E+00 | 3.200E+00 | 3.307E-03 | 2.044E-02 |
| 3.400E+00 | 3.400E+00 | 2.465E-03 | 1.523E-02 |
| 3.600E+00 | 3.600E+00 | 1.841E-03 | 1.137E-02 |
| 3.800E+00 | 3.800E+00 | 1.376E-03 | 8.504E-03 |
| 4.000E+00 | 4.000E+00 | 1.030E-03 | 6.364E-03 |
| 4.200E+00 | 4.200E+00 | 7.714E-04 | 4.767E-03 |
| 4.400E+00 | 4.400E+00 | 5.782E-04 | 3.573E-03 |
| 4.600E+00 | 4.600E+00 | 4.336E-04 | 2.679E-03 |
| 4.800E+00 | 4.800E+00 | 3.253E-04 | 2.010E-03 |
| 5.000E+00 | 5.000E+00 | 2.442E-04 | 1.509E-03 |
| 5.200E+00 | 5.200E+00 | 1.833E-04 | 1.133E-03 |
| 5.400E+00 | 5.400E+00 | 1.377E-04 | 8.508E-04 |
| 5.600E+00 | 5.600E+00 | 1.034E-04 | 6.392E-04 |
| 5.800E+00 | 5.800E+00 | 7.773E-05 | 4.803E-04 |
| 6.000E+00 | 6.000E+00 | 5.842E-05 | 3.610E-04 |
| 6.200E+00 | 6.200E+00 | 4.392E-05 | 2.714E-04 |
| 6.400E+00 | 6.400E+00 | 3.302E-05 | 2.040E-04 |
| 6.600E+00 | 6.600E+00 | 2.483E-05 | 1.534E-04 |
| 6.800E+00 | 6.800E+00 | 1.868E-05 | 1.154E-04 |
| 7.000E+00 | 7.000E+00 | 1.405E-05 | 8.684E-05 |
| 7.200E+00 | 7.200E+00 | 1.058E-05 | 6.536E-05 |
| 7.400E+00 | 7.400E+00 | 7.964E-06 | 4.921E-05 |
| 7.600E+00 | 7.600E+00 | 6.001E-06 | 3.708E-05 |
| 7.800E+00 | 7.800E+00 | 4.527E-06 | 2.797E-05 |
| 8.000E+00 | 8.000E+00 | 3.423E-06 | 2.115E-05 |

Program INCPD

INCPD is a program which computes velocity increment probability distributions of the non-gaussian turbulence model. The example given here shows the calculation of the u -gust increment distribution for a time lag of $0.1 L_w/U$ seconds. The correlation functions which were used in determining the input data are given in equations 51 through 54 of the report.

Card Decks

The following card decks were required to produce the sample calculation presented here:

INCPD

CF1

INVR

FFT

FFTRS

Input Data

The following nine data cards were used in producing this example. The meaning and format of these cards are explained by comments in the listing of program INCPD.

| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 |
|---|-----------|----|----|----|----|----|----|
| EXAMPLE USE OF INCPD TO COMPUTE U-GUST INCREMENT DIST., TAU=.1*L/U, R=1., SU=1. | | | | | | | |
| 2 | | | | | | | |
| .09516258 | | | | | | | |
| 1. | .95122942 | | | | | | |
| .95122942 | 1.0 | | | | | | |
| 1.0 | .95122942 | | | | | | |
| .95122942 | 1.0 | | | | | | |
| .5 | 0. | | | | | | |
| 0. | -.5 | | | | | | |

Results

The above named card decks were compiled, loaded, and executed on the University of Washington CDC-6400 computer system. Storage requirement including I/O buffers and all system programs was 20,476₈ words. Execution, including compilation and loading, required 5₁₀ seconds of central processor time. The following output was generated.

EXAMPLE USE OF INCPD TO COMPUTE U-GUST INCREMENT DIST., TAU=.1*L/U, R=1., SU=1.

COVARIANCE MATRIX OF 1ST GAUSSIAN VECTOR

| | |
|---------|---------|
| 1.00000 | .95123 |
| .95123 | 1.00000 |

COVARIANCE MATRIX OF 2ND GAUSSIAN VECTOR

| | |
|---------|---------|
| 1.00000 | .95123 |
| .95123 | 1.00000 |

FUNCTIONAL RELATION MATRIX

| | |
|---------|---------|
| .50000 | 0.00000 |
| 0.00000 | -.50000 |

STANDARD DEVIATION OF PROCESS = 3.778E-01

DETERMINANTS OF COVARIANCE MATRICIES

DETA = 9.51626E-02

DETB = 9.51626E-02

EXAMPLE USE OF INCPD TO COMPUTE U-GUST INCREMENT DIST., $\tau = .1 * L/U$, $R = 1.$, $SU = 1.$

| NORMALIZED VARIABLE X/SIGMA X | STOIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTOIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 0.000 | 4.118E-01 | 5.00000E-01 | 0. | 2.885E+00 |
| .050 | 4.112E-01 | 5.20579E-01 | 7.137E-03 | 2.881E+00 |
| .100 | 4.095E-01 | 5.41100E-01 | 1.427E-02 | 2.869E+00 |
| .150 | 4.066E-01 | 5.61506E-01 | 2.141E-02 | 2.849E+00 |
| .200 | 4.027E-01 | 5.81742E-01 | 2.855E-02 | 2.821E+00 |
| .250 | 3.976E-01 | 6.01754E-01 | 3.569E-02 | 2.786E+00 |
| .300 | 3.916E-01 | 6.21487E-01 | 4.282E-02 | 2.743E+00 |
| .350 | 3.845E-01 | 6.40893E-01 | 4.996E-02 | 2.694E+00 |
| .400 | 3.766E-01 | 6.59924E-01 | 5.710E-02 | 2.638E+00 |
| .450 | 3.678E-01 | 6.78536E-01 | 6.423E-02 | 2.576E+00 |
| .500 | 3.582E-01 | 6.96687E-01 | 7.137E-02 | 2.509E+00 |
| .550 | 3.479E-01 | 7.14341E-01 | 7.851E-02 | 2.437E+00 |
| .600 | 3.370E-01 | 7.31465E-01 | 8.565E-02 | 2.361E+00 |
| .650 | 3.255E-01 | 7.48030E-01 | 9.278E-02 | 2.280E+00 |
| .700 | 3.136E-01 | 7.64010E-01 | 9.992E-02 | 2.197E+00 |
| .750 | 3.013E-01 | 7.79385E-01 | 1.071E-01 | 2.111E+00 |
| .800 | 2.888E-01 | 7.94139E-01 | 1.142E-01 | 2.023E+00 |
| .850 | 2.760E-01 | 8.08259E-01 | 1.213E-01 | 1.934E+00 |
| .900 | 2.631E-01 | 8.21737E-01 | 1.285E-01 | 1.843E+00 |
| .950 | 2.502E-01 | 8.34669E-01 | 1.356E-01 | 1.753E+00 |
| 1.000 | 2.373E-01 | 8.46755E-01 | 1.427E-01 | 1.662E+00 |
| 1.050 | 2.245E-01 | 8.58298E-01 | 1.499E-01 | 1.572E+00 |
| 1.100 | 2.118E-01 | 8.69203E-01 | 1.570E-01 | 1.484E+00 |
| 1.150 | 1.994E-01 | 8.79481E-01 | 1.642E-01 | 1.397E+00 |
| 1.200 | 1.872E-01 | 8.89144E-01 | 1.713E-01 | 1.311E+00 |
| 1.250 | 1.754E-01 | 8.98207E-01 | 1.784E-01 | 1.228E+00 |
| 1.300 | 1.639E-01 | 9.06686E-01 | 1.856E-01 | 1.148E+00 |
| 1.350 | 1.528E-01 | 9.14601E-01 | 1.927E-01 | 1.070E+00 |
| 1.400 | 1.421E-01 | 9.21972E-01 | 1.998E-01 | 9.956E-01 |
| 1.450 | 1.319E-01 | 9.28820E-01 | 2.070E-01 | 9.240E-01 |
| 1.500 | 1.221E-01 | 9.35169E-01 | 2.141E-01 | 8.557E-01 |
| 1.550 | 1.129E-01 | 9.41042E-01 | 2.213E-01 | 7.907E-01 |
| 1.600 | 1.041E-01 | 9.46463E-01 | 2.284E-01 | 7.290E-01 |
| 1.650 | 9.575E-02 | 9.51457E-01 | 2.355E-01 | 6.708E-01 |
| 1.700 | 8.793E-02 | 9.56046E-01 | 2.427E-01 | 6.160E-01 |
| 1.750 | 8.058E-02 | 9.60257E-01 | 2.498E-01 | 5.645E-01 |
| 1.800 | 7.370E-02 | 9.64112E-01 | 2.569E-01 | 5.163E-01 |
| 1.850 | 6.729E-02 | 9.67635E-01 | 2.641E-01 | 4.714E-01 |
| 1.900 | 6.132E-02 | 9.70848E-01 | 2.712E-01 | 4.296E-01 |
| 1.950 | 5.578E-02 | 9.73774E-01 | 2.784E-01 | 3.908E-01 |
| 2.000 | 5.066E-02 | 9.76433E-01 | 2.855E-01 | 3.549E-01 |
| 2.050 | 4.593E-02 | 9.78846E-01 | 2.926E-01 | 3.217E-01 |
| 2.100 | 4.157E-02 | 9.81032E-01 | 2.998E-01 | 2.912E-01 |
| 2.150 | 3.757E-02 | 9.83009E-01 | 3.069E-01 | 2.632E-01 |
| 2.200 | 3.391E-02 | 9.84795E-01 | 3.140E-01 | 2.375E-01 |
| 2.250 | 3.056E-02 | 9.86405E-01 | 3.212E-01 | 2.141E-01 |
| 2.300 | 2.750E-02 | 9.87855E-01 | 3.283E-01 | 1.927E-01 |
| 2.350 | 2.472E-02 | 9.89160E-01 | 3.354E-01 | 1.732E-01 |
| 2.400 | 2.219E-02 | 9.90331E-01 | 3.426E-01 | 1.555E-01 |
| 2.450 | 1.990E-02 | 9.91383E-01 | 3.497E-01 | 1.394E-01 |
| 2.500 | 1.782E-02 | 9.92325E-01 | 3.569E-01 | 1.249E-01 |

EXAMPLE USE OF INCPD TO COMPUTE U-GUST INCREMENT DIST., $\tau = .1 * L/U$, $R=1.$, $SU=1.$

| NORMALIZED VARIABLE X/SIGMA X | STANDARDIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTANDARDIZED PROBABILITY DENSITY |
|-------------------------------------|--|--------------------------|-------------------------------|---|
| 5.000 | 4.341E-05 | 9.99982E-01 | 7.137E-01 | 3.041E-04 |
| 5.050 | 3.841E-05 | 9.99984E-01 | 7.209E-01 | 2.691E-04 |
| 5.100 | 3.398E-05 | 9.99986E-01 | 7.280E-01 | 2.381E-04 |
| 5.150 | 3.007E-05 | 9.99988E-01 | 7.351E-01 | 2.106E-04 |
| 5.200 | 2.660E-05 | 9.99989E-01 | 7.423E-01 | 1.863E-04 |
| 5.250 | 2.353E-05 | 9.99990E-01 | 7.494E-01 | 1.649E-04 |
| 5.300 | 2.082E-05 | 9.99991E-01 | 7.565E-01 | 1.459E-04 |
| 5.350 | 1.842E-05 | 9.99992E-01 | 7.637E-01 | 1.290E-04 |
| 5.400 | 1.630E-05 | 9.99993E-01 | 7.708E-01 | 1.142E-04 |
| 5.450 | 1.442E-05 | 9.99994E-01 | 7.780E-01 | 1.010E-04 |
| 5.500 | 1.276E-05 | 9.99995E-01 | 7.851E-01 | 8.937E-05 |
| 5.550 | 1.129E-05 | 9.99995E-01 | 7.922E-01 | 7.907E-05 |
| 5.600 | 9.985E-06 | 9.99996E-01 | 7.994E-01 | 6.995E-05 |
| 5.650 | 8.834E-06 | 9.99996E-01 | 8.065E-01 | 6.189E-05 |
| 5.700 | 7.816E-06 | 9.99997E-01 | 8.136E-01 | 5.475E-05 |
| 5.750 | 6.915E-06 | 9.99997E-01 | 8.208E-01 | 4.844E-05 |
| 5.800 | 6.118E-06 | 9.99998E-01 | 8.279E-01 | 4.286E-05 |
| 5.850 | 5.413E-06 | 9.99998E-01 | 8.351E-01 | 3.792E-05 |
| 5.900 | 4.789E-06 | 9.99998E-01 | 8.422E-01 | 3.355E-05 |
| 5.950 | 4.237E-06 | 9.99998E-01 | 8.493E-01 | 2.968E-05 |
| 6.000 | 3.748E-06 | 9.99998E-01 | 8.565E-01 | 2.626E-05 |
| 6.050 | 3.316E-06 | 9.99999E-01 | 8.636E-01 | 2.323E-05 |
| 6.100 | 2.934E-06 | 9.99999E-01 | 8.707E-01 | 2.055E-05 |
| 6.150 | 2.596E-06 | 9.99999E-01 | 8.779E-01 | 1.818E-05 |
| 6.200 | 2.297E-06 | 9.99999E-01 | 8.850E-01 | 1.609E-05 |
| 6.250 | 2.032E-06 | 9.99999E-01 | 8.921E-01 | 1.423E-05 |
| 6.300 | 1.798E-06 | 9.99999E-01 | 8.993E-01 | 1.259E-05 |
| 6.350 | 1.590E-06 | 9.99999E-01 | 9.064E-01 | 1.114E-05 |
| 6.400 | 1.407E-06 | 9.99999E-01 | 9.136E-01 | 9.857E-06 |
| 6.450 | 1.245E-06 | 9.99999E-01 | 9.207E-01 | 8.721E-06 |
| 6.500 | 1.101E-06 | 1.00000E+00 | 9.278E-01 | 7.716E-06 |
| 6.550 | 9.744E-07 | 1.00000E+00 | 9.350E-01 | 6.826E-06 |
| 6.600 | 8.621E-07 | 1.00000E+00 | 9.421E-01 | 6.039E-06 |
| 6.650 | 7.627E-07 | 1.00000E+00 | 9.492E-01 | 5.343E-06 |
| 6.700 | 6.748E-07 | 1.00000E+00 | 9.564E-01 | 4.727E-06 |
| 6.750 | 5.970E-07 | 1.00000E+00 | 9.635E-01 | 4.182E-06 |
| 6.800 | 5.282E-07 | 1.00000E+00 | 9.707E-01 | 3.703E-06 |
| 6.850 | 4.673E-07 | 1.00000E+00 | 9.778E-01 | 3.274E-06 |
| 6.900 | 4.134E-07 | 1.00000E+00 | 9.849E-01 | 2.896E-06 |
| 6.950 | 3.658E-07 | 1.00000E+00 | 9.921E-01 | 2.563E-06 |
| 7.000 | 3.236E-07 | 1.00000E+00 | 9.992E-01 | 2.267E-06 |
| 7.050 | 2.863E-07 | 1.00000E+00 | 1.006E+00 | 2.006E-06 |
| 7.100 | 2.533E-07 | 1.00000E+00 | 1.013E+00 | 1.775E-06 |
| 7.150 | 2.241E-07 | 1.00000E+00 | 1.021E+00 | 1.570E-06 |
| 7.200 | 1.983E-07 | 1.00000E+00 | 1.028E+00 | 1.389E-06 |
| 7.250 | 1.754E-07 | 1.00000E+00 | 1.035E+00 | 1.229E-06 |
| 7.300 | 1.552E-07 | 1.00000E+00 | 1.042E+00 | 1.087E-06 |
| 7.350 | 1.373E-07 | 1.00000E+00 | 1.049E+00 | 9.619E-07 |
| 7.400 | 1.215E-07 | 1.00000E+00 | 1.056E+00 | 8.511E-07 |
| 7.450 | 1.075E-07 | 1.00000E+00 | 1.063E+00 | 7.530E-07 |
| 7.500 | 9.509E-08 | 1.00000E+00 | 1.071E+00 | 6.662E-07 |

EXAMPLE USE OF INCPD TO COMPUTE U-GUST INCREMENT DIST., $\tau = .1 \cdot L/U$, $R=1.$, $SJ=1.$

| NORMALIZED VARIABLE X/SIGMA X | STOIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTOIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 2.500 | 1.782E-02 | 9.92325E-01 | 3.569E-01 | 1.249E-01 |
| 2.550 | 1.595E-02 | 9.93168E-01 | 3.640E-01 | 1.117E-01 |
| 2.600 | 1.425E-02 | 9.93922E-01 | 3.711E-01 | 9.986E-02 |
| 2.650 | 1.273E-02 | 9.94596E-01 | 3.783E-01 | 8.918E-02 |
| 2.700 | 1.136E-02 | 9.95198E-01 | 3.854E-01 | 7.958E-02 |
| 2.750 | 1.013E-02 | 9.95735E-01 | 3.925E-01 | 7.095E-02 |
| 2.800 | 9.023E-03 | 9.96213E-01 | 3.997E-01 | 6.321E-02 |
| 2.850 | 8.033E-03 | 9.96639E-01 | 4.068E-01 | 5.628E-02 |
| 2.900 | 7.147E-03 | 9.97018E-01 | 4.140E-01 | 5.007E-02 |
| 2.950 | 6.355E-03 | 9.97355E-01 | 4.211E-01 | 4.452E-02 |
| 3.000 | 5.648E-03 | 9.97655E-01 | 4.282E-01 | 3.957E-02 |
| 3.050 | 5.017E-03 | 9.97921E-01 | 4.354E-01 | 3.515E-02 |
| 3.100 | 4.455E-03 | 9.98158E-01 | 4.425E-01 | 3.121E-02 |
| 3.150 | 3.954E-03 | 9.98368E-01 | 4.496E-01 | 2.770E-02 |
| 3.200 | 3.507E-03 | 9.98554E-01 | 4.568E-01 | 2.457E-02 |
| 3.250 | 3.111E-03 | 9.98719E-01 | 4.639E-01 | 2.179E-02 |
| 3.300 | 2.758E-03 | 9.98866E-01 | 4.711E-01 | 1.932E-02 |
| 3.350 | 2.445E-03 | 9.98996E-01 | 4.782E-01 | 1.713E-02 |
| 3.400 | 2.166E-03 | 9.99111E-01 | 4.853E-01 | 1.518E-02 |
| 3.450 | 1.919E-03 | 9.99213E-01 | 4.925E-01 | 1.345E-02 |
| 3.500 | 1.700E-03 | 9.99303E-01 | 4.996E-01 | 1.191E-02 |
| 3.550 | 1.506E-03 | 9.99383E-01 | 5.067E-01 | 1.055E-02 |
| 3.600 | 1.333E-03 | 9.99454E-01 | 5.139E-01 | 9.341E-03 |
| 3.650 | 1.181E-03 | 9.99517E-01 | 5.210E-01 | 8.270E-03 |
| 3.700 | 1.045E-03 | 9.99572E-01 | 5.282E-01 | 7.322E-03 |
| 3.750 | 9.251E-04 | 9.99622E-01 | 5.353E-01 | 6.481E-03 |
| 3.800 | 8.189E-04 | 9.99665E-01 | 5.424E-01 | 5.737E-03 |
| 3.850 | 7.248E-04 | 9.99704E-01 | 5.496E-01 | 5.077E-03 |
| 3.900 | 6.414E-04 | 9.99738E-01 | 5.567E-01 | 4.494E-03 |
| 3.950 | 5.677E-04 | 9.99768E-01 | 5.638E-01 | 3.977E-03 |
| 4.000 | 5.023E-04 | 9.99795E-01 | 5.710E-01 | 3.519E-03 |
| 4.050 | 4.445E-04 | 9.99818E-01 | 5.781E-01 | 3.114E-03 |
| 4.100 | 3.933E-04 | 9.99839E-01 | 5.852E-01 | 2.756E-03 |
| 4.150 | 3.480E-04 | 9.99858E-01 | 5.924E-01 | 2.438E-03 |
| 4.200 | 3.080E-04 | 9.99874E-01 | 5.995E-01 | 2.157E-03 |
| 4.250 | 2.725E-04 | 9.99889E-01 | 6.067E-01 | 1.909E-03 |
| 4.300 | 2.411E-04 | 9.99902E-01 | 6.138E-01 | 1.689E-03 |
| 4.350 | 2.133E-04 | 9.99913E-01 | 6.209E-01 | 1.494E-03 |
| 4.400 | 1.887E-04 | 9.99923E-01 | 6.281E-01 | 1.322E-03 |
| 4.450 | 1.670E-04 | 9.99932E-01 | 6.352E-01 | 1.170E-03 |
| 4.500 | 1.477E-04 | 9.99940E-01 | 6.423E-01 | 1.035E-03 |
| 4.550 | 1.307E-04 | 9.99947E-01 | 6.495E-01 | 9.157E-04 |
| 4.600 | 1.156E-04 | 9.99953E-01 | 6.566E-01 | 8.102E-04 |
| 4.650 | 1.023E-04 | 9.99958E-01 | 6.638E-01 | 7.168E-04 |
| 4.700 | 9.052E-05 | 9.99963E-01 | 6.709E-01 | 6.342E-04 |
| 4.750 | 8.009E-05 | 9.99967E-01 | 6.780E-01 | 5.611E-04 |
| 4.800 | 7.086E-05 | 9.99971E-01 | 6.852E-01 | 4.964E-04 |
| 4.850 | 6.269E-05 | 9.99974E-01 | 6.923E-01 | 4.392E-04 |
| 4.900 | 5.546E-05 | 9.99977E-01 | 6.994E-01 | 3.886E-04 |
| 4.950 | 4.907E-05 | 9.99980E-01 | 7.066E-01 | 3.438E-04 |
| 5.000 | 4.341E-05 | 9.99982E-01 | 7.137E-01 | 3.041E-04 |

EXAMPLE USE OF INCPD TO COMPUTE U-GUST INCREMENT DIST., $\tau = .1 * L/U$, $R=1.$, $SU=1.$

| NORMALIZED VARIABLE X/SIGMA X | STDIZED PROBABILITY DENSITY | DISTRIBUTION FUNCTION | UNNORMALIZED VARIABLE X | NONSTDIZED PROBABILITY DENSITY |
|-------------------------------------|-----------------------------------|--------------------------|-------------------------------|--------------------------------------|
| 7.500 | 9.509E-08 | 1.00000E+00 | 1.071E+00 | 6.662E-07 |
| 7.550 | 8.413E-08 | 1.00000E+00 | 1.078E+00 | 5.894E-07 |
| 7.600 | 7.443E-08 | 1.00000E+00 | 1.085E+00 | 5.214E-07 |
| 7.650 | 6.585E-08 | 1.00000E+00 | 1.092E+00 | 4.613E-07 |
| 7.700 | 5.826E-08 | 1.00000E+00 | 1.099E+00 | 4.081E-07 |
| 7.750 | 5.154E-08 | 1.00000E+00 | 1.106E+00 | 3.611E-07 |
| 7.800 | 4.560E-08 | 1.00000E+00 | 1.113E+00 | 3.195E-07 |
| 7.850 | 4.035E-08 | 1.00000E+00 | 1.121E+00 | 2.826E-07 |
| 7.900 | 3.570E-08 | 1.00000E+00 | 1.128E+00 | 2.501E-07 |
| 7.950 | 3.158E-08 | 1.00000E+00 | 1.135E+00 | 2.212E-07 |
| 8.000 | 2.794E-08 | 1.00000E+00 | 1.142E+00 | 1.957E-07 |
| 8.050 | 2.472E-08 | 1.00000E+00 | 1.149E+00 | 1.732E-07 |
| 8.100 | 2.187E-08 | 1.00000E+00 | 1.156E+00 | 1.532E-07 |
| 8.150 | 1.935E-08 | 1.00000E+00 | 1.163E+00 | 1.356E-07 |
| 8.200 | 1.712E-08 | 1.00000E+00 | 1.170E+00 | 1.199E-07 |
| 8.250 | 1.515E-08 | 1.00000E+00 | 1.178E+00 | 1.061E-07 |
| 8.300 | 1.340E-08 | 1.00000E+00 | 1.185E+00 | 9.387E-08 |
| 8.350 | 1.185E-08 | 1.00000E+00 | 1.192E+00 | 8.305E-08 |
| 8.400 | 1.049E-08 | 1.00000E+00 | 1.199E+00 | 7.348E-08 |
| 8.450 | 9.279E-09 | 1.00000E+00 | 1.206E+00 | 6.501E-08 |
| 8.500 | 8.210E-09 | 1.00000E+00 | 1.213E+00 | 5.751E-08 |
| 8.550 | 7.263E-09 | 1.00000E+00 | 1.220E+00 | 5.088E-08 |
| 8.600 | 6.426E-09 | 1.00000E+00 | 1.228E+00 | 4.502E-08 |
| 8.650 | 5.685E-09 | 1.00000E+00 | 1.235E+00 | 3.983E-08 |
| 8.700 | 5.030E-09 | 1.00000E+00 | 1.242E+00 | 3.524E-08 |
| 8.750 | 4.450E-09 | 1.00000E+00 | 1.249E+00 | 3.118E-08 |
| 8.800 | 3.937E-09 | 1.00000E+00 | 1.256E+00 | 2.758E-08 |
| 8.850 | 3.483E-09 | 1.00000E+00 | 1.263E+00 | 2.440E-08 |
| 8.900 | 3.082E-09 | 1.00000E+00 | 1.270E+00 | 2.159E-08 |
| 8.950 | 2.727E-09 | 1.00000E+00 | 1.278E+00 | 1.910E-08 |
| 9.000 | 2.412E-09 | 1.00000E+00 | 1.285E+00 | 1.690E-08 |
| 9.050 | 2.134E-09 | 1.00000E+00 | 1.292E+00 | 1.495E-08 |
| 9.100 | 1.888E-09 | 1.00000E+00 | 1.299E+00 | 1.323E-08 |
| 9.150 | 1.671E-09 | 1.00000E+00 | 1.306E+00 | 1.170E-08 |
| 9.200 | 1.478E-09 | 1.00000E+00 | 1.313E+00 | 1.035E-08 |
| 9.250 | 1.308E-09 | 1.00000E+00 | 1.320E+00 | 9.160E-09 |
| 9.300 | 1.157E-09 | 1.00000E+00 | 1.328E+00 | 8.104E-09 |
| 9.350 | 1.023E-09 | 1.00000E+00 | 1.335E+00 | 7.170E-09 |
| 9.400 | 9.055E-10 | 1.00000E+00 | 1.342E+00 | 6.343E-09 |
| 9.450 | 8.011E-10 | 1.00000E+00 | 1.349E+00 | 5.612E-09 |
| 9.500 | 7.088E-10 | 1.00000E+00 | 1.356E+00 | 4.966E-09 |
| 9.550 | 6.270E-10 | 1.00000E+00 | 1.363E+00 | 4.393E-09 |
| 9.600 | 5.548E-10 | 1.00000E+00 | 1.370E+00 | 3.887E-09 |
| 9.650 | 4.908E-10 | 1.00000E+00 | 1.377E+00 | 3.438E-09 |
| 9.700 | 4.343E-10 | 1.00000E+00 | 1.385E+00 | 3.042E-09 |
| 9.750 | 3.841E-10 | 1.00000E+00 | 1.392E+00 | 2.691E-09 |
| 9.800 | 3.398E-10 | 1.00000E+00 | 1.399E+00 | 2.381E-09 |
| 9.850 | 3.007E-10 | 1.00000E+00 | 1.406E+00 | 2.107E-09 |
| 9.900 | 2.660E-10 | 1.00000E+00 | 1.413E+00 | 1.864E-09 |
| 9.950 | 2.353E-10 | 1.00000E+00 | 1.420E+00 | 1.648E-09 |
| 10.000 | 2.083E-10 | 1.00000E+00 | 1.427E+00 | 1.459E-09 |

Program EIGU

EIGU is a program which computes eigenvalues and eigenfunctions of linear system responses to the longitudinal component of the non-gaussian turbulence model. The example presented here shows the calculation of the first eigensolution for a system having the transfer function

$$H(s) = \frac{1+i}{s-(1+i)} + \frac{1-i}{s-(1-i)} \quad (A1)$$

The parameters of the problem are:

$$\sigma_u = .3048 \text{ (m/sec)}$$

$$L_u = 200. \text{ (m)} \quad (A2)$$

$$U = 100. \text{ (m/sec)}$$

Card Decks

The following card decks were required to produce the sample calculation presented here:

EIGU

attached subroutines.

(All of the decks required by program EIGU have been given the identification tag EIGU and will be found listed consecutively with program EIGU at the end of this appendix.)

Input Data

The following five data cards were used to produce the example presented here. The meaning and format of these cards is described by comments in program EIGU.

| | | | | | | | |
|-------|-----------------|-----------------|-----------------|------------------|------------------|------------------|------------------|
| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 |
| F | | | | | | | |
| 1 | 1.000000000E+00 | 1.000000000E+00 | 1.000000000E+00 | -1.000000000E+00 | -1.000000000E+00 | -1.000000000E+00 | -1.000000000E+00 |
| .3848 | 100. | 200. | | | | | |
| 1 | | | | | | | |

Results

The above named card decks were compiled, loaded, and executed on the University of Washington CDC-6400 computer system. Storage requirement including I/O buffers and all system programs was 17,664₈ words. Execution, including compilation and loading, required 11₁₀ seconds of central processor time. The following printer output was generated.

BEGIN ITERATION FOR EIGENVALUE AND EIGENFUNCTION NO.1

TURBULENCE PARAMETERS:

STD. DEV. = .305 MTAS = 100.000 SCALE LENGTH = 200.000

VEHICLE PARAMETERS:

| COEFFICIENTS | | POLES | |
|--------------|------------|------------|------------|
| 1.0000E+00 | 1.0000E+00 | 1.0000E+00 | 1.0000E+00 |

SYSTEM RESPONSE VARIANCE = 3.144411E-01

| | | | | |
|---------------|----|-------------------------|----------------|------------------|
| ITERATION NO. | 1. | ESTIMATED EIGENVALUE IS | 1.1279131E-01. | (DEV = 1.000E+00 |
| ITERATION NO. | 2. | ESTIMATED EIGENVALUE IS | 2.3169420E-01. | (DEV = 1.054E+00 |
| ITERATION NO. | 3. | ESTIMATED EIGENVALUE IS | 5.5995409E-01. | (DEV = 1.417E+00 |
| ITERATION NO. | 4. | ESTIMATED EIGENVALUE IS | 5.6586583E-01. | (DEV = 1.056E-02 |
| ITERATION NO. | 5. | ESTIMATED EIGENVALUE IS | 5.6579235E-01. | (DEV = 1.299E-04 |
| ITERATION NO. | 6. | ESTIMATED EIGENVALUE IS | 5.6578378E-01. | (DEV = 1.513E-05 |
| ITERATION NO. | 6. | ESTIMATED EIGENVALUE IS | 9.9804141E-01. | (DEV = 7.640E-01 |
| ITERATION NO. | 7. | ESTIMATED EIGENVALUE IS | 5.5910282E-01. | (DEV = 4.398E-01 |
| ITERATION NO. | 8. | ESTIMATED EIGENVALUE IS | 5.5924255E-01. | (DEV = 2.499E-04 |
| ITERATION NO. | 9. | ESTIMATED EIGENVALUE IS | 5.5924316E-01. | (DEV = 1.078E-06 |

EI = 8.866E+50)
 EI = 1.720E-02)
 EI = 1.183E-02)
 EI = 3.261E-04)
 EI = 2.309E-05)
 EI = 1.754E-06)
 EI = 1.758E-06)
 EI = 2.277E-02)
 EI = 2.842E-05)
 EI = 1.799E-06)

EIGENVALUE NO. 1 IS 5.5924316E-01

FRACTION OF RESPONSE VARIANCE DUE TO THIS EIGENVALUE IS .99463
 FRACTION OF RESPONSE VARIANCE DUE TO FIRST 1 EIGENVALUES IS .99463

| X | EIGENFUNCTION |
|---------|------------------|
| 0.0000 | 0. |
| .0100 | 7.0285872829E-03 |
| .0200 | 1.4055442114E-02 |
| .0300 | 2.1078851805E-02 |
| .0450 | 3.1603814298E-02 |
| .0688 | 4.8233469059E-02 |
| .1588 | 1.1050040718E-01 |
| .3925 | 2.6064371720E-01 |
| .7532 | 4.3826169502E-01 |
| 1.2436 | 5.5772357582E-01 |
| 1.8600 | 5.6061854671E-01 |
| 2.6071 | 4.7467481916E-01 |
| 3.4827 | 3.6776402814E-01 |
| 4.4830 | 2.7839939679E-01 |
| 5.6153 | 2.0852833382E-01 |
| 6.8713 | 1.5266666688E-01 |
| 8.2603 | 1.0799291389E-01 |
| 9.7720 | 7.4001491704E-02 |
| 11.4176 | 4.9038754620E-02 |
| 13.1918 | 3.1470795013E-02 |
| 15.0874 | 1.9593125325E-02 |
| 17.1183 | 1.1792418679E-02 |
| 19.2695 | 6.8870511724E-03 |
| 21.5571 | 3.8874500871E-03 |
| 23.9732 | 2.1248914355E-03 |
| 26.5083 | 1.1274420928E-03 |
| 29.1811 | 5.7796094088E-04 |
| 31.9719 | 2.8766772923E-04 |
| 34.9013 | 1.3830176211E-04 |
| 37.9594 | 6.4387963348E-05 |
| 41.1340 | 2.9115511114E-05 |
| 44.4487 | 1.2712580531E-05 |
| 47.8791 | 5.3924807584E-06 |
| 51.4505 | 2.2081584150E-06 |
| 55.1365 | 8.7865881544E-07 |
| 58.9646 | 3.3743817791E-07 |
| 62.9212 | 1.2548984137E-07 |
| 66.9911 | 4.5365498615E-08 |
| 71.2044 | 1.5822408221E-08 |
| 75.5300 | 5.3656784205E-09 |
| 80.0000 | 1.7551071753E-09 |

FIRST 1 EIGENVALUES ARE:

1 5.5924316E-01

REMAINING EIGENVALUES SQUARED SUM TO 1.6881512E-03
 THE ABOVE EIGENVALUES ACCOUNT FOR .99463 OF THE RESPONSE VAR.
 LARGEST POSSIBLE REMAINING EIGENVALUE IS 4.1087118E-02

Program EIGVW

EIGVW is a program which computes eigenvalues and adjoint eigenfunctions of linear system responses to the vertical or lateral components of the non-gaussian turbulence model. The example presented here shows the calculation of the first eigensolution for a system having the transfer function

$$H(s) = \frac{1+i}{s - (1+i)} + \frac{1-i}{s - (1-i)} \quad (A3)$$

The parameters of the problem are

$$\sigma_{\omega} = .3048 \text{ (m/sec)}$$

$$L_{\omega} = 200. \text{ (m)} \quad (A4)$$

$$U = 100. \text{ (m/sec)}$$

Card Decks

The following card decks were required to produce the sample calculation presented here:

EIGVW

attached subroutines.

(All of the decks required by program EIGVW have been given the identification tag EIGVW and will be found listed consecutively with program EIGVW at the end of this appendix.)

Input Data

The following five data cards were used to produce the example presented here. The meaning and format of these cards is described by

EIGENVALUE NO. 1 IS 5.3054256E-01
 EVP = 5.3037603E-01 EVQ = 5.3070909E-01 RELATIVE DIFFERENCE = 6.278E-04

FRACTION OF RESPONSE VARIANCE DUE TO THIS EIGENVALUE IS .98848
 FRACTION OF RESPONSE VARIANCE DUE TO FIRST 1 EIGENVALUES IS .98848

| X | FUNCTION OF 1ST ARG | FUNCTION OF 2ND ARG |
|---------|---------------------|---------------------|
| 0.0000 | 0. | 0. |
| .0100 | 6.7324511024E-03 | 1.0461591109E-02 |
| .0200 | 1.3482343875E-02 | 2.0894505585E-02 |
| .0300 | 2.0247587919E-02 | 3.1296288211E-02 |
| .0450 | 3.0419692969E-02 | 4.6835310860E-02 |
| .0688 | 4.6571979135E-02 | 7.1268468976E-02 |
| .1588 | 1.0724141104E-01 | 1.6144901928E-01 |
| .3925 | 2.5942980425E-01 | 3.6958022178E-01 |
| .7532 | 4.4146159714E-01 | 5.9018308049E-01 |
| 1.2436 | 5.5866644871E-01 | 6.8767121324E-01 |
| 1.8600 | 5.5238306103E-01 | 5.9129268019E-01 |
| 2.6071 | 4.6575693583E-01 | 3.7291355406E-01 |
| 3.4827 | 3.6835976265E-01 | 1.5671227976E-01 |
| 4.4830 | 2.8544936694E-01 | 4.9083324351E-03 |
| 5.6153 | 2.1516724290E-01 | -8.6185086779E-02 |
| 6.8713 | 1.5702144891E-01 | -1.3371281284E-01 |
| 8.2603 | 1.1086763418E-01 | -1.5010589158E-01 |
| 9.7720 | 7.5981065944E-02 | -1.4449874249E-01 |
| 11.4176 | 5.0358752968E-02 | -1.2579504215E-01 |
| 13.1918 | 3.2317565084E-02 | -1.0150526678E-01 |
| 15.0874 | 2.0120179993E-02 | -7.7014503724E-02 |
| 17.1183 | 1.2109644039E-02 | -5.5263694482E-02 |
| 19.2695 | 7.0723185987E-03 | -3.7788198842E-02 |
| 21.5571 | 3.9920256440E-03 | -2.4638804756E-02 |
| 23.9732 | 2.1820527423E-03 | -1.5377986136E-02 |
| 26.5083 | 1.1577712014E-03 | -9.2228821484E-03 |
| 29.1811 | 5.9350855992E-04 | -5.3027323990E-03 |
| 31.9719 | 2.9540622495E-04 | -2.9380489023E-03 |
| 34.9013 | 1.4202219191E-04 | -1.5632781479E-03 |
| 37.9594 | 6.6120051893E-05 | -8.0106806971E-04 |
| 41.1340 | 2.9898742026E-05 | -3.9662702230E-04 |
| 44.4487 | 1.3054559279E-05 | -1.8885713453E-04 |
| 47.8791 | 5.5375428735E-06 | -8.6993337604E-05 |
| 51.4505 | 2.2675596710E-06 | -3.8557179289E-05 |
| 55.1365 | 9.0229545170E-07 | -1.6547612754E-05 |
| 58.9646 | 3.4651553914E-07 | -6.8355563869E-06 |
| 62.9212 | 1.2886562009E-07 | -2.7268259085E-06 |
| 66.9911 | 4.6585867398E-08 | -1.0544685683E-06 |
| 71.2044 | 1.6248035647E-08 | -3.9257897507E-07 |
| 75.5300 | 5.5190195309E-09 | -1.4176732168E-07 |
| 80.0000 | 1.8023209848E-09 | -4.9291119477E-08 |

FIRST 1 EIGENVALUES ARE
 1 5.3054256E-01

REMAINING EIGENVALUES SQUARED SUM TO 3.2806552E-03
 THE ABOVE EIGENVALUES ACCOUNT FOR .98848 OF THE RESPONSE VAR.
 LARGEST POSSIBLE REMAINING EIGENVALUE IS 5.7277004E-02

Program RLEVX

RLEVX is a program which computes the level crossing frequency of a linear system response to the non-gaussian turbulence model. The example presented here, in order to be as simple and compact as possible, utilizes the first and second eigensolutions of the response and its first derivative which were presented in the numerical example section of this report. These solutions were assumed to represent the total non-gaussian portion of the vehicle response.

Card Decks

The following card decks were required to produce the sample calculation presented here:

RLEVX

CF2

COEF

FFT

INRPDT

INVR

SCALE

Input Data

The 175 data cards listed on the next three pages of this appendix were used to produce the example presented here. The meaning and format of these cards is described by comments in program RLEVX.

Results

The above named card decks were compiled, loaded, and executed on the University of Washington CDC 6400 computer system. Storage require-

ment including I/O buffers and all system programs was 44,550₈ words. Execution, including compilation and loading, required 53₁₀ seconds of central processor time. Printer output generated by the program is presented following the listing of input data on the next few pages.

| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 |
|--|----|----|-----------------------|----|----|-----------------------|----|
| EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT | | | | | | | |
| 41 | | | | | | | |
| 2 | | | | | | | |
| EIGENVALUE AND CONJUGATE EIGENFUNCTIONS NO. 1 3.1700000000E+00 | | | | | | | |
| 0. | | | 0. | | | 0. | |
| 1.0000000000000E-02 | | | 1.03380453484144E-04 | | | -1.72991714847168E-04 | |
| 2.9000000000000E-02 | | | 4.12091576661881E-04 | | | -6.87407578743960E-04 | |
| 3.0000000000000E-02 | | | 9.23896341785153E-04 | | | -1.53631177532787E-03 | |
| 4.4999999999999E-02 | | | 2.06720167699628E-03 | | | -3.42136455657666E-03 | |
| 6.81428683949825E-02 | | | 4.69721641553622E-03 | | | -7.71643578813392E-03 | |
| 1.53285815554841E-01 | | | 2.28626732999262E-02 | | | -3.66087563417696E-02 | |
| 3.72590895436845E-01 | | | 1.17667551347099E-01 | | | -1.77411931788454E-01 | |
| 7.10029610127151E-01 | | | 3.15235689867819E-01 | | | -4.41306397306692E-01 | |
| 1.16810150566051E+00 | | | 5.06897531032767E-01 | | | -6.52671271597395E-01 | |
| 1.74342597764184E+00 | | | 5.67526255624035E-01 | | | -6.49494662899258E-01 | |
| 2.44026468882656E+00 | | | 5.19197439608305E-01 | | | -4.85131711607991E-01 | |
| 3.25E70747957044E+00 | | | 4.24516551865992E-01 | | | -2.63762803987694E-01 | |
| 4.18508044493492E+00 | | | 3.27771101732861E-01 | | | -5.85279676770651E-02 | |
| 5.24429005133021E+00 | | | 2.49079990695487E-01 | | | 8.41333427699911E-02 | |
| 6.41454877398570E+00 | | | 1.86462553522839E-01 | | | 1.52833075683970E-01 | |
| 7.70852519603235E+00 | | | 1.31750567381850E-01 | | | 1.66427982508313E-01 | |
| 9.11666967597887E+00 | | | 8.35926886090079E-02 | | | 1.51907809431097E-01 | |
| 1.06494129136769E+01 | | | 4.41659736179496E-02 | | | 1.29219411572128E-01 | |
| 1.23017602309341E+01 | | | 1.60219729957691E-02 | | | 1.06441974604521E-01 | |
| 1.40669532042637E+01 | | | -5.48702502495294E-04 | | | 8.44795620987124E-02 | |
| 1.59580673371723E+01 | | | -7.66173407083739E-03 | | | 6.31065303437834E-02 | |
| 1.79E11460E77929E+01 | | | -8.63893748285316E-03 | | | 4.38186274857890E-02 | |
| 2.00910270163529E+01 | | | -6.82849982042144E-03 | | | 2.8151469994369E-02 | |
| 2.23405120444720E+01 | | | -4.46049667514021E-03 | | | 1.68717553016897E-02 | |
| 2.47006392684760E+01 | | | -2.55025463732295E-03 | | | 9.55632383823490E-03 | |
| 2.7188911122464E+01 | | | -1.29766408007102E-03 | | | 5.15262972619396E-03 | |
| 2.97869040935412E+01 | | | -5.90696512745958E-04 | | | 2.68500152170235E-03 | |
| 3.25139227529630E+01 | | | -2.33464304622154E-04 | | | 1.35755209985893E-03 | |
| 3.53605454919441E+01 | | | -7.40024255770607E-05 | | | 6.71330037414992E-04 | |
| 3.83156069666220E+01 | | | -1.27131811524769E-05 | | | 3.25952700191411E-04 | |
| 4.14009965212542E+01 | | | 5.83078018676558E-06 | | | 1.53963513657933E-04 | |
| 4.45939437532236E+01 | | | 8.23567267220282E-06 | | | 7.05683858937510E-05 | |
| 4.79181001235074E+01 | | | 6.09255924741253E-06 | | | 3.08558623572342E-05 | |
| 5.13489331127671E+01 | | | 3.60552463379568E-06 | | | 1.27738830239340E-05 | |
| 5.491185E2987023E+01 | | | 1.85513385142841E-06 | | | 4.88950261394951E-06 | |
| 5.85943835641965E+01 | | | 8.57487222683428E-07 | | | 1.69291201392801E-06 | |
| 6.23822650468401E+01 | | | 3.61763259176369E-07 | | | 5.08360093577582E-07 | |
| 6.63235591279859E+01 | | | 1.38814280111638E-07 | | | 1.15665820127314E-07 | |
| 7.032932E3E79197E+01 | | | 4.87069025402085E-08 | | | 8.57255448554467E-09 | |
| 7.44893872E47162E+01 | | | 1.58616695717021E-08 | | | -1.10905148192891E-08 | |
| EIGENVALUE AND CONJUGATE EIGENFUNCTIONS NO. 2 1.0600000000E+00 | | | | | | | |
| 0. | | | 0. | | | 0. | |
| 1.0000000000000E-02 | | | -8.69519530805193E-06 | | | -2.52815052205637E-05 | |
| 2.0000000000000E-02 | | | -3.39523493750350E-05 | | | -1.00459167608672E-04 | |
| 3.0000000000000E-02 | | | -7.45416537000987E-05 | | | -2.24520718071720E-04 | |
| 4.4999999999999E-02 | | | -1.61536342966318E-04 | | | -5.00016835807367E-04 | |
| 6.81428683949825E-02 | | | -3.48931473193238E-04 | | | -1.12808927923015E-03 | |
| 1.53285815554841E-01 | | | -1.39148673268830E-03 | | | -5.35521121283800E-03 | |
| 3.72590895436845E-01 | | | -3.93038362827093E-03 | | | -2.61890546807597E-02 | |
| 7.10029610127151E-01 | | | -3.83589175539736E-03 | | | -6.73452998806918E-02 | |
| 1.16810150566051E+00 | | | -6.78939134014542E-03 | | | -1.06560156451943E-01 | |
| 1.74342597764184E+00 | | | -2.14490278886953E-02 | | | -1.17581773707561E-01 | |
| 2.44026468882656E+00 | | | -3.83616057015668E-02 | | | -1.01221433202409E-01 | |
| 3.25E70747957044E+00 | | | -3.91568443081112E-02 | | | -6.20101607080183E-02 | |
| 4.18508044493492E+00 | | | -7.63839777097708E-03 | | | 9.92956566809651E-04 | |

| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 |
|----------------------|----|----|-----------------------|----|----|-----------------------|----|
| 5.24429005133021E+00 | | | 3.97177922571035E-02 | | | 6.57123429445154E-02 | |
| 6.41454877398573E+00 | | | 6.06885777644510E-02 | | | 9.03789788032769E-02 | |
| 7.70852519603235E+00 | | | 4.50805539134367E-02 | | | 4.98002894663261E-02 | |
| 9.11666967597887E+00 | | | 4.71604951463764E-02 | | | -5.94319491016844E-02 | |
| 1.06494129136769E+01 | | | 1.35591320666842E-01 | | | -2.22167775229246E-01 | |
| 1.23017602309341E+01 | | | 2.94485384218987E-01 | | | -3.63392695078555E-01 | |
| 1.40669532042637E+01 | | | 4.08035325734462E-01 | | | -3.68788789597764E-01 | |
| 1.59580673371723E+01 | | | 3.89297788794009E-01 | | | -2.11617096227440E-01 | |
| 1.79611460677929E+01 | | | 2.71570497335306E-01 | | | 1.93767290027855E-03 | |
| 2.00910270163529E+01 | | | 1.44661071328695E-01 | | | 1.52781694393146E-01 | |
| 2.23405120444720E+01 | | | 6.21005711383047E-02 | | | 2.03027824667095E-01 | |
| 2.47006392684760E+01 | | | 2.49596625136401E-02 | | | 1.83762358792857E-01 | |
| 2.71889111224644E+01 | | | 1.36646736902045E-02 | | | 1.37931865751325E-01 | |
| 2.97869040935412E+01 | | | 1.19885313164789E-02 | | | 9.26571524561961E-02 | |
| 3.25139227529630E+01 | | | 1.17493632958510E-02 | | | 5.74945188032219E-02 | |
| 3.53605454919441E+01 | | | 1.04811163248553E-02 | | | 3.33096610587029E-02 | |
| 3.83156069666220E+01 | | | 8.22929508429632E-03 | | | 1.78103196585160E-02 | |
| 4.14009965212542E+01 | | | 5.66523355253881E-03 | | | 8.37137425128397E-03 | |
| 4.45939437532236E+01 | | | 3.399989958075259E-03 | | | 3.10800793111082E-03 | |
| 4.79181001235074E+01 | | | 1.72421306391670E-03 | | | 5.22663851835921E-04 | |
| 5.13489331127671E+01 | | | 6.91156204792816E-04 | | | -4.51110542564829E-04 | |
| 5.49118562987023E+01 | | | 1.65660254513518E-04 | | | -6.07999512135440E-04 | |
| 5.85943835641965E+01 | | | -3.42273569713914E-05 | | | -4.51907425055079E-04 | |
| 6.23022650468641E+01 | | | -7.21755676678982E-05 | | | -2.53514867514097E-04 | |
| 6.63035591279859E+01 | | | -5.36292503554574E-05 | | | -1.11782167060860E-04 | |
| 7.03293263679197E+01 | | | -2.83773231187326E-05 | | | -3.69582286494171E-05 | |
| 7.44893872647162E+01 | | | -1.11984106958818E-05 | | | -6.06986937417661E-06 | |

11.1725

2
EIGENVALUE AND CONJUGATE EIGENFUNCTIONS NO. 1 2.3700000000E+00

| 0. | 0. | 0. |
|----------------------|----------------------|-----------------------|
| 1.00000000000000E-02 | 2.67833760736946E-02 | -5.15251505856056E-02 |
| 2.00000000000000E-02 | 5.30625583253603E-02 | -1.01874912801769E-01 |
| 3.00000000000000E-02 | 7.88074957739657E-02 | -1.51024263681225E-01 |
| 4.49999999999999E-02 | 1.16363508544427E-01 | -2.22449038406235E-01 |
| 6.81428683949825E-02 | 1.71640383766994E-01 | -3.27104143804830E-01 |
| 1.53285815954841E-01 | 3.44118671938734E-01 | -6.52202540402399E-01 |
| 3.72599895436945E-01 | 5.63937095410314E-01 | -1.07314077204266E+00 |
| 7.10029610127151E-01 | 5.42510706785940E-01 | -9.31857530905500E-01 |
| 1.16810150566051E+00 | 4.74721832016675E-01 | -3.44354249413085E-01 |
| 1.74342597764184E+00 | 4.73797897477160E-01 | 9.31581358611679E-02 |
| 2.44026468882656E+00 | 4.43802726360362E-01 | 2.43951203098911E-01 |
| 3.25670747957044E+00 | 3.86559791690964E-01 | 2.63555657191302E-01 |
| 4.18908044493492E+00 | 3.12958534477563E-01 | 2.13692339808374E-01 |
| 5.24429005133021E+00 | 2.35792396071104E-01 | 1.36507751973840E-01 |
| 6.41454877398570E+00 | 1.69581710798080E-01 | 6.99000293998200E-02 |
| 7.70852519603235E+00 | 1.18287975089846E-01 | 2.74275187917110E-02 |
| 9.11666967597887E+00 | 8.06506340959054E-02 | 7.11411209514043E-03 |
| 1.06494129136769E+01 | 5.34101524019166E-02 | 2.79028766096090E-04 |
| 1.23017602309341E+01 | 3.42755108763746E-02 | -7.30310853090402E-04 |
| 1.40669532042637E+01 | 2.13381902207938E-02 | -3.29130633516623E-04 |
| 1.59580673371723E+01 | 1.28467170758664E-02 | -7.49930794329804E-05 |
| 1.79611460677929E+01 | 7.50617661303865E-03 | -1.15083437043135E-04 |
| 2.00910270163529E+01 | 4.23652766018562E-03 | -2.38522654276351E-04 |
| 2.23405120444720E+01 | 2.31319394395975E-03 | -3.05141127892416E-04 |
| 2.47006392684760E+01 | 1.22477581756603E-03 | -2.93680566403198E-04 |
| 2.71889111224644E+01 | 6.25973554386275E-04 | -2.36295385150925E-04 |
| 2.97869040935412E+01 | 3.10409732592443E-04 | -1.68059995933396E-04 |
| 3.25139227529630E+01 | 1.48686869868019E-04 | -1.08633922450165E-04 |

| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 |
|----------------------|----------------------|-----------|-----------------------|-----------|-------|-----------------------|----|
| | 3.53605454919441E+01 | | 6.88880832720551E-05 | | | -6.50379689448506E-05 | |
| | 3.83156069666220E+01 | | 3.10271683566852E-05 | | | -3.65438871071716E-05 | |
| | 4.14009965212542E+01 | | 1.35044074250793E-05 | | | -1.93267633277824E-05 | |
| | 4.45939437532236E+01 | | 5.71725849849897E-06 | | | -9.71388073449906E-06 | |
| | 4.79181001235074E+01 | | 2.34019483343113E-06 | | | -4.63334657245312E-06 | |
| | 5.13489331127671E+01 | | 9.32443248999940E-07 | | | -2.11472332925064E-06 | |
| | 5.49118562987023E+01 | | 3.59237586111701E-07 | | | -9.20258869335057E-07 | |
| | 5.85943835641965E+01 | | 1.34276644205201E-07 | | | -3.83590936288736E-07 | |
| | 6.23822650468401E+01 | | 4.88683144455833E-08 | | | -1.53910883719772E-07 | |
| | 6.63035591279859E+01 | | 1.71882282036868E-08 | | | -5.91161300058141E-08 | |
| | 7.03293263679197E+01 | | 5.88997291890889E-09 | | | -2.19123949843889E-08 | |
| | 7.44893872647162E+01 | | 1.94660630625993E-09 | | | -7.78565074227341E-09 | |
| EIGENVALUE | ANC | CONJUGATE | EIGEN | FUNCTIONS | NO. 2 | 1.510000000000E+00 | |
| 0. | | | 0. | | | 0. | |
| 1.00000000000000E-02 | | | 3.88063033243247E-02 | | | -1.53375986019278E-02 | |
| 2.00000000000000E-02 | | | 7.69740139635919E-02 | | | -3.03047007599950E-02 | |
| 3.00000000000000E-02 | | | 1.14474934736473E-01 | | | -4.48748452392091E-02 | |
| 4.00000000000000E-02 | | | 1.69419051382761E-01 | | | -6.5933887847412E-02 | |
| 6.81428683949825E-02 | | | 2.50947334564264E-01 | | | -9.64078598023085E-02 | |
| 1.53285815554841E-01 | | | 5.13737271069587E-01 | | | -1.85101714699446E-01 | |
| 3.72596895436845E-01 | | | 9.04404597705405E-01 | | | -2.55852619159532E-01 | |
| 7.10029610127151E-01 | | | 8.68124792815401E-01 | | | -2.25153091492161E-01 | |
| 1.16810150566051E+00 | | | 3.45374241005247E-01 | | | -3.98656499854033E-01 | |
| 1.74342597764184E+00 | | | -1.19196350875066E-01 | | | -5.96410665852027E-01 | |
| 2.4402646882656E+00 | | | -2.89263748237742E-01 | | | -5.56663489728083E-01 | |
| 3.25670747957944E+00 | | | -3.16233332528343E-01 | | | -4.01300153150739E-01 | |
| 4.18508044493492E+00 | | | -2.70586456071879E-01 | | | -2.02056678239782E-01 | |
| 5.24429005133021E+00 | | | -2.94741585567772E-01 | | | -1.22435114209345E-02 | |
| 6.41454877398570E+03 | | | -1.54446341675700E-01 | | | 1.20590482074696E-01 | |
| 7.70852519603235E+00 | | | -1.21557713204129E-01 | | | 1.85838982527843E-01 | |
| 9.11666967597817E+00 | | | -9.56485908048741E-02 | | | 1.95629438964109E-01 | |
| 1.06494129136769E+01 | | | -7.02727245954553E-02 | | | 1.70558467181611E-01 | |
| 1.23117602309341E+01 | | | -4.65354077224842E-02 | | | 1.33611176688984E-01 | |
| 1.40669532042637E+01 | | | -2.76058455786113E-02 | | | 9.78085087346914E-02 | |
| 1.5958067371723E+01 | | | -1.46736948501758E-02 | | | 6.83093814454936E-02 | |
| 1.79611460677929E+01 | | | -7.07182723300015E-03 | | | 4.60521438393322E-02 | |
| 2.00910270163529E+01 | | | -3.08817934231163E-03 | | | 2.99068302948385E-02 | |
| 2.23405120444720E+01 | | | -1.21779445260214E-03 | | | 1.87003140159376E-02 | |
| 2.47006392684760E+01 | | | -4.20119922575086E-04 | | | 1.12643125056804E-02 | |
| 2.71888911122464E+01 | | | -1.10100155145177E-04 | | | 6.50912988355490E-03 | |
| 2.97869040935412E+01 | | | -7.32254500619177E-06 | | | 3.62384652648332E-03 | |
| 3.25139227529630E+01 | | | 1.70496169813352E-05 | | | 1.93639245862539E-03 | |
| 3.53605454919441E+01 | | | 1.61936227279608E-05 | | | 9.95800845775822E-04 | |
| 3.83156069666220E+01 | | | 1.03182196228569E-05 | | | 4.94464848243826E-04 | |
| 4.14009965212542E+01 | | | 5.45645183054567E-06 | | | 2.35989938640799E-04 | |
| 4.45939437532236E+01 | | | 2.55315094911578E-06 | | | 1.08908574204372E-04 | |
| 4.79181001235074E+01 | | | 1.07570451409445E-06 | | | 4.83474632175509E-05 | |
| 5.13489331127671E+01 | | | 4.13740633153342E-07 | | | 2.07787870576726E-05 | |
| 5.49118562987023E+01 | | | 1.43811405615230E-07 | | | 8.59495597945543E-06 | |
| 5.85943835641965E+01 | | | 4.46115937752297E-08 | | | 3.43323763466789E-06 | |
| 6.23822650468401E+01 | | | 1.18891375860130E-08 | | | 1.32943857945384E-06 | |
| 6.63035591279859E+01 | | | 2.33059539609191E-09 | | | 4.95607023768681E-07 | |
| 7.03293263679197E+01 | | | 3.44089527994805E-11 | | | 1.79212157290131E-07 | |
| 7.44893872647162E+01 | | | -2.33971875959158E-10 | | | 6.23918134784871E-08 | |
| 7.8970 | | | | | | | |
| 1.0 | | | | | | | |

RESULTS OF SCALING
EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT

VARIABLE NO. 1
GAUSSIAN VARIANCE INCLUDING CORRECTION FOR NEGLECTED EIGENVALUES = 5.58625E+00
SCALED EIGENVALUES
2.24153E+00
7.49533E-01

VARIABLE NO. 2
GAUSSIAN VARIANCE INCLUDING CORRECTION FOR NEGLECTED EIGENVALUES = 3.94850E+00
SCALED EIGENVALUES
1.67584E+00
1.06773E+00

SIGMA RATIO OF TURBULENCE MODEL = 1.000

VARIANCE CHECK, VARIABLE NO. 1
CORRECT TOTAL VARIANCE = 1.11725E+01
SUM OF SCALED EIGENVALUES SQUARED = 5.58625E+00
GAUSSIAN VARIANCE = 5.58625E+00
TOTAL VARIANCE = 1.11725E+01

VARIANCE CHECK, VARIABLE NO. 2
CORRECT TOTAL VARIANCE = 7.89700E+00
SUM OF SCALED EIGENVALUES SQUARED = 3.94850E+00
GAUSSIAN VARIANCE = 3.94850E+00
TOTAL VARIANCE = 7.89700E+00

COVARIANCE MATRIX FOR P VARIABLES

EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT

| | | | |
|----------|----------|----------|----------|
| 1.000000 | 0.000000 | .943717 | -.261535 |
| 0.000000 | 1.000000 | .058157 | -.090917 |
| .943717 | .058157 | 1.000000 | 0.000000 |
| -.261535 | -.090917 | 0.000000 | 1.000000 |

COVARIANCE MATRIX FOR Q VARIABLES

EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT

| | | | |
|----------|----------|----------|----------|
| 1.000000 | 0.000000 | .171608 | .940547 |
| 0.000000 | 1.000000 | .032251 | -.063324 |
| .171608 | .032251 | 1.000000 | 0.000000 |
| .940547 | -.063324 | 0.000000 | 1.000000 |

FUNCTIONAL DEPENDENCE MATRIX

EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT
VARIABLE NO. 1

| | |
|----------|----------|
| 2.241528 | 0.000000 |
| 0.000000 | .749533 |

FUNCTIONAL DEPENDENCE MATRIX

EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT
VARIABLE NO. 2

| | |
|----------|----------|
| 1.675843 | 0.000000 |
| 0.000000 | 1.067731 |

CORRELATION COEFFICIENT OF RESPONSE AND ITS FIRST DERIVATIVE = 2.831E-03
THIS COEFFICIENT SHOULD BE MUCH LESS THAN 1.0

DETERMINANTS OF COVARIANCE MATRICES

DETA = 3.43320E-02
DETB = 8.25690E-02

INCREMENTS:

DX(1) = 1.0027587E+00 DX(2) = 8.4304804E-01
DF(1) = 1.5582014E-02 DF(2) = 1.8533938E-02

FIRST ROW, FIRST COLUMN, AND DIAGONAL OF JOINT CHARACTERISTIC FUNCTION

| | | | |
|--------------|--------------|--------------|--------------|
| 1.000E+00 0. | 9.484E-01 0. | 8.140E-01 0. | 6.410E-01 0. |
| 4.698E-01 0. | 3.238E-01 0. | 2.111E-01 0. | 1.307E-01 0. |
| 7.693E-02 0. | 4.310E-02 0. | 2.298E-02 0. | 1.167E-02 0. |
| 5.639E-03 0. | 2.594E-03 0. | 1.136E-03 0. | 4.730E-04 0. |
| 1.874E-04 0. | 7.064E-05 0. | 2.531E-05 0. | 8.626E-06 0. |
| 2.794E-06 0. | 8.603E-07 0. | 2.517E-07 0. | 6.999E-08 0. |
| 1.849E-08 0. | 4.640E-09 0. | 1.106E-09 0. | 2.504E-10 0. |
| 5.383E-11 0. | 1.099E-11 0. | 2.132E-12 0. | 4.035E-13 0. |
| 1.370E-13 0. | 4.035E-13 0. | 2.132E-12 0. | 1.099E-11 0. |
| 5.383E-11 0. | 2.504E-10 0. | 1.106E-09 0. | 4.640E-09 0. |
| 1.849E-08 0. | 6.999E-08 0. | 2.517E-07 0. | 8.603E-07 0. |
| 2.794E-06 0. | 8.626E-06 0. | 2.531E-05 0. | 7.064E-05 0. |
| 1.874E-04 0. | 4.730E-04 0. | 1.136E-03 0. | 2.594E-03 0. |
| 5.639E-03 0. | 1.167E-02 0. | 2.298E-02 0. | 4.310E-02 0. |
| 7.693E-02 0. | 1.307E-01 0. | 2.111E-01 0. | 3.238E-01 0. |
| 4.698E-01 0. | 6.410E-01 0. | 8.140E-01 0. | 9.484E-01 0. |
| 1.000E+00 0. | 9.483E-01 0. | 8.122E-01 0. | 6.354E-01 0. |
| 4.599E-01 0. | 3.112E-01 0. | 1.985E-01 0. | 1.200E-01 0. |
| 6.892E-02 0. | 3.769E-02 0. | 1.965E-02 0. | 9.765E-03 0. |
| 4.628E-03 0. | 2.091E-03 0. | 9.007E-04 0. | 3.697E-04 0. |
| 1.446E-04 0. | 5.383E-05 0. | 1.909E-05 0. | 6.441E-06 0. |
| 2.068E-06 0. | 6.319E-07 0. | 1.836E-07 0. | 5.071E-08 0. |
| 1.332E-08 0. | 3.324E-09 0. | 7.886E-10 0. | 1.777E-10 0. |
| 3.806E-11 0. | 7.742E-12 0. | 1.497E-12 0. | 2.823E-13 0. |
| 9.561E-14 0. | 2.823E-13 0. | 1.497E-12 0. | 7.742E-12 0. |
| 3.806E-11 0. | 1.777E-10 0. | 7.886E-10 0. | 3.324E-09 0. |
| 1.332E-08 0. | 5.071E-08 0. | 1.836E-07 0. | 6.319E-07 0. |
| 2.068E-06 0. | 6.441E-06 0. | 1.909E-05 0. | 5.383E-05 0. |
| 1.446E-04 0. | 3.697E-04 0. | 9.007E-04 0. | 2.091E-03 0. |
| 4.628E-03 0. | 9.765E-03 0. | 1.965E-02 0. | 3.769E-02 0. |
| 6.892E-02 0. | 1.200E-01 0. | 1.985E-01 0. | 3.112E-01 0. |
| 4.599E-01 0. | 6.354E-01 0. | 8.122E-01 0. | 9.483E-01 0. |
| 1.000E+00 0. | 9.802E-01 0. | 6.709E-01 0. | 4.308E-01 0. |
| 2.452E-01 0. | 1.253E-01 0. | 5.780E-02 0. | 2.413E-02 0. |
| 9.086E-03 0. | 3.095E-03 0. | 9.527E-04 0. | 2.648E-04 0. |
| 6.646E-05 0. | 1.505E-05 0. | 3.077E-06 0. | 5.675E-07 0. |
| 9.441E-08 0. | 1.417E-08 0. | 1.917E-09 0. | 2.338E-10 0. |
| 2.571E-11 0. | 2.549E-12 0. | 2.277E-13 0. | 1.833E-14 0. |
| 1.329E-15 0. | 8.683E-17 0. | 5.110E-18 0. | 2.709E-19 0. |
| 1.293E-20 0. | 5.558E-22 0. | 2.153E-23 0. | 7.820E-25 0. |
| 8.065E-26 0. | 7.820E-25 0. | 2.153E-23 0. | 5.558E-22 0. |
| 1.293E-20 0. | 2.709E-19 0. | 5.110E-18 0. | 8.683E-17 0. |
| 1.329E-15 0. | 1.833E-14 0. | 2.277E-13 0. | 2.549E-12 0. |
| 2.571E-11 0. | 2.338E-10 0. | 1.917E-09 0. | 1.417E-08 0. |
| 9.441E-08 0. | 5.675E-07 0. | 3.077E-06 0. | 1.505E-05 0. |
| 6.646E-05 0. | 2.648E-04 0. | 9.527E-04 0. | 3.095E-03 0. |
| 9.086E-03 0. | 2.410E-02 0. | 5.780E-02 0. | 1.253E-01 0. |
| 2.452E-01 0. | 4.308E-01 0. | 6.709E-01 0. | 9.802E-01 0. |

COMPUTED VALUES OF X AND N(X)

| | |
|---------------|---------------|
| 0. | 1.3474656E-01 |
| 1.0027587E+00 | 1.2746600E-01 |
| 2.0055174E+00 | 1.0203034E-01 |
| 3.0082761E+00 | 8.2746242E-02 |
| 4.0110348E+00 | 5.8122990E-02 |
| 5.0137935E+00 | 3.8195555E-02 |
| 6.0165522E+00 | 2.4032236E-02 |
| 7.0193109E+00 | 1.4803461E-02 |
| 8.0220696E+00 | 9.0793143E-03 |
| 9.0248283E+00 | 5.5959660E-03 |
| 1.0027587E+01 | 3.4756023E-03 |
| 1.1030346E+01 | 2.1737180E-03 |
| 1.2033104E+01 | 1.3666137E-03 |
| 1.3035863E+01 | 8.6235751E-04 |
| 1.4038622E+01 | 5.4557255E-04 |
| 1.5041380E+01 | 3.4580208E-04 |
| 1.6044139E+01 | 2.1948558E-04 |
| 1.7046898E+01 | 1.3945934E-04 |
| 1.8049657E+01 | 8.8685815E-05 |
| 1.9052415E+01 | 5.6435779E-05 |
| 2.0055174E+01 | 3.5933260E-05 |
| 2.1057933E+01 | 2.2889875E-05 |
| 2.2060691E+01 | 1.4587248E-05 |
| 2.3063450E+01 | 9.3000929E-06 |
| 2.4066209E+01 | 5.9323970E-06 |
| 2.5068967E+01 | 3.7874420E-06 |
| 2.6071726E+01 | 2.4222495E-06 |
| 2.7074485E+01 | 1.5552427E-06 |
| 2.8077243E+01 | 1.0978336E-06 |
| 2.9080002E+01 | 6.6737529E-07 |
| 3.0082761E+01 | 4.6386707E-07 |
| 3.1085520E+01 | 3.5549707E-07 |
| 3.2088278E+01 | 3.2001740E-07 |
| 3.3091037E+01 | 3.5016234E-07 |
| 3.4093796E+01 | 4.5215746E-07 |

LEVEL CROSSINGS
 EXAMPLE OF PROGRAM RLEVX USING 4 EIGENSOLUTIONS FROM NUMERICAL EXAMPLE OF REPORT

| DIMENSIONAL LEVEL X | NON-DIMENSIONAL LEVEL X/SIGMA X | CROSSINGS PER UNIT TIME | CROSSINGS PER ZERO CROSSING |
|---------------------------|---------------------------------------|----------------------------|--------------------------------|
| 0. | 0. | 1.347E-01 | 1.000E+00 |
| 6.685E-01 | 2.000E-01 | 1.315E-01 | 9.755E-01 |
| 1.337E+00 | 4.000E-01 | 1.221E-01 | 9.059E-01 |
| 2.006E+00 | 6.000E-01 | 1.080E-01 | 8.017E-01 |
| 2.674E+00 | 8.000E-01 | 9.140E-02 | 6.783E-01 |
| 3.343E+00 | 1.000E+00 | 7.421E-02 | 5.507E-01 |
| 4.011E+00 | 1.200E+00 | 5.812E-02 | 4.314E-01 |
| 4.680E+00 | 1.400E+00 | 4.420E-02 | 3.280E-01 |
| 5.348E+00 | 1.600E+00 | 3.285E-02 | 2.438E-01 |
| 6.017E+00 | 1.800E+00 | 2.403E-02 | 1.784E-01 |
| 6.685E+00 | 2.000E+00 | 1.742E-02 | 1.292E-01 |
| 7.354E+00 | 2.200E+00 | 1.257E-02 | 9.332E-02 |
| 8.022E+00 | 2.400E+00 | 9.079E-03 | 6.738E-02 |
| 8.691E+00 | 2.600E+00 | 6.570E-03 | 4.876E-02 |
| 9.359E+00 | 2.800E+00 | 4.770E-03 | 3.540E-02 |
| 1.003E+01 | 3.000E+00 | 3.476E-03 | 2.579E-02 |
| 1.070E+01 | 3.200E+00 | 2.540E-03 | 1.885E-02 |
| 1.136E+01 | 3.400E+00 | 1.861E-03 | 1.381E-02 |
| 1.203E+01 | 3.600E+00 | 1.367E-03 | 1.014E-02 |
| 1.270E+01 | 3.800E+00 | 1.005E-03 | 7.460E-03 |
| 1.337E+01 | 4.000E+00 | 7.402E-04 | 5.493E-03 |
| 1.404E+01 | 4.200E+00 | 5.456E-04 | 4.049E-03 |
| 1.471E+01 | 4.400E+00 | 4.025E-04 | 2.987E-03 |
| 1.538E+01 | 4.600E+00 | 2.972E-04 | 2.205E-03 |
| 1.604E+01 | 4.800E+00 | 2.195E-04 | 1.629E-03 |
| 1.671E+01 | 5.000E+00 | 1.622E-04 | 1.204E-03 |
| 1.738E+01 | 5.200E+00 | 1.199E-04 | 8.900E-04 |
| 1.805E+01 | 5.400E+00 | 8.869E-05 | 6.582E-04 |
| 1.872E+01 | 5.600E+00 | 6.561E-05 | 4.869E-04 |
| 1.939E+01 | 5.800E+00 | 4.855E-05 | 3.603E-04 |
| 2.006E+01 | 6.000E+00 | 3.593E-05 | 2.667E-04 |
| 2.072E+01 | 6.200E+00 | 2.660E-05 | 1.974E-04 |
| 2.139E+01 | 6.400E+00 | 1.970E-05 | 1.462E-04 |
| 2.206E+01 | 6.600E+00 | 1.459E-05 | 1.083E-04 |
| 2.273E+01 | 6.800E+00 | 1.081E-05 | 8.019E-05 |
| 2.340E+01 | 7.000E+00 | 8.005E-06 | 5.941E-05 |
| 2.407E+01 | 7.200E+00 | 5.932E-06 | 4.403E-05 |
| 2.473E+01 | 7.400E+00 | 4.398E-06 | 3.264E-05 |
| 2.540E+01 | 7.600E+00 | 3.262E-06 | 2.421E-05 |
| 2.607E+01 | 7.800E+00 | 2.422E-06 | 1.798E-05 |
| 2.674E+01 | 8.000E+00 | 1.802E-06 | 1.337E-05 |

DECK LISTINGS

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SUBROUTINE CF1(ANS,F)
C
C A SUBROUTINE TO EVALUATE THE CHARACTERISTIC FUNCTION OF THE
C NON-GAUSSIAN RESPONSE FOR THE CASE OF CORRELATED GAUSSIAN COMPONENTS
C SUBROUTINE INTEGRATES FUNCTION OF THE FORM
C  $(X(T)*A(-1)*X/2 + Y(T)*B(-1)*Y/2 + I*PI2*F*X(T)*C*Y)*DX*DY$ 
C FROM -INF TO +INF, WHERE X,Y ARE N VECTORS, (T) DENOTES TRANSPOSE,
C (-1) DENOTES INVERSE, A,B ARE SYMMETRIC COVARIANCE MATRICIES,
C AND I DENOTES SCRT(-1.).
C
C ANS = ANSWER
C A = INPUT CORRELATION MATRIX FOR FIRST SET OF GAUSSIAN PROCESSES
C A1,B1,C1 = SCRATCH MATRICIES USED IN RECURSIVE COMPUTATION
C A2 = .5*(INVERSE OF CORRELATION MATRIX A)
C B = INPUT CORRELATION MATRIX FOR SECOND SET OF GAUSSIAN PROCESSES
C B2 = .5*(INVERSE OF CORRELATION MATRIX B)
C C = FUNCTIONAL MATRIX
C DETA = DETERMINANT OF A
C DETB = DETERMINANT OF B
C F = INDEPENDENT VARIABLE OF CHARACTERISTIC FUNCTION
C N = NUMBER OF MODIFIED BESSEL PROCESSES
C COMMON /COR/ A(8,8),B(8,8),C(8,8),N,ISIZE,DETA,DET8,
C A1(8,8),B1(8,8),C1(8,8),A2(8,8),B2(8,8)
C DATA PI2/6.28318530717958/
C
C COPY MATRICIES
  DO 5 I=1,N
  DO 5 J=I,N
  A1(I,J)=A2(I,J)
  5 B1(I,J)=B2(I,J)
  F2=F*PI2
  DO 6 I=1,N
  DO 6 J=1,N
  6 C1(I,J)=C(I,J)*F2
C
C COMPUTE ANSWER FOR FIRST PAIR OF INTEGRATIONS
  A11=A1(1,1) $ B11=B1(1,1) $ C11=C1(1,1)
  DIV=4.*A11*B11+C11**2 $ ANS=1./(DETA*DET8*DIV)
  IF (N-1) 160,105,10
C
C COMPUTE ANSWER FOR ADDITIONAL INTEGRATIONS
  10 DO 100 NI=2,N
  K=NI-1
  DO 60 I=NI,N
  A1KI=A1(K,I) $ B1KI=B1(K,I) $ C1KI=C1(K,I) $ C1IK=C1(I,K)
  DO 60 J=NI,N
  A1KJ=A1(K,J) $ B1KJ=B1(K,J) $ C1KJ=C1(K,J) $ C1JK=C1(J,K)
  IF (J.LT.I) GO TO 50
  A1(I,J)=A1(I,J)-(4.*B11*A1KI*A1KJ+(C11*(A1KI*C1JK+C1IK*A1KJ)-
  1A11*C1IK*C1JK))/DIV
  B1(I,J)=B1(I,J)-(4.*A11*B1KI*B1KJ+(C11*(B1KI*C1KJ+C1KI*B1KJ)-
  1B11*C1KI*C1KJ))/DIV
  50 C1(I,J)=C1(I,J)-(4.*(A11*C1IK*B1KJ+B11*A1KI*C1KJ-C11*A1KI*B1KJ)+
  1C11*C1IK*C1KJ)/DIV
  60 CONTINUE
  A11=A1(NI,NI) $ B11=B1(NI,NI) $ C11=C1(NI,NI)
  DIV=4.*A11*B11+C11**2
  100 ANS=ANS/DIV
  105 ANS=SQRT(ANS)
  106 RETURN
C
C

```

```

CF100001
CF100002
CF100003
CF100004
CF100005
CF100006
CF100007
CF100008
CF100009
CF100010
CF100011
CF100012
CF100013
CF100014
CF100015
CF100016
CF100017
CF100018
CF100019
CF100020
CF100021
CF100022
CF100023
CF100024
CF100025
CF100026
CF100027
CF100028
CF100029
CF100030
CF100031
CF100032
CF100033
CF100034
CF100035
CF100036
CF100037
CF100038
CF100039
CF100040
CF100041
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CF100043
CF100044
CF100045
CF100046
CF100047
CF100048
CF100049
CF100050
CF100051
CF100052
CF100053
CF100054
CF100055
CF100056
CF100057
CF100058
CF100059
CF100060
CF100061
CF100062
CF100063

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C COMPUTE INVERSE AND DETERMINANT OF CORRELATION MATRICIES          CF100064
  ENTRY INVRS                                                         CF100065
  IF (N-1) 160,110,110                                              CF100066
110 DO 130 I=1,N                                                    CF100067
    DO 130 J=1,N                                                    CF100068
    A2(I,J)=A(I,J)                                                  CF100069
130 B2(I,J)=B(I,J)                                                 CF100070
    CALL INVR(A2,N,DETA,ISIZE,ISIZE)                                CF100071
    CALL INVR(B2,N,DETB,ISIZE,ISIZE)                               CF100072
    DO 140 I=1,N                                                    CF100073
    DO 140 J=1,N                                                    CF100074
    A2(I,J)=A2(I,J)/2.                                             CF100075
140 B2(I,J)=B2(I,J)/2.                                             CF100076
    IF ((DETA.GT.0.).AND.(DETB.GT.0.)) RETURN                       CF100077
    WRITE (6,150) OETA,DETB                                         CF100078
150 FORMAT (1H ,5X,*DETERMINANT .LE. ZERO, DETA =*,E11.3,*, DETB =*, CF100079
    1E11.3)                                                         CF100080
    GO TO 180                                                       CF100081
160 WRITE (6,170) N                                               CF100082
170 FORMAT (1H ,5X,*MATRIX DIMENSION .LE. ZERO, N =*,I3)        CF100083
180 STOP $ END                                                     CF100084

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          SUBROUTINE CF2(ANS,F1,F2)                                   CF200001
C                                                                    CF200002
C A SUBROUTINE TO EVALUATE THE 2-D CHARACTERISTIC FUNCTION OF THE  CF200003
C NON-GAUSSIAN RESPONSE FOR THE CASE OF CORRELATED GAUSSIAN COMPONENTS  CF200004
C SUBROUTINE INTEGRATES FUNCTION OF THE FORM                          CF200005
C (X(T)*A(-1)*X + Y(T)*B(-1)*Y + I*PI2*(X(T)*C*Y*F1 + X(T)*D*Y*F2)*DX*DY  CF200006
C FROM -INF TO +INF, WHERE X,Y ARE N VECTORS, (T) DENOTES TRANSPOSE,  CF200007
C (-1) DENOTES INVERSE/2, A,B, ARE COVARIANCE MATRICIES, I IS SQRT(-1.) CF200008
C                                                                    CF200009
C      COMMON /COR/ A(8,8),B(8,8),C(8,8),D(8,8),N,ISIZE,DETA,DETB,  CF200010
C      1A1(8,8),B1(8,8),C1(8,8),A2(8,8),B2(8,8)                   CF200011
C      DATA PI2/6.28318530717958/                                  CF200012
C                                                                    CF200013
C A = INPUT CORRELATION MATRIX FOR FIRST SET OF GAUSSIAN PROCESSES  CF200014
C ANS = ANSWER                                                       CF200015
C A2 = .5*(INVERSE OF CORRELATION MATRIX A)                         CF200016
C A1,B1,C1 = SCRATCH ARRAYS USED FOR RECURSIVE COMPUTATION        CF200017
C B = INPUT CORRELATION MATRIX FOR SECOND SET OF GAUSSIAN PROCESSES  CF200018
C B2 = .5*(INVERSE OF CORRELATION MATRIX B)                         CF200019
C C = FUNCTIONAL MATRIX FOR F1                                       CF200020
C D = FUNCTIONAL MATRIX FOR F2                                       CF200021
C DETA = DETERMINANT OF A                                           CF200022
C DETB = DETERMINANT OF B                                           CF200023
C F1,F2 = INDEPENDENT VARIABLES OF CHARACTERISTIC FUNCTION        CF200024
C N = NUMBER OF MODIFIED BESSEL PROCESSES                           CF200025
C                                                                    CF200026
C                                                                    CF200027
C      IF (N) 160,107,4                                             CF200028
C COPY MATRICIES                                                    CF200029
  4 DO 5 I=1,N                                                       CF200030
    DO 5 J=I,N                                                       CF200031
    A1(I,J)=A2(I,J)                                                 CF200032
  5 B1(I,J)=B2(I,J)                                                 CF200033
    DO 6 I=1,N                                                       CF200034
    DO 6 J=1,N                                                       CF200035
    C1(I,J)=(C(I,J)*F1+D(I,J)*F2)*PI2                               CF200036
C                                                                    CF200037
C COMPUTE ANSWER FOR FIRST PAIR OF INTEGRATIONS                    CF200038

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```

A11=A1(1,1) $ B11=B1(1,1) $ C11=C1(1,1)
DIV=4.*A11*B11+C11**2 $ DIV1=DETA*DETB/DIV
IF (N-1) 160,105,10
C
C COMPUTE ANSWER FOR ADDITIONAL INTEGRATIONS
10 DO 100 NI=2,N
   K=NI-1
   DO 60 I=NI,N
     A1KI=A1(K,I) $ C1IK=C1(I,K) $ B1KI=B1(K,I) $ C1KI=C1(K,I)
     DO 60 J=NI,N
       C1KJ=C1(K,J) $ B1KJ=B1(K,J)
       IF (J.LT.I) GO TO 55
       A1KJ=A1(K,J) $ C1JK=C1(J,K)
       A1(I,J)=A1(I,J)-(4.*B11*A1KI*A1KJ+(C11*(A1KI*C1JK+C1IK*A1KJ)
1-A11*C1IK*C1JK)/DIV
       B1(I,J)=B1(I,J)-(4.*A11*B1KI*B1KJ+(C11*(B1KI*C1KJ+C1KI*B1KJ)
1-B11*C1KI*C1KJ)/DIV
55 C1(I,J)=C1(I,J)-(4.*A11*C1IK*B1KJ+4.*B11*A1KI*C1KJ-4.*C11*A1KI*
1B1KJ+C11*C1IK*C1KJ)/DIV
60 CONTINUE
   A11=A1(NI,NI) $ B11=B1(NI,NI) $ C11=C1(NI,NI)
   DIV=4.*A11*B11+C11**2
100 DIV1=DIV1*DIV
105 ANS=1./SQRT(DIV1)
106 RETURN
C
107 ANS=1.0
   GO TO 106
C
C COMPUTE INVERSE AND DETERMINANT OF CORRELATION MATRICIES
ENTRY INVR
IF (N-1) 160,110,110
110 DO 130 I=1,N
   DO 130 J=1,N
     A2(I,J)=A(I,J)
130 B2(I,J)=B(I,J)
   CALL INVR(A2,N,DETA,ISIZE,ISIZE)
   CALL INVR(B2,N,DETB,ISIZE,ISIZE)
   DO 140 I=1,N
     DO 140 J=1,N
       A2(I,J)=A2(I,J)/2.
140 B2(I,J)=B2(I,J)/2.
   IF ((DETA.GT.0.).AND.(DETB.GT.0.)) GO TO 106
   WRITE (6,150) DETA,DETB
150 FORMAT (1H ,5X,*DETERMINANT .LE. ZERO, DETA =*,E11.3,*, DETB =*,
1E11.3)
   GO TO 180
160 WRITE (6,170) N
170 FORMAT (1H ,5X,*MATRIX DIMENSION .LE. ZERO, N =*,I3)
180 STOP $ END
CF200039
CF200040
CF200041
CF200042
CF200043
CF200044
CF200045
CF200046
CF200047
CF200048
CF200049
CF200050
CF200051
CF200052
CF200053
CF200054
CF200055
CF200056
CF200057
CF200058
CF200059
CF200060
CF200061
CF200062
CF200063
CF200064
CF200065
CF200066
CF200067
CF200068
CF200069
CF200070
CF200071
CF200072
CF200073
CF200074
CF200075
CF200076
CF200077
CF200078
CF200079
CF200080
CF200081
CF200082
CF200083
CF200084
CF200085
CF200086
CF200087
CF200088
CF200089

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```

SUBROUTINE COEF(I,A,C)
C A SUBROUTINE TO FILL C ARRAY WITH THE COEFFICIENTS OF THE
C POLYNOMIAL REPRESENTING A(X) BETWEEN X(I) AND X(I+1). NPOLY IS THE
C THE ORDER OF THE POLYNOMIAL
C
DIMENSION A(1),C(1)
COMMON /ARRAYS/ E(8),EFP(45,8),EFQ(45,8),X(45),FF1(45),FF2(45),
1NP,NPOLY
DATA NPOLY/3/
IF (I.GT.1) GO TO 10
N=1 $ GO TO 30
10 IF (I.LE.NP-2) GO TO 20
N=NP-3 $ GO TO 30
20 N=I-1
30 F1=A(N) $ X1=X(N) $ N=N+1 $ F2=A(N) $ X2=X(N)
N=N+1 $ F3=A(N) $ X3=X(N)
N=N+1 $ F4=A(N) $ X4=X(N)
F1=F1/(X1-X2)/(X1-X3)/(X1-X4)
F2=F2/(X2-X1)/(X2-X3)/(X2-X4)
F3=F3/(X3-X1)/(X3-X2)/(X3-X4)
F4=F4/(X4-X1)/(X4-X2)/(X4-X3)
C(4)=F1+F2+F3+F4
C(3)=-F1*(X2+X3+X4)-F2*(X1+X3+X4)-F3*(X1+X2+X4)-F4*(X1+X2+X3)
C(2)=F1*(X2*(X3+X4)+X3*X4)+F2*(X1*(X3+X4)+X3*X4)+F3*(X1*(X2+X4)+
1X2*X4)+F4*(X1*(X2+X3)+X2*X3)
C(1)=-F1*X2*X3*X4-F2*X1*X3*X4-F3*X1*X2*X4-F4*X1*X2*X3
RETURN $ END
COEF0001
COEF0002
COEF0003
COEF0004
COEF0005
COEF0006
COEF0007
COEF0008
COEF0009
COEF0010
COEF0011
COEF0012
COEF0013
COEF0014
COEF0015
COEF0016
COEF0017
COEF0018
COEF0019
COEF0020
COEF0021
COEF0022
COEF0023
COEF0024
COEF0025
COEF0026
COEF0027

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PROGRAM EIGU(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=
1PUNCH)
EIGU0001
EIGU0002
EIGU0003
C THIS IS A PROGRAM TO GENERATE EIGENVALUES AND EIGENFUNCTIONS FOR THE
C NON-GAUSSIAN TURBULENCE MODEL RESPONSE CALCULATIONS: THE EIGEN-
C FUNCTIONS ARE REPRESENTED BY PIECEWISE POLYNOMIAL FUNCTIONS OVER
C THE RANGE OF INTEGRATION, AND ALL INTEGRATIONS ARE EXACT WITHIN
C THIS APPROXIMATION.
EIGU0004
EIGU0005
EIGU0006
EIGU0007
C THIS PROGRAM IS FOR LONGITUDINAL GUSTS ONLY
EIGU0008
EIGU0009
C P,Q ARE EIGENFUNCTION ARRAYS FOR ITERATIVE PROCESS
EIGU0010
C EFP IS ARRAY FOR STORAGE OF EIGENFUNCTIONS
EIGU0011
C ER1 IS ERROR TOLERANCE ARRAY FOR EIGENVALUES USING 21 POINT EIGFN APP.
EIGU0012
C ER2 IS ERROR TOLERANCE ARRAY FOR EIGENVALUES USING 41 POINT EIGFN APP.
EIGU0013
C X(I) IS ARRAY CONTAINING ABSCISSAE OF EIGENFUNCTIONS
EIGU0014
C NP IS THE MAXIMUM NUMBER OF ABSCISSAE POINTS FOR EIGENFUNCTIONS.
EIGU0015
C CURRENTLY SET TO 41, VALUES .GT. 45 REQUIRE REDIMENSIONING, NP MUST
EIGU0016
C BE AN ODD NUMBER.
EIGU0017
C SHOULD COMPLEX POLES OCCUR, ONLY ONE OF EACH CONJUGATE PAIR
EIGU0018
C IS TO BE USED
EIGU0019
C
EIGU0020
C INPUT DATA:
EIGU0021
C RESTR1 (L1) LOGICAL INDICATOR FOR RESTART OPTION. IF RESTR1 IS
EIGU0022
C TRUE DATA DECK PUNCHED BY PREVIOUS RUN CAN BE USED
EIGU0023
C TO RESTART COMPUTATION.
EIGU0024
C
EIGU0025
C IF RESTR1 = .FALSE. DATA DECK IS AS FOLLOWS
EIGU0026
C NPOL (I1) NUMBER OF SYSTEM POLES TO BE READ. IF COMPLEX POLES
EIGU0027
C OCCUR, ONLY ONE OF EACH CONJUGATE PAIR IS USED.
EIGU0028
C (CI(I),AI(I),I=1,NPOL) (4E20,10) NUMERATORS AND POLES RESPECTIVELY
EIGU0029
C OF SYSTEM TRANSFER FUNCTION PARTIAL FRACTION EXPANSION
EIGU0030
C IN COMPLEX FORM. IF COMPLEX POLES OCCUR, ONLY ONE OF
EIGU0031
C THE PARTIAL FRACTION COMPLEX CONJUGATE TERMS IS USED.
EIGU0032

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C          REAL PART OF POLES MUST BE NEGATIVE FOR STABLE SYSTEM. EIGU0033
C          SU,U,UL (3F10.5) TURBULENCE VELOCITY STANDARD DEVIATION, EIGU0034
C          VEHICLE MEAN TRUE AIRSPEED, AND TURBULENCE GUST EIGU0035
C          VELOCITY SCALE LENGTH. EIGU0036
C          NEV (I1) TOTAL NUMBER OF EIGENVALUES/EIGENFUNCTIONS REQUIRED. EIGU0037
C          NEV MUST BE LESS THAN OR EQUAL TO 5. EIGU0038
C          EIGU0039
C          IF RESTRT = .TRUE. DATA DECK IS AS FOLLOWS EIGU0040
C          (ALL BUT LAST THREE CARDS (EOR,CONT,NEV) ARE PUNCHED BY EIGU0041
C          PREVIOUS RUN) EIGU0042
C          SU,U,UL (FORMAT 17) SEE DESCRIPTION ABOVE EIGU0043
C          B,B1,NPOL,(CI(I),AI(I),I=1,NPOL) (FORMAT 6) EIGU0044
C          NP,LO,HI (FORMAT 7) EIGU0045
C          J,E(J),(X(K),EFP(K,J),K=1,NP) (FORMAT 13) EIGU0046
C          END OF RECORD INDICATES END OF EIGENVALUE/EIGENFUNCTION DATA. EIGU0047
C          CONT (L1) LOGICAL VARIABLE, IF TRUE IT IS ASSUMED THAT ITERATION EIGU0048
C          OF THE LAST EIGENSOLUTION IS TO CONTINUE, IF FALSE IT IS ASSUMED EIGU0049
C          THAT THE LAST EIGENSOLUTION IS ACCURATE AND ITERATION OF THE EIGU0050
C          NEXT SOLUTION IS TO BEGIN. EIGU0051
C          NEV (I1) TOTAL NUMBER OF EIGENVALUES/EIGENFUNCTIONS SOUGHT (.LE.5) EIGU0052
C          EIGU0053
C          DIMENSION E(5),ER1(5),ER2(5) EIGU0054
C          COMMON /ARRAYS/ X(45),P(45),Q(45),NP,NSKIP,NPOLY,EFP(45,5),C(4), EIGU0055
C          1D(4) EIGU0056
C          COMMON /GCNST/ CI(5),AI(5),NPOL,B,B1 EIGU0057
C          COMMON /EVAL/ EVP EIGU0058
C          COMPLEX CI,AI $ REAL LO $ LOGICAL RESTRT,CONT EIGU0059
C          DATA ER1/1.E-3/ EIGU0060
C          DATA ER1,ER2,NP/1.E-4,1.E-4,5.E-4,1.E-3,1.E-2,1.E-5,1.E-5,1.E-4, EIGU0061
C          15.E-4,1.E-3,41/ EIGU0062
C          DATA (X(I),I=1,41)/0.,.01,.02,.03,.045,.06,.08,.1071,.1341, EIGU0063
C          1.1612,.1882,.2153,.2424,.2694,.2965,.3235,.3506,.3776,.4047, EIGU0064
C          2.4318,.4588,.4859,.5129,.5400,.5671,.5941,.6212,.6482,.6753, EIGU0065
C          3.7024,.7294,.7565,.7835,.8106,.8376,.8647,.8918,.9188,.9459, EIGU0066
C          4.9729,1./ EIGU0067
C          EIGU0068
C          1 FORMAT (I1) EIGU0069
C          2 FORMAT (4E20.10) EIGU0070
C          3 FORMAT (3F10.5) EIGU0071
C          4 FORMAT (I10) EIGU0072
C          5 FORMAT (L1) EIGU0073
C          6 FORMAT (3X,E20.12,5X,E20.12,11X,I2,/, (4E20.12)) EIGU0074
C          7 FORMAT (4X,I3,27X,2E20.11) EIGU0075
C          8 FORMAT (8F10.5) EIGU0076
C          11 FORMAT (*B =*,E20.12,1X,*B1 =*,E20.12,5X,*NPOL =*,I2,/, (4E20.12)) EIGU0077
C          12 FORMAT (*NP =*,I3,5X,*RANGE OF INTEGRATION =*,2E20.11) EIGU0078
C          13 FORMAT (43X,I2,E20.11,/, (2E25.14)) EIGU0079
C          14 FORMAT (*SU =*,E15.7,1X,*MTAS =*,E15.7,1X,*UL =*,E15.7) EIGU0080
C          17 FORMAT (4X,E15.7,7X,E15.7,5X,E15.7) EIGU0081
C          EIGU0082
C          READ (5,5) RESTRT EIGU0083
C          IF(.NOT.RESTRT) GO TO 20 EIGU0084
C          EIGU0085
C          RESTART IS TRUE, READ DATA FROM PREVIOUS RUN EIGU0086
C          REAC (5,17) SU,U,UL EIGU0087
C          READ (5,6) B,B1,NPOL,(CI(I),AI(I),I=1,NPOL) EIGU0088
C          READ (5,7) NP,LO,HI EIGU0089
C          DO 15 I1=1,6 EIGU0090
C          REAC (5,13) J,E(J),(X(K),EFP(K,J),K=1,NP) EIGU0091
C          IF (EOR,5) 16,15 EIGU0092
C          15 CONTINUE EIGU0093
C          GO TO 800 EIGU0094
C          16 REAC (5,5) CONT EIGU0095

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      READ (5,1) NEV
      I=I1-1
      DO 18 K=1,NP
18     P(K)=EFP(K,I)
      IF (.NOT.CONT) GO TO 95
      I1=I1-1 $ I=I1-1
      GO TO 95
C
C RESTART IS FALSE, READ DATA DEFINING NEW PROBLEM.
20 READ (5,1) NPOL
      IF (NPOL) 900,900,21
21 READ (5,2) (CI(I),AI(I),I=1,NPOL)
C CHANGE SIGN OF EXPONENT SINCE ALL SUBROUTINES EXPECT NEGATIVE OF
C TRUE EXPONENT.
      DO 22 I=1,NPOL
22 AI(I)=-AI(I)
      READ (5,3) SU,U,UL
      B=U/2./UL $ HI=20./B $ B1=SU*U/UL
      IF (HI.LE.1.) GO TO 26
C
C SCALE X VALUES TO COVER RANGE OF INTEGRATION, USE EI,EVP,EROR FOR
C TEMPORARY STORAGE.
C EI IS LOWER LIMIT OF POWER LAW SCALING.
C NPWR IS POWER OF SCALING, HIGHER VALUES WILL PLACE MORE X VALUES NEAR
C ORIGIN. LOW VALUES WILL PROVIDE MORE EVEN DISTRIBUTION OF X VALUES
C OVER RANGE OF INTEGRATION.
      EI=.05 $ NPWR=1
      EROP=((HI-EI)/(1.-EI)-1.)/(1.-EI)**NPWR
      DO 25 I=1,NP
      EVP=X(I)
      IF (EVP.LT.EI) GO TO 25
      EVP=EVP-EI $ EVP=EVP*(1.+EROR*EVP**NPWR)+EI
25 X(I)=EVP
C
C PUNCH PARAMETERS ON CARDS
26 WRITE (7,14) SU,U,UL
      WRITE (7,11) B,B1,NPOL,(CI(I),AI(I),I=1,NPOL)
      WRITE (7,12) NP,X(1),X(NP)
      I=0 $ I1=1
90 READ (5,1) NEV
      CONT=.F.
95 IF (NEV-I1) 800,100,100
C
C COMPUTE EXACT SYSTEM RESPONSE VARIANCE
100 CALL VARU(SU,UL,U,SR2)
      CALL STARAY(L0,L0)
      IF (.NOT.CONT) GO TO 110
      EROR=ER2(I1) $ NSKIP=1 $ GO TO 111
C
C GENERATE INITIAL GUESS OF EIGENFUNCTION P(ALPHA)
110 EROR=ER1(I1) $ NSKIP=2
      CALL GUESS(I1)
C
111 WRITE (6,101) I1,SU,U,UL,(CI(J),AI(J),J=1,NPOL)
101 FOR*AT (1F1,5X,*BEGIN ITERATION FOR EIGENVALUE AND EIGENFUNCTION NE
10.*I2,/,,6X,*TURBULENCE PARAMETERS*,,/,,6X,*STD. DEV. =*,F6.3,5X,
2*MTAS =*,F8.3,5X,*SCALE LENGTH =*,F9.3,/,,6X,*VEHICLE PARAMETERS:
3*,,/,,12X,*COEFFICIENTS*,26X,*POLES*,/,(4X,2E13.4,8X,2E13.4))
      WRITE (6,102) SR2
102 FOR*AT (1H0,5X,*SYSTEM RESPONSE VARIANCE =*,E13.6,/,,1H0)
      J=1 $ EVP=EI=1.E50 $ SEV=1. $ GO TO 210
C
200 CALL FILL

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EIGU0096
EIGU0097
EIGU0098
EIGU0099
EIGU0100
EIGU0101
EIGU0102
EIGU0103
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EIGU0125
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EIGU0158

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      EROR=ER2(I1)
C
C IF (I.GT.0) EIGENFUNCTION MUST BE ORTHOGONAL TO ALL PREVIOUS FNS
210 IF (I.LE.0) GO TO 300
      CALL ORTHOG(I)
C
C COMPUTE NORMALIZATION FACTOR
300 CALL NORMF(F)
C
C ESTIMATE EIGENVALUE
400 EI=EI*F
401 DEVP=ABS(ABS(1./EVP/F)-1.)
      EVP=SEV/F
      WRITE (6,405) J,EVP,DEVP,EI
405 FORMAT (1H,5X,*ITERATION NO.*,I3,*, ESTIMATED EIGENVALUE IS*,
      1E15.7,*, (DEVP =*,E10.3,2X,*EI =*,E10.3,*)*)
C
C NORMALIZE EIGENFUNCTION
500 CALL NORM(F)
      IF ((J.LT.30).OR.((J/5)*5.NE.J)) GO TO 520
      WRITE (6,510) (X(JJ),P(JJ),JJ=1,NP)
510 FORMAT (1H,2E25.14)
      WRITE (7,511) I1
511 FORMAT (43X,I2)
      WRITE (7,707) (X(JJ),P(JJ),P(JJ),JJ=1,NP)
C
C IF DEV IS SUFFICIENTLY SMALL, ACCEPT EIGENFUNCTION
520 IF ((DEVP.LE.ERCR).AND.(EI.LE.ERI)) GO TO 700
      IF ((J.GE.25).AND.(NSKIP.NE.1)) GO TO 200
C
C EIGENFUNCTION NOT YET FOUND, CONTINUE ITERATION PROCESS
600 CALL ITERAT(EI,SEV)
      J=J+1
      GO TO 210
C
C ERROR IS BELOW TOLERANCE, CHECK TO SEE THAT MAX. NO. OF POINTS USED
700 IF (NSKIP.NE.1) GO TO 200
C ACCEPT EIGENFUNCTION
      I=I+1 $ I1=I+1
      E(I)=EVP $ FS=EVP**2/SR2 $ TFS=0.
      DO 702 J=1,I
702 TFS=TFS+E(J)**2 $ TFS=TFS/SR2
      WRITE (7,706) I,EVP
706 FORMAT (*EIGENVALUE AND CONJUGATE EIGENFUNCTIONS NO.*,I2,E20.11)
      WRITE (7,707) (X(J),P(J),P(J),J=1,NP)
707 FORMAT (3E25.14)
      WRITE (6,708) I,EVP,FS,I,TFS,(X(J),P(J),J=1,NP)
708 FORMAT (1H1,5X,*EIGENVALUE NO.*,I2,* IS*,E14.7,2(/),6X,*FRACTION OF
2F RESPONSE VARIANCE DUE TO THIS EIGENVALUE IS*,F7.5,/,6X,
3*FRACTION OF RESPONSE VARIANCE DUE TO FIRST*,I2,* EIGENVALUES IS*,
4F7.5,/,/,9X,*X*,11X,*EIGENFUNCTION*,/, (6X,F8.4,5X,E17.10))
      IF (I.GE.NEV) GO TO 800
      CALL STCR(I)
      CONT=.F.
      GO TO 110
C
800 WRITE (6,805) NEV,(J,E(J),J=1,I)
805 FORMAT (1H1,5X,*FIRST*,I2,* EIGENVALUES ARE:*,/, (10X,I2,2X,E15.7))
      FS=SR2*(1.-TFS)
      WRITE (6,810) FS,TFS
810 FORMAT (1H0,5X,*REMAINING EIGENVALUES SQUARED SUM TO*,E15.7,/,6X,
1*THE ABOVE EIGENVALUES ACCOUNT FOR*,F7.5,* OF THE RESPONSE VAR.*)
      IF (FS.LT.0.) GO TO 900

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EIGU0159
 EIGU0160
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 EIGU0220
 EIGU0221

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      FS=SQRT(FS)
      WRITE (6,812) FS
812  FORMAT (1H,5X,*LARGEST POSSIBLE REMAINING EIGENVALUE IS*,E14.7)
900  STOP $ END
      SUBROUTINE VARU(SU,UL,U,S2)
C
C A SUBROUTINE TO COMPUTE THE RESPONSE VARIANCE OF A LINEAR SYSTEM
C SUBJECTED TO RANDOM INPUT WITH DRYDEN U-GUST SPECTRUM
C
C SU = TURBULENCE STANDARD DEVIATION
C UL = TURBULENCE SCALE LENGTH
C U = VEHICLE MEAN TRUE AIRSPEED
C AI = VEHICLE IMPULSE RESPONSE EXPONENTS (MUST HAVE POSITIVE REAL PART)
C CI = VEHICLE IMPULSE RESPONSE CONSTANTS
C IF CI OR AI IS COMPLEX, ONLY ONE OF EACH CONJUGATE PAIR IS TO BE USED
C
      COMMON /CCNST/ CI(5),AI(5),NPOL
      LOGICAL RL1,RL2
      C=U/UL $ S2=0.
      DO 20 K=1,NPOL
      C4=C. $ AK=AI(K) $ CK=CI(K)
      RL1=AIMAG(AK).EQ.0..AND.AIMAG(CK).EQ.0.
5     DO 10 J=1,NPOL
      C3=C. $ AJ=AI(J) $ CJ=CI(J)
      RL2=AIMAG(AJ).EQ.0..AND.AIMAG(CJ).EQ.0.
6     C2=1./(C-AJ)
      C2=C2*(1./(AJ+AK)/(AJ+C)-.5/C/(AK+C))
      C2=C2*CJ*CK
      C3=C3+C2
      IF (RL2) GO TO 10
      AJ=CONJG(AJ) $ CJ=CONJG(CJ) $ RL2=.T. $ GO TO 6
10    C4=C4+C3
      IF (RL1) GO TO 20
      AK=CONJG(AK) $ CK=CONJG(CK) $ RL1=.T. $ GO TO 5
20    S2=S2+REAL(C4)
      S2=S2*2.*C*SU**2
      RETURN $ END
      SUBROUTINE FILL
C
C THIS SUBROUTINE INTERPOLATES THE 21 POINT EIGENFUNCTION TO FORM THE
C FIRST APPROXIMATION TO THE 41 POINT EIGENFUNCTION.
C
      COMMON /ARRAYS/ X(45),P(45),I,I1,K,XP,C(41),NP,NSKIP
      I1=NSKIP/2+1
      DO 10 I=I1,NP,NSKIP
      XP=X(I) $ K=I+1-I1
      CALL COEF(K,P,C)
10    P(I)=((C(4)*XP+C(3))*XP+C(2))*XP+C(1)
      NSKIP=NSKIP/2
      RETURN $ END
      SUBROUTINE GUESS(I)
C
C THIS SUBROUTINE GUESSES THE NEXT EIGENFUNCTION TO BE THE
C DERIVATIVE OF THE PROVIOUS RESULT OR X*EXP(-3.*X) FOR THE FIRST.
C
      COMMON /ARRAYS/ X(45),P(45),J,XJ,K,DX,DP,Q(40),NP,NSKIP
      IF (NSKIP.LE.1) GO TO 70
      IF (I.GT.1) GO TO 50
      DO 10 J=1,NP,NSKIP
      XJ=X(J)
10    P(J)=XJ*EXP(-3.*XJ)
      RETURN

```

EIGU0222
 EIGU0223
 EIGU0224
 EIGU0225
 EIGU0226
 EIGU0227
 EIGU0228
 EIGU0229
 EIGU0230
 EIGU0231
 EIGU0232
 EIGU0233
 EIGU0234
 EIGU0235
 EIGU0236
 EIGU0237
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 EIGU0240
 EIGU0241
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 EIGU0270
 EIGU0271
 EIGU0272
 EIGU0273
 EIGU0274
 EIGU0275
 EIGU0276
 EIGU0277
 EIGU0278
 EIGU0279
 EIGU0280
 EIGU0281
 EIGU0282
 EIGU0283
 EIGU0284

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C
50 K=NSKIP+1
DO 60 J=K,NF,NSKIP
DP=F(J)-P(J-1) $ DX=X(J)-X(J-1) $ XJ=X(J)
XJ=XJ*EXP(-3.*XJ)
60 P(J)=XJ*DP/DX
RETURN
C
70 WRITE (6,71) NSKIP
71 FORMAT (1I,5X,*NSKIP =*,I10,* NOT ALLOWED IN GUESS*)
STOP $ END
SUBROUTINE NORMF(F)
C
C COMPUTES NORM OF THE FUNCTION P(X)
C THE NORM IS THE ESTIMATED EIGENVALUE, THE INVERSE OF THE NORM IS THE
C NORMALIZATION FACTOR
COMMON /ARRAYS/ X(45),P(45),C(4),F1,NP1,J,I,X1,X2,II,CII,IJ,K,CK,
1X12,X22,JJ,C(27),NP,NSKIP,NPOLY
F1=0. $ NP1=NPOLY*1 $ J=NP-1
DO 60 I=1,J,NSKIP
F = 0.
CALL COEF(I,P,C)
12 X1=X(I) $ X2=X(I+NSKIP)
IF (X2.GT.X(NP)) X2=X(NP)
DO 40 II=1,NPOLY
CII=C(II) $ IJ=II+1 $ K=II+IJ-1 $ CK=FLOAT(K)
X12=X1**K $ X22=X2**K
DO 40 JJ=IJ,NP1
F = F+CII*C(JJ)*(X22-X12)/CK
CK=CK+1. $ X12=X12*X1
40 X22=X22*X2
F=F*2. $ X12=X1**2 $ X22=X2**2 $ CK=1.
DO 50 II=1,NP1
F=F+(X2-X1)*(C(II)**2)/CK $ CK=CK+2. $ X2=X2*X2
50 X1=X1*X12
60 F1=F1+F
F=SGRT(1./F1)
RETURN
C
C NORMALIZE EIGENFUNCTION P
ENTRY NCRM
DO 100 I=1,NP,NSKIP
100 P(I)=P(I)*F
RETURN $ END
SUBROUTINE CRTHOG(I)
C
C A SUBROUTINE TO ORTHOGONALIZE P(X) WITH RESPECT TO THE FIRST I EGNFNS
COMMON /ARRAYS/ X(45),P(45),Q(45),NP,NSKIP,NPOLY,EFP(45,5),C(4),
1D(4)
NP1 = NPOLY*1
DO 50 J=1,I
DO 10 JJ=1,NP,NSKIP
10 C(JJ)=EFP(JJ,J)
K=NP-1 $ F=0.
DO 40 JJ=1,K,NSKIP
CALL COEF(JJ,P,C)
CALL COEF(JJ,Q,D)
X1=X(JJ) $ II=JJ+NSKIP $ IF (II.GT.NP) II=NP $ X2=X(II)
DO 40 II=1,NP1
X12=X1**II $ X22=X2**II $ CII=C(II) $ CL=FLOAT(II)
DO 40 KK=1,NP1
EIGU0285
EIGU0286
EIGU0287
EIGU0288
EIGU0289
EIGU0290
EIGU0291
EIGU0292
EIGU0293
EIGU0294
EIGU0295
EIGU0296
EIGU0297
EIGU0298
EIGU0299
EIGU0300
EIGU0301
EIGU0302
EIGU0303
EIGU0304
EIGU0305
EIGU0306
EIGU0307
EIGU0308
EIGU0309
EIGU0310
EIGU0311
EIGU0312
EIGU0313
EIGU0314
EIGU0315
EIGU0316
EIGU0317
EIGU0318
EIGU0319
EIGU0320
EIGU0321
EIGU0322
EIGU0323
EIGU0324
EIGU0325
EIGU0326
EIGU0327
EIGU0328
EIGU0329
EIGU0330
EIGU0331
EIGU0332
EIGU0333
EIGU0334
EIGU0335
EIGU0336
EIGU0337
EIGU0338
EIGU0339
EIGU0340
EIGU0341
EIGU0342
EIGU0343
EIGU0344
EIGU0345
EIGU0346
EIGU0347

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```

F=F*CI1*D(KK)*(X22-X12)/CL $ CL=CL+1. $ X12=X12*X1
40 X22 = X22*X2
DO 50 JJ=1, NP, NSKIP
50 P(JJ)=P(JJ)-F*Q(JJ)
RETURN $ END
FUNCTION SUM(I,J)
C
C FORMS PRODUCT (J-I+1)*(J-I+2).....(J)
C
N=1
IF(I) 20,20,10
10 K=J-I+1
DO 15 L=K,J
15 N=N*L
20 SUM=FLOAT(N)
RETURN $ END
SUBROUTINE COEF(I,A,C)
C A SUBROUTINE TO FILL C ARRAY WITH THE COEFFICIENTS OF THE
C POLYNOMIAL REPRESENTING A(X) BETWEEN X(I) AND X(I+NSKIP). NPOLY IS
C THE ORDER OF THE POLYNOMIAL
C POLYNOMIAL IS C(1) + C(2)*X + C(3)*X**2 + C(4)*X**3
C
DIMENSION A(1),C(1)
COMMON /ARRAYS/ X(45),P(45),Q(45),NP,NSKIP,NPOLY
DATA NPCLY/3/
IF (I.GT.NSKIP) GO TO 10
N=1 $ GC TO 30
10 IF (I.LE.NP-2*NSKIP) GO TO 20
N=NP-3*NSKIP $ GO TO 30
20 N=I-NSKIP
30 F1=A(N) $ X1=X(N) $ N=N+NSKIP $ F2=A(N) $ X2=X(N)
N=N+NSKIP $ F3=A(N) $ X3=X(N)
N=N+NSKIP $ F4=A(N) $ X4=X(N)
F1=F1/(X1-X2)/(X1-X3)/(X1-X4)
F2=F2/(X2-X1)/(X2-X3)/(X2-X4)
F3=F3/(X3-X1)/(X3-X2)/(X3-X4)
F4=F4/(X4-X1)/(X4-X2)/(X4-X3)
C(4)=F1+F2+F3+F4
C(3)=-F1*(X2+X3+X4)-F2*(X1+X3+X4)-F3*(X1+X2+X4)-F4*(X1+X2+X3)
C(2)=F1*(X2*(X3+X4)+X3*X4)+F2*(X1*(X3+X4)+X3*X4)+F3*(X1*(X2+X4)+
1X2*X4)+F4*(X1*(X2+X3)+X2*X3)
C(1)=-F1*X2*X3*X4-F2*X1*X3*X4-F3*X1*X2*X4-F4*X1*X2*X3
RETURN $ END
SUBROUTINE C1FC
C INTEGRATE COMPLEX TERMS OF TRANSFER FUNCTION
C
EQUIVALENCE (E,ER),(XE,XER),(XE1,XE1R),(D1,D1R)
COMMON /DIF/ X,E,N,D1
COMPLEX E,XE,XE1,D1
N1=N+1 $ D1=0.
IF (N) 70,10,20
C
C CONSTANT TERM OF POLYNOMIAL
10 D1=1. $ GC TO 30
C
C POWER TERM OF POLYNOMIAL
20 IF (X.NE.0.) GO TO 25
D1=(-1./E)**N*SUM(N,N)
GO TO 30
25 XN=X**N
XE1=X*E $ XE=1. $ S1=1. $ S2=FLOAT(N)
DO 28 I=1,N1
D1=XN/XE*S1+D1 $ XE=XE*XE1 $ S1=S1*S2 $ S2=S2-1.
EIGU0348
EIGU0349
EIGU0350
EIGU0351
EIGU0352
EIGU0353
EIGU0354
EIGU0355
EIGU0356
EIGU0357
EIGU0358
EIGU0359
EIGU0360
EIGU0361
EIGU0362
EIGU0363
EIGU0364
EIGU0365
EIGU0366
EIGU0367
EIGU0368
EIGU0369
EIGU0370
EIGU0371
EIGU0372
EIGU0373
EIGU0374
EIGU0375
EIGU0376
EIGU0377
EIGU0378
EIGU0379
EIGU0380
EIGU0381
EIGU0382
EIGU0383
EIGU0384
EIGU0385
EIGU0386
EIGU0387
EIGU0388
EIGU0389
EIGU0390
EIGU0391
EIGU0392
EIGU0393
EIGU0394
EIGU0395
EIGU0396
EIGU0397
EIGU0398
EIGU0399
EIGU0400
EIGU0401
EIGU0402
EIGU0403
EIGU0404
EIGU0405
EIGU0406
EIGU0407
EIGU0408
EIGU0409
EIGU0410

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28 XN=-XN
30 D1=D1*CEXF(E*X)/E
70 RETURN
C
C INTEGRATE REAL TERMS OF TRANSFER FUNCTION
ENTRY DIFR
N1=N+1 $ D1=0.
IF (N) 170,110,120
C
C CONSTANT TERM OF POLYNOMIAL
110 D1R=1. $ GO TO 130
C
C POWER TERM OF POLYNOMIAL
120 IF (X.NE.0.) GO TO 125
D1R=(-1./ER)**N*SUN(N,N)
GO TO 130
125 XN=X**N
XE1R=X*ER $ XER=1. $ S1=1. $ S2=FLOAT(N)
DO 128 I=1,N1
D1R=XN/XER*S1+D1R $ XER=XER*XE1R $ S1=S1*S2 $ S2=S2-1.
128 XN=-XN
130 D1=CMPLX(D1R*EXP(ER*X)/ER,0.0)
170 RETURN $ END
SUBROUTINE NTGRAL(V,ALPHA)
C
C INTEGRATES P(X)*H(ALPHA,X)
C
COMMON /ARRAYS/ X(45),P(45),Q(45),NP,NSKIP,NPOLY,EFP(225),C(4)
COMMON /CONST/ CI(5),AI(5),NPOL,B,B1
COMMON /DIF/ X1,E,NPOL,D1C
DIMENSION D1S(4),D2S(4)
DIMENSION D1A(45,4),D1CA(45,4,2),CA(180)
COMPLEX D1CA
COMPLEX AI,CI,E,C1,C2,V1,V2,E2,D2S,D1C
LOGICAL RL
EQUIVALENCE (E,ER),(D1,D1C)
V=0. $ E1=EXP(-B*ALPHA)
C
C CYCLE THROUGH PCLES OF TRANSFER FUNCTION
DO 100 IPCL=1,NPOL
C1=CI(IPCL)/(AI(IPCL)-2.*B)
E2=CEXP(-ALPHA*(AI(IPCL)-B))
V1=0. $ V2=0.
C
C CYCLE THROUGH X INTERVALS
DO 10 NX=1,NP01
D1S(NX)=D1A(1,NX)
10 D2S(NX)=D1CA(1,NX,IPOL)
I=NSKIP+1 $ NXI=1
DO 80 NX=I,NP,NSKIP
X1=X(NX)
C
C CYCLE THROUGH TERMS OF POLYNOMIAL
DO 60 NPO=1,NP01
D1=D1A(NX,NPO) $ E=D1-D1S(NPO) $ V1=E1*E
20 D1S(NPO)=C1
IF (X1-ALPHA) 21,21,40
21 D1C=D1CA(NX,NPO,IPOL)
V1=V1-E1*(D1C-D2S(NPO))
25 D2S(NPO)=D1C
GO TO 59
40 V1=V1-E2*E
59 V2=V1*CA(NXII)+V2
EIGU0411
EIGU0412
EIGU0413
EIGU0414
EIGU0415
EIGU0416
EIGU0417
EIGU0418
EIGU0419
EIGU0420
EIGU0421
EIGU0422
EIGU0423
EIGU0424
EIGU0425
EIGU0426
EIGU0427
EIGU0428
EIGU0429
EIGU0430
EIGU0431
EIGU0432
EIGU0433
EIGU0434
EIGU0435
EIGU0436
EIGU0437
EIGU0438
EIGU0439
EIGU0440
EIGU0441
EIGU0442
EIGU0443
EIGU0444
EIGU0445
EIGU0446
EIGU0447
EIGU0448
EIGU0449
EIGU0450
EIGU0451
EIGU0452
EIGU0453
EIGU0454
EIGU0455
EIGU0456
EIGU0457
EIGU0458
EIGU0459
EIGU0460
EIGU0461
EIGU0462
EIGU0463
EIGU0464
EIGU0465
EIGU0466
EIGU0467
EIGU0468
EIGU0469
EIGU0470
EIGU0471
EIGU0472
EIGU0473

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60 NXI=NXI+1
80 CONTINUE
   V2=V2*C1
   IF ((AIMAG(C1).EQ.0.).AND.(AIMAG(E2).EQ.0.)) GO TO 100
   V2=2.*REAL(V2)
100 V=V+V2
   V=V*B1
   GO TO 500
C
C
ENTRY STARAY
NPO1=NPOLY+1 $ ER=-B
DO 200 NX=1,NP
  X1=X(NX)
  DO 200 NPO=1,NPO1
    NPOH=NPO-1
    CALL DIFR
200 D1A(NX,NPC)=D1
    DO 300 IPCL=1,NPOL
      E=B-AI(IPCL) $ RL=AIMAG(E).EQ.0.
      DO 300 NPO=1,NPO1
        NPOH=NPO-1
        DO 300 NX=1,NP
          X1=X(NX) $ IF(RL) GO TO 250 $ CALL DIFC $ GO TO 300
250 CALL DIFR
300 D1CA(NX,NFO,IPOL)=D1C
   GO TO 500
C
C GENERATE TABLE OF POLYNOMIAL COEFFICIENTS
ENTRY GENCOE
NPO=0
DO 400 NXI=1,NP,NSKIP
  IF (NXI-NP) 350,400,400
350 CALL COEF(NXI,P,C)
   DO 380 NPO1=1,4
     NPO=NPO+1
380 CA(NPO)=C(NPO1)
400 CONTINUE
   NPO1=NPOLY+1
500 RETURN $ END
SUBROUTINE ITERAT(E,SEV)
C
C A SUBROUTINE TO ITERATE THE EIGENFUNCTION P(X)
C
COMMON /ARRAYS/ X(45),P(45),Q(45),NP,NSKIP,NPOLY
COMMON /EVAL/ EVP
CALL GENCOE(Q,X)
Q(1)=0. $ E=0. $ I1=NSKIP+1
DO 50 I=I1,NP,NSKIP
30 QI=P(I)*EVP
   CALL NTGRAL(Q(I),X(I))
50 E=E+(ABS(QI)-ABS(Q(I)))**2
   E=SQRT(E)/FLOAT(NP) $ S1=S2=0.
   DO 60 I=1,NP,NSKIP
     S1=S1+ABS(P(I)+Q(I)) $ S2=S2+ABS(P(I)-Q(I))
60 P(I)=Q(I)
   IF (S1-S2) 70,80,90
70 SEV=-1. $ GO TO 100
80 WRITE (6,81) $ STOP
81 FORMAT (1H ,5X,*ZERO EIGFN. COMPUTED BY ITERAT*)
90 SEV=1.
100 RETURN $ END
SUBROUTINE STOR(I)
EIGU0474
EIGU0475
EIGU0476
EIGU0477
EIGU0478
EIGU0479
EIGU0480
EIGU0481
EIGU0482
EIGU0483
EIGU0484
EIGU0485
EIGU0486
EIGU0487
EIGU0488
EIGU0489
EIGU0490
EIGU0491
EIGU0492
EIGU0493
EIGU0494
EIGU0495
EIGU0496
EIGU0497
EIGU0498
EIGU0499
EIGU0500
EIGU0501
EIGU0502
EIGU0503
EIGU0504
EIGU0505
EIGU0506
EIGU0507
EIGU0508
EIGU0509
EIGU0510
EIGU0511
EIGU0512
EIGU0513
EIGU0514
EIGU0515
EIGU0516
EIGU0517
EIGU0518
EIGU0519
EIGU0520
EIGU0521
EIGU0522
EIGU0523
EIGU0524
EIGU0525
EIGU0526
EIGU0527
EIGU0528
EIGU0529
EIGU0530
EIGU0531
EIGU0532
EIGU0533
EIGU0534
EIGU0535
EIGU0536

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C
C STORES ITH EIGENFUNCTION
C
COMMON /ARRAYS/ X(45),P(45),J,DUM(44),NP,NSKIP,NPOLY, EFP(45,5)
DO 10 J=1,NP
10 EFP(J,I)=P(J)
RETURN $ END
EIGU0537
EIGU0538
EIGU0539
EIGU0540
EIGU0541
EIGU0542
EIGU0543

PROGRAM EIGVM(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=
1PUNCH)
EIGVM000
EIGVM001
EIGVM002
EIGVM003
EIGVM004
EIGVM005
EIGVM006
EIGVM007
EIGVM008
EIGVM009
EIGVM010
EIGVM011
EIGVM012
EIGVM013
EIGVM014
EIGVM015
EIGVM016
EIGVM017
EIGVM018
EIGVM019
EIGVM020
EIGVM021
EIGVM022
EIGVM023
EIGVM024
EIGVM025
EIGVM026
EIGVM027
EIGVM028
EIGVM029
EIGVM030
EIGVM031
EIGVM032
EIGVM033
EIGVM034
EIGVM035
EIGVM036
EIGVM037
EIGVM038
EIGVM039
EIGVM040
EIGVM041
EIGVM042
EIGVM043
EIGVM044
EIGVM045
EIGVM046
EIGVM047
EIGVM048
EIGVM049
EIGVM050
EIGVM051

C THIS IS A PROGRAM TO GENERATE EIGENVALUES AND EIGENFUNCTIONS FOR THE
C NON-GAUSSIAN TURBULENCE MODEL RESPONSE CALCULATIONS: THE EIGEN-
C FUNCTIONS ARE REPRESENTED BY PIECEWISE POLYNOMIAL FUNCTIONS OVER
C THE RANGE OF INTEGRATION, AND ALL INTEGRATIONS ARE EXACT WITHIN
C THIS APPROXIMATION.
C THIS PROGRAM IS FOR VERTICAL AND LATERAL GUSTS ONLY
C P(ALPHA),Q(BETA) ARE EIGENFUNCTION ARRAYS FOR ITERATIVE PROCESS
C EFP,EFQ ARE ARRAYS FOR STORAGE OF EIGENFUNCTIONS
C ER1 IS ERROR TOLERANCE ARRAY FOR EIGENVALUES USING 21 POINT EIGFN APP.
C ER2 IS ERROR TOLERANCE ARRAY FOR EIGENVALUES USING 41 POINT EIGFN APP.
C X(I) IS APRAY CONTAINING ABCISSAE OF EIGENFUNCTIONS
C NP IS THE MAXIMUM NUMBER OF ABCISSAE POINTS FOR EIGENFUNCTIONS.
C CURRENTLY SET TO 41, VALUES .GT. 45 REQUIRE REDIMENSIONING, NP MUST
C BE AN ODD NUMBER.
C E IS THE EIGENVALUE ARRAY
C HI IS THE UPPER LIMIT OF INTEGRATION
C NPOL IS THE NUMBER OF POLES IN THE LINEAR SYSTEM TRANSFER FUNCTION
C SHOULD COMPLEX POLES OCCUR, ONLY ONE OF EACH CONJUGATE PAIR
C IS TO BE USED
C INPUT DATA:
C RESTRT (L1) LOGICAL INDICATOR FOR RESTART OPTION. IF RESTRT IS
C TRUE DATA DECK PUNCHED BY PREVIOUS RUN CAN BE USED
C TO RESTART COMPUTATION.
C IF RESTRT = .FALSE. DATA DECK IS AS FOLLOWS
C NPOL (I1) NUMBER OF SYSTEM POLES TO BE READ. IF COMPLEX POLES
C OCCUR, ONLY ONE OF EACH CONJUGATE PAIR IS USED.
C (CI(I),AI(I),I=1,NPOL) (4E20.10) NUMERATORS AND POLES RESPECTIVELY
C CF SYSTEM TRANSFER FUNCTION PARTIAL FRACTION EXPANSION
C IN COMPLEX FORM. IF COMPLEX POLES OCCUR, ONLY ONE OF
C THE PARTIAL FRACTION COMPLEX CONJUGATE TERMS IS USED.
C REAL PART OF POLES MUST BE NEGATIVE FOR STABLE SYSTEM.
C SW,U,WL (3F10.5) TURBULENCE VELOCITY STANDARD DEVIATION,
C VEHICLE MEAN TRUE AIRSPEED, AND TURBULENCE GUST
C VELOCITY SCALE LENGTH.
C NEV (I1) .TOTAI NUMBER OF EIGENVALUES/EIGENFUNCTIONS REQUIRED.
C NEV MUST BE LESS THAN OR EQUAL TO 5.
C IF RESTRT = .TRUE. DATA DECK IS AS FOLLOWS
C (ALL BUT LAST THREE CARDS (EOR,CONT,NEV) ARE PUNCHED BY
C PREVIOUS RUN)
C SW,U,WL (FORMAT 17) SEE DESCRIPTION ABOVE.
C B,B1,NPOL,(CI(I),AI(I),I=1,NPOL) (FORMAT 6)
C NP,LO,HI (FCRMT 7)
C J,E(J),(X(K),EFP(K,J),EFQ(K,J),K=1,NP) (FORMAT 13)
C END OF RECORD INDICATES END OF EIGENVALUE/EIGENFUNCTION DATA.
C CONT (L1) LOGICAL VARIABLE, IF TRUE IT IS ASSUMED THAT ITERATION
C OF THE LAST EIGENSOLUTION IS TO CONTINUE, IF FALSE IT IS ASSUMED

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DO 22 I=1,NPOL
22 AI(I)=-AI(I)
READ (5,3) SW,U,HL
B=U/2./HL $ HI=20./B $ B1=SW*U/HL*SQRT(2.)
IF (HI.LE.1.) GO TO 26
C
C SCALE X VALUES TO COVER RANGE OF INTEGRATION, USE EI,EVP,EVQ FOR
C TEMPORARY STORAGE.
C EI IS LOWER LIMIT OF POWER LAW SCALING.
C NPWR IS POWER OF SCALING, HIGHER VALUES WILL PLACE MORE X VALUES NEAR
C ORIGIN. LOW VALUES WILL PROVIDE MORE EVEN DISTRIBUTION OF X VALUES
C OVER RANGE OF INTEGRATION.
EI=.05 $ NPWR=1
EVQ=((HI-EI)/(1.-EI)-1.)/(1.-EI)**NPWR
DO 25 I=1,NF
EVP=X(I)
IF (EVP.LT.EI) GO TO 25
EVP=EVP-EI $ EVP=EVP*(1.+EVQ*EVP**NPWR)+EI
25 X(I)=EVP
C
C PUNCH PARAMETERS ON CARDS
26 WRITE (7,14) SW,U,HL
WRITE (7,11) B,B1,NPOL,(CI(I),AI(I),I=1,NPOL)
WRITE (7,12) NP,X(1),X(NP)
I=0 $ I1=1
90 READ (5,1) NEV
CONT=.F.
95 IF (NEV-I1) 800,100,100
C
C COMPUTE EXACT SYSTEM RESPONSE VARIANCE
100 CALL VARW(SW,HL,U,SR2)
CALL STARAY(LO,LO)
IF (.NOT.CONT) GO TO 110
EROR=ER2(I1) $ NSKIP=1 $ USEP=.T. $ GO TO 111
C
C GENERATE INITIAL GUESS OF EIGENFUNCTION P(ALPHA)
110 EROR=ER1(I1) $ NSKIP=2 $ USEP=.T.
CALL GUESS(I1)
C
111 WRITE (6,101) I1,SW,U,HL,(CI(J),AI(J),J=1,NPOL)
101 FORMAT (1H1,5X,*BEGIN ITERATION FOR EIGENVALUE AND EIGENFUNCTION NEIGVH155
10.*I2,/,/,6X,*TURBULENCE PARAMETERS*/,/,6X,*STD. DEV. =*,F6.3,5X,EIGVH156
2*MTAS =*,F8.3,5X,*SCALE LENGTH =*,F9.3,/,/,6X,*VEHICLE PARAMETERS: EIGVH157
3*,/,12X,*COEFFICIENTS*,26X,*POLES*/,/(4X,2E13.4,8X,2E13.4))
WRITE (6,102) SR2
102 FORMAT (1H0,5X,*SYSTEM RESPONSE VARIANCE =*,E13.6,/,1H0)
J=1 $ EVP=EVQ=EI=DEVQ=1.E50 $ GO TO 210
C
200 CALL FILL
EROR=ER2(I1)
C
C IF (I.GT.0) EIGENFUNCTION MUST BE ORTHOGONAL TO ALL PREVIOUS FNS
210 IF (I.LE.0) GO TO 300
CALL ORTHCG(I)
C
C COMPUTE NORMALIZATION FACTOR
300 CALL NORMF(F)
C
C ESTIMATE EIGENVALUE
400 EI=EI*F
IF (USEP) 401,410
401 DEVP=ABS(1./EVP/F-1.)
EVP=1./F
EIGVH115
EIGVH116
EIGVH117
EIGVH118
EIGVH119
EIGVH120
EIGVH121
EIGVH122
EIGVH123
EIGVH124
EIGVH125
EIGVH126
EIGVH127
EIGVH128
EIGVH129
EIGVH130
EIGVH131
EIGVH132
EIGVH133
EIGVH134
EIGVH135
EIGVH136
EIGVH137
EIGVH138
EIGVH139
EIGVH140
EIGVH141
EIGVH142
EIGVH143
EIGVH144
EIGVH145
EIGVH146
EIGVH147
EIGVH148
EIGVH149
EIGVH150
EIGVH151
EIGVH152
EIGVH153
EIGVH154
EIGVH155
EIGVH156
EIGVH157
EIGVH158
EIGVH159
EIGVH160
EIGVH161
EIGVH162
EIGVH163
EIGVH164
EIGVH165
EIGVH166
EIGVH167
EIGVH168
EIGVH169
EIGVH170
EIGVH171
EIGVH172
EIGVH173
EIGVH174
EIGVH175
EIGVH176
EIGVH177

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WRITE (6,405) J,EVP,DEVP,EI
405 FORMAT (1H ,5X,*ITERATION NO.*,I3,*, ESTIMATED P EIGENVALUE IS*,
1E15.7,*, (DEVP =*,E10.3,2X,*EI =*,E10.3,*))
GO TO 500
410 DEVP= ABS(1./EVQ/F-1.)
EVQ=1./F
WRITE (6,415) J,EVQ,DEVP,EI
415 FORMAT (1H ,5X,*ITERATION NO.*,I3,*, ESTIMATED Q EIGENVALUE IS*,
1E15.7,*, (DEVP =*,E10.3,2X,*EI =*,E10.3,*))
C
C NORMALIZE EIGENFUNCTION
500 CALL NORM(F)
IF ((J.LT.30).OR.((J/5)*5.NE.J)) GO TO 520
WRITE (6,510) (X(JJ),P(JJ),Q(JJ),JJ=1,NP)
510 FORMAT (1H ,3E25.14)
WRITE (7,511) I1
511 FORMAT (43X,I2)
WRITE (7,707) (X(JJ),P(JJ),Q(JJ),JJ=1,NP)
C
C IF DEV IS SUFFICIENTLY SMALL, ACCEPT EIGENFUNCTION
520 IF ((DEVP.LE.ERROR).AND.(DEVP.LE.ERROR).AND.(EI.LE.ERI)) GO TO 700
IF ((J.GE.25).AND.(NSKIP.NE.1)) GO TO 200
C
C EIGENFUNCTION NOT YET FOUND, CONTINUE ITERATION PROCESS
600 CALL ITERAT(EI)
J=J+1
GO TO 210
C
C ERROR IS BELOW TOLERANCE, CHECK TO SEE THAT MAX. NO. OF POINTS USED
700 IF (NSKIP.NE.1) GO TO 200
C ACCEPT EIGENFUNCTIONS
I=I+1 $ I1=I+1
DEVP=ABS(EVP-EVQ) $ DEVP=(EVP+EVQ)/2. $ DEVP=DEVP/DEVP
E(I)=DEVP $ FS=DEVP**2/SR2 $ TFS=0.
DO 702 J=1,I
702 TFS=TFS+E(J)**2 $ TFS=TFS/SR2
WRITE (7,706) I,DEVP
706 FORMAT (*EIGENVALUE AND CONJUGATE EIGENFUNCTIONS NO.*,I2,E20.11)
WRITE (7,707) (X(J),P(J),Q(J),J=1,NP)
707 FORMAT (3E25.14)
WRITE (6,708) I,DEVP,EVP,EVQ,DEVP,FS,I,TFS,(X(J),P(J),Q(J),J=1,NP)
708 FORMAT (1H1,5X,*EIGENVALUE NO.*,I2,* IS*,E14.7,/,6X,*EVP =*,E14.7,
1* EVQ =*,E14.7,* RELATIVE DIFFERENCE =*,E10.3,/,/,6X,*FRACTION OF
2* RESPONSE VARIANCE DUE TO THIS EIGENVALUE IS*,F7.5,/,6X,
3*FRACTION OF RESPONSE VARIANCE DUE TO FIRST*,I2,* EIGENVALUES IS*,
4*F7.5,/,/,9X,*X*,8X,*FUNCTION OF 1ST ARG*,3X,*FUNCTION OF 2ND ARG*,
5/, (6X,F7.4,5X,E17.10,5X,E17.10))
IF (I.GE.NEV) GO TO 800
CALL STOR(I)
CONT=.F.
GO TO 110
C
800 WRITE (6,805) NEV,(J,E(J),J=1,I)
805 FORMAT (1H1,5X,*FIRST*,I2,* EIGENVALUES ARE*,/, (10X,I2,2X,E15.7))
FS=SR2*(1.-TFS)
WRITE (6,810) FS,TFS
810 FORMAT (1H0,5X,*REMAINING EIGENVALUES SQUARED SUM TO*,E15.7,/,6X,
1*THE ABOVE EIGENVALUES ACCOUNT FOR*,F7.5,* OF THE RESPONSE VAR.*)
IF (FS.LT.0.) GO TO 900
FS=SQRT(FS)
WRITE (6,812) FS
812 FORMAT (1H ,5X,*LARGEST POSSIBLE REMAINING EIGENVALUE IS*,E14.7)
900 STOP $ END

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EIGVH178
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EIGVH235
EIGVH236
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EIGVH238
EIGVH239
EIGVH240

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SUBROUTINE VARH(SH,HL,U,S2)
C
C A SUBROUTINE TO COMPUTE THE RESPONSE VARIANCE OF A LINEAR SYSTEM
C SUBJECTED TO RANDOM INPUT WITH DRYDEN W-GUST SPECTRUM
C
C SW = TURBULENCE STANDARD DEVIATION
C HL = TURBULENCE SCALE LENGTH
C U = VEHICLE MEAN TRUE AIRSPEED
C AI = NEGATIVE OF VEHICLE IMPULSE RESPONSE EXPONENTS (POSITIVE REAL PT)
C CI = VEHICLE IMPULSE RESPONSE CONSTANTS
C IF CI OR AI IS COMPLEX, ONLY ONE OF EACH CONJUGATE PAIR IS TO BE USED
C
COMMON /CCNST/ CI(5),AI(5),NPOL
COMPLEX CI,AI,AJ,AK,CJ,CK,C2,C3,C4,D1,D2
LOGICAL RL1,RL2
C=U/HL $ S2=0.
DO 20 K=1,NPOL
AK=AI(K) $ CK=CI(K) $ C4=0.
RL1=AIMAG(AK).EQ.0..AND.AIMAG(CK).EQ.0.
5 D1=C+AK
DO 10 J=1,NPOL
AJ=AI(J) $ CJ=CI(J) $ C3=0.
RL2=AIMAG(AJ).EQ.0..AND.AIMAG(CJ).EQ.0.
6 D2=C-AJ
C2=(C*(C+(AK-AJ)/2.)/D1/D2-1.)/D1/D2
C2=C2+C*(C**2-3.*AJ**2)/(AK+AJ)/(AJ+C)**2/(AJ-C)**2
C2=C2*CJ*CK $ C3=C3+C2 $ IF (RL2) GO TO 10
AJ=CONJG(AJ) $ CJ=CONJG(CJ) $ RL2=.T. $ GO TO 6
10 C4=C4+C3
IF (RL1) GO TO 20
AK=CONJG(AK) $ CK=CONJG(CK) $ RL1=.T. $ GO TO 5
20 S2=S2+REAL(C4) $ S2=S2*SW**2 $ RETURN $ END
SUBROUTINE FILL
C
C THIS SUBROUTINE INTERPOLATES THE 21 POINT EIGENFUNCTION TO FORM THE
C FIRST APPROXIMATION TO THE 41 POINT EIGENFUNCTION.
C
COMMON /ARRAYS/ X(45),P(45),Q(45),I,I1,K,XP,C(41),NP,NSKIP
I1=NSKIP/2+1
DO 10 I=I1,NP,NSKIP
XP=X(I) $ K=I+1-I1
CALL COEF(K,P,C)
P(I)=((C(4)*XP+C(3))*XP+C(2))*XP+C(1)
CALL COEF(K,Q,C)
10 Q(I)=((C(4)*XP+C(3))*XP+C(2))*XP+C(1)
NSKIP=NSKIP/2
RETURN $ END
SUBROUTINE GUESS(I)
C
C THIS SUBROUTINE GUESSES THE NEXT EIGENFUNCTION TO BE THE
C DERIVATIVE OF THE PREVIOUS RESULT OR X*EXP(-3.*X) FOR THE FIRST.
C
COMMON /ARRAYS/ X(45),P(45),Q(45),J,XJ,K,DX,CP,DUH(40),NP,NSKIP
IF (NSKIP.LE.1) GO TO 70
IF (I.GT.1) GO TO 50
DO 10 J=1,NP,NSKIP
Q(J)=0. $ XJ=X(J)
10 P(J)=XJ*EXP(-3.*XJ)
RETURN
C
50 K=NSKIP+1
DO 60 J=K,NP,NSKIP
Q(J)=0. $ DP=P(J)-P(J-1) $ DX=X(J)-X(J-1) $ XJ=X(J)

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EIGVH301
EIGVH302
EIGVH303

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XJ=XJ*EXP(-3.*XJ)
60 P(J)=XJ*DP/DX
RETURN
70 WRITE (6,71) NSKIP
71 FORMAT (1H ,5X,*NSKIP =*,I10,*NOT ALLOWED IN GUESS*)
STOP $ END
SUBROUTINE NORMF(F)
C
C COMPUTES NORM OF P(X) OR Q(X) DEPENDING UPON USEP TRUE OR FALSE
C THE NCRM OF P OF Q IS THE ESTIMATED EV., THE INVERSE IS THE NORM. FAC.
C
COMMON /ARRAYS/ X(45),P(45),Q(45),C(4),F1,NP1,J,I,X1,X2,II,CII,
1IJ,K,CK,X12,X22,JJ,DUM(27),NP,NSKIP,NPOLY,USEP
LOGICAL USEP
F1=0. $ NP1=NPOLY+1 $ J=NP-1
DO 60 I=1,J,NSKIP
F = 0.
IF(USEP) 10,11
10 CALL COEF(I,P,C)
GO TO 12
11 CALL COEF(I,Q,C)
12 X1=X(I) $ X2=X(I+NSKIP)
IF (X2.GT.X(NP)) X2=X(NP)
DO 40 II=1,NPOLY
CII=C(II) $ IJ=II+1 $ K=II+IJ-1 $ CK=FLOAT(K)
X12=X1**K $ X22=X2**K
DO 40 JJ=IJ,NP1
F = F+CII*C(IJ)*X22-X12/CK
CK=CK+1. $ X12=X12*X1
40 X22=X22*X2
F=F*2. $ X12=X12**2 $ X22=X22**2 $ CK=1.
DO 50 II=1,NP1
F=F+(X2-X1)**2*(C(II)**2)/CK $ CK=CK+2. $ X2=X2*X2
50 X1=X1*X12
60 F1=F1+F
F=SQRT(1./F1)
RETURN
C
C NORMALIZE EIGENFUNCTION P OR Q DEPENDING UPON USEP TRUE OR FALSE.
ENTRY NCRM
DO 100 I=1,NP,NSKIP
IF (USEP) 95,96
95 P(I)=P(I)*F
GO TO 100
96 Q(I)=Q(I)*F
100 CONTINUE
RETURN $ END
SUBROUTINE ORTHOG(I)
C
C A SUBROUTINE TO ORTHOGONALIZE P(X) OR Q(X) WITH RESPECT TO THE FIRST
C EIGENFUNCTIONS DEPENDING UPON USEP TRUE OR FALSE.
C
COMMON /ARRAYS/ X(45),P(45),Q(45),DUM(45),NP,NSKIP,NPOLY,USEP,
1EFP(45,5),EFQ(45,5),C(4),D(4)
LOGICAL USEP
NP1 = NPOLY+1
DO 50 J=1,I
DO 10 JJ=1,NP,NSKIP
IF (USEP) 5,6
5 DUM(JJ)=EFP(JJ,J)
GO TO 10
6 DUM(JJ)=EFQ(JJ,J)
10 CONTINUE
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EIGVH364
EIGVH365
EIGVH366

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      K=NP-1 $ F=0.
      DO 40 JJ=1,K,NSKIP
      IF (USEP) 20,21
20  CALL COEF(JJ,P,C)
      GO TO 22
21  CALL COEF(JJ,Q,C)
22  CALL COEF(JJ,DUM,O)
      X1=X(JJ) $ II=JJ+NSKIP $ IF (II.GT.NP) II=NP $ X2=X(II)
      DO 40 II=1,NP1
      X12=X1**II $ X22=X2**II $ CII=C(II) $ CL=FLOAT(II)
      DO 40 KK=1,NP1
      F=F+CII*D(KK)*(X22-X12)/CL $ CL=CL+1. $ X12=X12*X1
40  X22 = X22*X2
      DO 50 JJ=1,MP,NSKIP
      IF (USEP) 45,46
45  P(JJ)=P(JJ)-F*DUM(JJ)
      GO TO 50
46  Q(JJ)=Q(JJ)-F*DUM(JJ)
50  CONTINUE
      RETURN $ END
      FUNCTION SUM(I,J)
C
C FORMS PRODUCT (J-I+1)*(J-I+2).....(J)
C
      N=1
      IF(I) 20,20,10
10  K=J-I+1
      DO 15 L=K,J
15  N=N*L
20  SUM=FLOAT(N)
      RETURN $ END
      SUBROUTINE COEF(I,A,C)
C A SUBROUTINE TO FILL C ARRAY WITH THE COEFFICIENTS OF THE
C POLYNOMIAL REPRESENTING A(X) BETWEEN X(I) AND X(I+NSKIP). NPOLY IS
C THE ORDER OF THE POLYNOMIAL
C POLYNOMIAL IS C(1) + C(2)*X + C(3)*X**2 + C(4)*X**3
C
      DIMENSION A(1),C(1)
      COMMON /ARRAYS/ X(45),P(45),Q(45),DUM(45),NP,NSKIP,NPOLY
      DATA NPOLY/3/
      IF (I.GT.NSKIP) GO TO 10
      N=1 $ GO TO 30
10  IF (I.LE.NP-2*NSKIP) GO TO 20
      N=NP-3*NSKIP $ GO TO 30
20  N=I-NSKIP
30  F1=A(N) $ X1=X(N) $ N=N+NSKIP $ F2=A(N) $ X2=X(N)
      N=N+NSKIP $ F3=A(N) $ X3=X(N)
      N=N+NSKIP $ F4=A(N) $ X4=X(N)
      F1=F1/(X1-X2)/(X1-X3)/(X1-X4)
      F2=F2/(X2-X1)/(X2-X3)/(X2-X4)
      F3=F3/(X3-X1)/(X3-X2)/(X3-X4)
      F4=F4/(X4-X1)/(X4-X2)/(X4-X3)
      C(4)=F1+F2+F3+F4
      C(3)=-F1*(X2+X3+X4)-F2*(X1+X3+X4)-F3*(X1+X2+X4)-F4*(X1+X2+X3)
      C(2)=F1*(X2*(X3+X4)+X3*X4)+F2*(X1*(X3+X4)+X3*X4)+F3*(X1*(X2+X4)+
1 X2*X4)+F4*(X1*(X2+X3)+X2*X3)
      C(1)=-F1*X2*X3*X4-F2*X1*X3*X4-F3*X1*X2*X4-F4*X1*X2*X3
      RETURN $ END
      SUBROUTINE DIFC
C INTEGRATE COMPLEX TERMS OF TRANSFER FUNCTION
C
      EQUIVALENCE (E,ER),(XE,XER),(XE1,XE1R),(D1,D1R)
      COMMON /DIF/ X,E,N,D1

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 EIGVH424
 EIGVH425
 EIGVH426
 EIGVH427
 EIGVH428
 EIGVH429

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COMPLEX E,XE,XE1,D1
N1=N+1 $ D1=0.
IF (N) 70,10,20
C
C CONSTANT TERM OF POLYNOMIAL
10 D1=1. $ GC TO 30
C
C POWER TERM OF POLYNOMIAL
20 IF (X.NE.0.) GO TO 25
D1=(-1./E)**N*SUM(N,N)
GO TO 30
25 XN=X**N
XE1=X*E $ XE=1. $ S1=1. $ S2=FLOAT(N)
DO 26 I=1,N1
D1=XN/XE*S1+D1 $ XE=XE*XE1 $ S1=S1*S2 $ S2=S2-1.
28 XN=-XN
30 D1=D1*CEXP(E*X)/E
70 RETURN
C
C INTEGRATE REAL TERMS OF TRANSFER FUNCTION
ENTRY DIFR
N1=N+1 $ D1=0.
IF (N) 170,110,120
C
C CONSTANT TERM OF POLYNOMIAL
110 D1R=1. $ GO TO 130
C
C POWER TERM OF POLYNOMIAL
120 IF (X.NE.0.) GO TO 125
D1R=(-1./E)**N*SUM(N,N)
GO TO 130
125 XN=X**N
XE1R=X*ER $ XER=1. $ S1=1. $ S2=FLOAT(N)
DO 128 I=1,N1
D1R=XN/XER*S1+D1R $ XER=XER*XE1R $ S1=S1*S2 $ S2=S2-1.
128 XN=-XN
130 D1=CMPLX(D1R*EXP(ER*X)/ER,0.0)
170 RETURN $ END
SUBROUTINE NTGRAL(V,XAB)
C
C INTEGRATES P(X)*H(X,XAB) OR Q(X)*H(XAB,X) DEPENDING UPON USEP TRUE
C OR FALSE.
C
COMMON /ARRAYS/ X(45),P(45),Q(45),E1,E2,C1,C2,C02,C03,C04,C06,IPOL
1,NX,NPO,NF01,IA,ICA,D1S(4),D2S(4),D1S1(4),D1S2(4),NP,NSKIP,NPOLY,
2USEP,EFP(450),C(4)
COMMON /CONST/ CI(5),AI(5),NPOL,B,B1
COMMON /DIF/ X1,E,NPOM,D1C
DIMENSION D1A(225),D1CA(900),CA(180)
COMPLEX D1CA,AI,CI,E,C1,C2,V1,V2,E2,D1C,D1S1,D1S2,C01,C02,C03,
1C04,C05,CC6,D1C1,D1C2
LOGICAL RL,USEP
EQUIVALENCE (E,ER),(D1,D2,D1C),(C01,C02),(C04,C05),(D1C1,D1C2)
V=0. $ E1=EXP(-B*XAB) $ ICA=0
C
C CYCLE THROUGH PCLES OF TRANSFER FUNCTION
DO 100 IPCL=1,NPOL
E2=CEXP(-XAB*(AI(IPOL)-B)) $ C1=B1*CI(IPOL)/(AI(IPOL)-2.*B)
C2=C1*B $ C1=C1*(AI(IPOL)-B)/(AI(IPOL)-2.*B) $ C03=C2*E1
IF (USEP) 5,6
5 G01=(C1-XAB*C2)*E1 $ C05=C1*E2 $ GO TO 7
6 C02=(C1+XAB*C2)*E2 $ G04=C2*(E2-E1) $ C06=C1*E1
7 V1=0. $ V2=0. $ IA=1 $ ICA=ICA+1
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EIGVH491
EIGVH492

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C
C CYCLE THROUGH X INTERVALS
DO 15 NPO=1,NP01
  D1S(NPO)=D1A(IA) $ IA=IA+1 $ D1S1(NPO)=D1CA(ICA) $ ICA=ICA+1
  IF (USEP) 12,13
12 D1S2(NPO)=D1CA(ICA)
  GO TO 15
13 D2S(NPO)=D1A(IA)
15 CONTINUE
  I=NSKIP+1 $ NXI=1
  DO 80 NX=I, NP, NSKIP
  X1=X(NX) $ IA=IA+IINC $ ICA=ICA+IINC
C
C CYCLE THROUGH TERMS OF POLYNOMIAL
IF (USEP) 16,25
C
C COMPUTE NEW Q (BETA) EIGENFUNCTION
16 DO 20 NPO=1,NP01
  D1=D1A(IA) $ IA=IA+1 $ E=D1-D1S(NPO) $ V1=C01*E
  D1S(NPO)=D1
  IF (X1-XAB) 17,17,18
17 D1C1=D1CA(ICA)
  ICA=ICA+1 $ E=D1C1-D1S1(NPO) $ V1=V1-C01*E $ D1S1(NPO)=D1C1
  D1C2=D1CA(ICA) $ E=D1C2-D1S2(NPO) $ V1=V1-C03*E
  D1S2(NPO)=D1C2
  GO TO 19
18 V1=V1-C05*E
  ICA=ICA+1
19 V2=V2+V1*CA(NXI)
20 NXI=NXI+1
  GO TO 80
C
C COMPUTE NEW P (ALPHA) EIGENFUNCTION
25 DO 40 NPO=1,NP01
  D1=D1A(IA) $ IA=IA+1 $ E=D1-D1S(NPO) $ V1=C06*E
30 D1S(NPO)=D1
  IF (X1-XAB) 31,31,38
31 D1C1=D1CA(ICA)
  ICA=ICA+1 $ E=D1C1-D1S1(NPO) $ V1=V1-C06*E
35 D1S1(NPO)=D1C1
  D2=D1A(IA) $ E=D2-D2S(NPO) $ V1=V1-C03*E $ D2S(NPO)=D2
  GO TO 39
38 V1=V1-C02*E
  D2=D1A(IA) $ E=D2-D2S(NPO) $ V1=V1+C04*E $ D2S(NPO)=D2
  ICA=ICA+1
39 V2=V2+V1*CA(NXI)
40 NXI=NXI+1
80 CONTINUE
  IF ((AIMAG(C1).EQ.0.).AND.(AIMAG(E2).EQ.0.)) GO TO 100
  V2=2.*REAL(V2)
100 V=V+V2
  GO TO 500
C
C
C SET UP D1A AND D1CA ARRAYS
ENTRY STARAY
NP01=NPPLY+2 $ ER=-8 $ I=1
DO 200 NX=1,NP
  X1=X(NX)
DO 200 NPO=1,NP01
  NP0M=NP0-1 $ CALL DIFR $ D1A(I)=D1
200 I=I+1 $ I=1
  DO 300 IPOL=1,NPOL

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EIGVM493
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E=B-AI(IPCL) $ RL=AIMAG(E).EQ.0.
DO 300 NX=1,NP
X1=X(NX)
DO 300 NPO=1,NPO1
NFOH=NPO-1 $ IF (RL) GO TO 250
CALL DIFC
GO TO 298
250 CALL DIFR
290 D1CA(I)=D1C
300 I=I+1
GO TO 500
C
C GENERATE TABLE OF POLYNOMIAL COEFFICIENTS, CA, FOR EIGENFUNCTION P OR Q
C Q DEPENDING UPON USEP TRUE OR FALSE
ENTRY GENCOE
NPO=0
DO 400 NXI=1,NP,NSKIP
IF (NXI-NP) 350,400,400
350 IF (USEP) 360,370
360 CALL COEF(NXI,P,C)
GO TO 375
370 CALL COEF(NXI,Q,C)
375 DO 380 NPO1=1,4
NPO=NPO+1
380 CA(NPO1)=C(NPO1)
400 CONTINUE
NFO1=NPOLY+1
IINC=(NSKIP-1)*(NPOLY+2)+1
500 RETURN $ END
SUBROUTINE ITERAT(E)
C
C A SUBROUTINE TO ITERATE THE EIGENFUNCTION P(X) OR Q(X) DEPENDING
C UPON USEP TRUE OR FALSE.
C
COMMON /ARRAYS/ X(45),P(45),Q(45),DUM(45),NP,NSKIP,NPOLY,USEP
COMMON /EVAL/ EVP,EVQ
EQUIVALENCE (PI,QI)
LOGICAL USEP
CALL GENCOE(Q,X)
Q(1)=0. $ P(1)=0. $ E=0. $ I1=NSKIP+1
DO 50 I=I1,NP,NSKIP
IF (USEP) 30,40
30 QI=Q(I)*EVQ
CALL NTGRAL(Q(I),X(I))
E=E+(ABS(QI)-ABS(Q(I)))**2
GO TO 50
40 PI=P(I)*EVP
CALL NTGRAL(P(I),X(I))
E=E+(ABS(PI)-ABS(P(I)))**2
50 CONTINUE
E=SQRT(E)/FLOAT(NP)
USEP=.NOT.USEP
RETURN $ END
SUBROUTINE STOR(I)
C
C STORES ITH EIGENFUNCTION
C P(X) IN EFP(X,I), Q(X) IN EFQ(X,I)
C
COMMON /ARRAYS/ X(45),P(45),Q(45),J,DUM(44),NP,NSKIP,NPOLY,USEP,
1EFP(45,5),EFQ(45,5)
DO 10 J=1,NP
EFP(J,I)=P(J)
10 EFQ(J,I)=Q(J)
EIGVM556
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EIGVM618

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RETURN \$ END

EIGVM619

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SUBROUTINE FFT (A,M,INV,S,IFSET,IFERR)
C
C FAST FOURIER TRANSFORM FOR COMPLEX FUNCTIONS OF UP TO THREE DIMENSIONS
C WRITTEN BY DUANE HARDER, FEB. 1969
C AVAILABLE THROUGH VIM INC. (CDC USER ORGANIZATION)
C
DIMENSION A(1),INV(1),S(1),N(3),M(3),NP(3),W(2),W2(2),W3(2),
EQUIVALENCE (N1,N(1)), (N2,N(2)), (N3,N(3))
10 IF (IABS(IFSET)-1) 610,610,20
20 MTT=MAX0(M(1),M(2),M(3))-2
   ROOT2=SQRT(2.)
   IF (MTT-MT) 40,40,30
30 IFERR=1
   RETURN
40 IFERR=0
   M1=M(1)
   M2=M(2)
   M3=M(3)
   N1=2**M1
   N2=2**M2
   N3=2**M3
50 IF (IFSET) 50,50,70
   NX=N1*N2*N3
   FN=NX
   DO 60 I=1,NX
   A(2*I-1)=A(2*I-1)/FN
60 A(2*I)=-A(2*I)/FN
70 NP(1)=N1*2
   NP(2)=NP(1)*N2
   NP(3)=NP(2)*N3
   DO 330 ID=1,3
   IL=NP(3)-NP(ID)
   IL1=IL+1
   MI=M(ID)
80 IF (MI) 330,330,80
   IDIF=NP(ID)
   KBIT=NP(ID)
   MEV=2*(MI/2)
   IF (MI-MEV) 120,120,90
90 KBIT=KBIT/2
   KL=KBIT-2
   DO 100 I=1,IL1,IDIF
   KLAST=KL+I
   DO 100 K=I,KLAST,2
   KD=K+KBIT
   T=A(KD)
   A(KD)=A(K)-T
   A(K)=A(K)+T
   T=A(KD+1)
   A(KD+1)=A(K+1)-T
100 A(K+1)=A(K+1)+T
   IF (MI-1) 320,330,110
110 LFIRST=3
   JLAST=1
   GO TO 130
120 LFIRST=2
   JLAST=0
130 DO 320 L=LFIRST,MI,2

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FFT00001
FFT00002
FFT00003
FFT00004
FFT00005
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FFT00058

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JJDIF=KBIT
KBIT=KBIT/4
KL=KBIT-2
DO 140 I=1,IL1,IDIF
KLAST=I+KL
DO 140 K=I,KLAST,2
K1=K+KBIT
K2=K1+KBIT
K3=K2+KBIT
T=A(K2)
A(K2)=A(K)-T
A(K)=A(K)+T
T=A(K2+1)
A(K2+1)=A(K+1)-T
A(K+1)=A(K+1)+T
T=A(K3)
A(K3)=A(K1)-T
A(K1)=A(K1)+T
T=A(K3+1)
A(K3+1)=A(K1+1)-T
A(K1+1)=A(K1+1)+T
T=A(K1)
A(K1)=A(K)-T
A(K)=A(K)+T
T=A(K1+1)
A(K1+1)=A(K+1)-T
A(K+1)=A(K+1)+T
R=-A(K3+1)
T=A(K3)
A(K3)=A(K2)-R
A(K2)=A(K2)+R
A(K3+1)=A(K2+1)-T
140 A(K2+1)=A(K2+1)+T
IF (JLAST) 310,310,150
150 JJ=JJDIF+1
ILAST=IL+JJ
DO 160 I=JJ,ILAST,IDIF
KLAST=KL+I
DO 160 K=I,KLAST,2
K1=K+KBIT
K2=K1+KBIT
K3=K2+KBIT
R=-A(K2+1)
T=A(K2)
A(K2)=A(K)-R
A(K)=A(K)+R
A(K2+1)=A(K+1)-T
A(K+1)=A(K+1)+T
AHR=A(K1)-A(K1+1)
AHI=A(K1+1)+A(K1)
R=-A(K3)-A(K3+1)
T=A(K3)-A(K3+1)
A(K3)=(AHR-R)/ROOT2
A(K3+1)=(AHI-T)/ROOT2
A(K1)=(AHR+R)/ROOT2
A(K1+1)=(AHI+T)/ROOT2
T=A(K1)
A(K1)=A(K)-T
A(K)=A(K)+T
T=A(K1+1)
A(K1+1)=A(K+1)-T
A(K+1)=A(K+1)+T
R=-A(K3+1)

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FFT00119
FFT00120
FFT00121

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| | | |
|-----|-----------------------------|-----------|
| | T=A(K3) | FFT 00122 |
| | A(K3)=A(K2)-R | FFT 00123 |
| | A(K2)=A(K2)+R | FFT 00124 |
| | A(K3+1)=A(K2+1)-T | FFT 00125 |
| 160 | A(K2+1)=A(K2+1)+T | FFT 00126 |
| | IF (JLAST-1) 310,310,170 | FFT 00127 |
| 170 | JJ=JJ+JJOIF | FFT 00128 |
| | DO 300 J=2,JLAST | FFT 00129 |
| | I=INV(J+1) | FFT 00130 |
| | IC=NT-I | FFT 00131 |
| | W(1)=S(IC) | FFT 00132 |
| | W(2)=S(I) | FFT 00133 |
| | I2=2*I | FFT 00134 |
| | I2C=NT-I2 | FFT 00135 |
| | IF (I2C) 200,190,180 | FFT 00136 |
| 180 | W2(1)=S(I2C) | FFT 00137 |
| | W2(2)=S(I2) | FFT 00138 |
| | GO TO 210 | FFT 00139 |
| 190 | W2(1)=0. | FFT 00140 |
| | W2(2)=1. | FFT 00141 |
| | GO TO 210 | FFT 00142 |
| 200 | I2CC=I2C+NT | FFT 00143 |
| | I2C=-I2C | FFT 00144 |
| | W2(1)=-S(I2C) | FFT 00145 |
| | W2(2)=S(I2CC) | FFT 00146 |
| 210 | I3=I+I2 | FFT 00147 |
| | I3C=NT-I3 | FFT 00148 |
| | IF (I3C) 240,230,220 | FFT 00149 |
| 220 | W3(1)=S(I3C) | FFT 00150 |
| | W3(2)=S(I3) | FFT 00151 |
| | GO TO 280 | FFT 00152 |
| 230 | W3(1)=0. | FFT 00153 |
| | W3(2)=1. | FFT 00154 |
| | GO TO 280 | FFT 00155 |
| 240 | I3CC=I3C+NT | FFT 00156 |
| | IF (I3CC) 270,260,250 | FFT 00157 |
| 250 | I3C=-I3C | FFT 00158 |
| | W3(1)=-S(I3C) | FFT 00159 |
| | W3(2)=S(I3CC) | FFT 00160 |
| | GO TO 280 | FFT 00161 |
| 260 | W3(1)=-1. | FFT 00162 |
| | W3(2)=0. | FFT 00163 |
| | GO TO 280 | FFT 00164 |
| 270 | I3CCC=NT+I3CC | FFT 00165 |
| | I3CC=-I3CC | FFT 00166 |
| | W3(1)=-S(I3CCC) | FFT 00167 |
| | W3(2)=-S(I3CC) | FFT 00168 |
| 280 | ILAST=IL+JJ | FFT 00169 |
| | DO 290 I=JJ,ILAST,I0IF | FFT 00170 |
| | KLAST=KL+I | FFT 00171 |
| | DO 290 K=I,KLAST,2 | FFT 00172 |
| | K1=K+KBIT | FFT 00173 |
| | K2=K1+KBIT | FFT 00174 |
| | K3=K2+KBIT | FFT 00175 |
| | R=A(K2)*W2(1)-A(K2+1)*W2(2) | FFT 00176 |
| | T=A(K2)*W2(2)+A(K2+1)*W2(1) | FFT 00177 |
| | A(K2)=A(K)-R | FFT 00178 |
| | A(K)=A(K)+R | FFT 00179 |
| | A(K2+1)=A(K+1)-T | FFT 00180 |
| | A(K+1)=A(K+1)+T | FFT 00181 |
| | R=A(K3)*W3(1)-A(K3+1)*W3(2) | FFT 00182 |
| | T=A(K3)*W3(2)+A(K3+1)*W3(1) | FFT 00183 |
| | AHR=A(K1)*W(1)-A(K1+1)*W(2) | FFT 00184 |

| | | |
|-----|-----------------------------|----------|
| | AWI=A(K1)*W(2)+A(K1+1)*W(1) | FFT00185 |
| | A(K3)=AHR-R | FFT00186 |
| | A(K3+1)=A+I-T | FFT00187 |
| | A(K1)=AHR+R | FFT00188 |
| | A(K1+1)=A+I+T | FFT00189 |
| | T=A(K1) | FFT00190 |
| | A(K1)=A(K)-T | FFT00191 |
| | A(K)=A(K)+T | FFT00192 |
| | T=A(K1+1) | FFT00193 |
| | A(K1+1)=A(K+1)-T | FFT00194 |
| | A(K+1)=A(K+1)+T | FFT00195 |
| | R=-A(K3+1) | FFT00196 |
| | T=A(K3) | FFT00197 |
| | A(K3)=A(K2)-R | FFT00198 |
| | A(K2)=A(K2)+R | FFT00199 |
| | A(K3+1)=A(K2+1)-T | FFT00200 |
| 290 | A(K2+1)=A(K2+1)+T | FFT00201 |
| 300 | JJ=JJDIF+JJ | FFT00202 |
| 310 | JLAST=4*JLAST+3 | FFT00203 |
| 320 | CONTINUE | FFT00204 |
| 330 | CONTINUE | FFT00205 |
| | NTSQ=NT*NT | FFT00206 |
| | M3MT=M3-MT | FFT00207 |
| | IF (M3MT) 350,340,340 | FFT00208 |
| 340 | IG03=1 | FFT00209 |
| | N3VNT=N3/NT | FFT00210 |
| | MINN3=NT | FFT00211 |
| | GO TO 360 | FFT00212 |
| 350 | IG03=2 | FFT00213 |
| | N3VNT=1 | FFT00214 |
| | NTVN3=NT/N3 | FFT00215 |
| | MINN3=N3 | FFT00216 |
| 360 | JJD3=NTSQ/N3 | FFT00217 |
| | M2MT=M2-MT | FFT00218 |
| | IF (M2MT) 380,370,370 | FFT00219 |
| 370 | IG02=1 | FFT00220 |
| | N2VNT=N2/NT | FFT00221 |
| | MINN2=NT | FFT00222 |
| | GO TO 390 | FFT00223 |
| 380 | IG02=2 | FFT00224 |
| | N2VNT=1 | FFT00225 |
| | NTVN2=NT/N2 | FFT00226 |
| | MINN2=N2 | FFT00227 |
| 390 | JJD2=NTSQ/N2 | FFT00228 |
| | M1MT=M1-MT | FFT00229 |
| | IF (M1MT) 410,400,400 | FFT00230 |
| 400 | IG01=1 | FFT00231 |
| | N1VNT=N1/NT | FFT00232 |
| | MINN1=NT | FFT00233 |
| | GO TO 420 | FFT00234 |
| 410 | IG01=2 | FFT00235 |
| | N1VNT=1 | FFT00236 |
| | NTVN1=NT/N1 | FFT00237 |
| | MINN1=N1 | FFT00238 |
| 420 | JJD1=NTSQ/N1 | FFT00239 |
| | JJ3=1 | FFT00240 |
| | J=1 | FFT00241 |
| | DO 570 JPF3=1,N3VNT | FFT00242 |
| | IPP3=INV(JJ3) | FFT00243 |
| | DO 560 JP3=1,MINN3 | FFT00244 |
| | GO TO (430,440), IG03 | FFT00245 |
| 430 | IP3=INV(JP3)*N3VNT | FFT00246 |
| | GO TO 450 | FFT00247 |

| | | |
|-----|---------------------------------|----------|
| 440 | IP3=INV(JP3)/NTVN3 | FFT00248 |
| 450 | I3=(IPP3+IP3)*N2 | FFT00249 |
| | JJ2=1 | FFT00250 |
| | DO 560 JPF2=1,N2VNT | FFT00251 |
| | IPP2=INV(JJ2)+I3 | FFT00252 |
| | DO 550 JP2=1,MINN2 | FFT00253 |
| | GO TO (460,470), IGO2 | FFT00254 |
| 460 | IP2=INV(JP2)*N2VNT | FFT00255 |
| | GO TO 480 | FFT00256 |
| 470 | IP2=INV(JP2)/NTVN2 | FFT00257 |
| 480 | I2=(IPP2+IP2)*N1 | FFT00258 |
| | JJ1=1 | FFT00259 |
| | DO 550 JPF1=1,N1VNT | FFT00260 |
| | IPP1=INV(JJ1)+I2 | FFT00261 |
| | DO 540 JP1=1,MINN1 | FFT00262 |
| | GO TO (490,500), IGO1 | FFT00263 |
| 490 | IP1=INV(JP1)*N1VNT | FFT00264 |
| | GO TO 510 | FFT00265 |
| 500 | IP1=INV(JP1)/NTVN1 | FFT00266 |
| 510 | I=2*(IPF1+IP1)+1 | FFT00267 |
| | IF (J-I) 520,530,530 | FFT00268 |
| 520 | T=A(I) | FFT00269 |
| | A(I)=A(J) | FFT00270 |
| | A(J)=T | FFT00271 |
| | T=A(I+1) | FFT00272 |
| | A(I+1)=A(J+1) | FFT00273 |
| | A(J+1)=T | FFT00274 |
| 530 | CONTINUE | FFT00275 |
| 540 | J=J+2 | FFT00276 |
| 550 | JJ1=JJ1+JJD1 | FFT00277 |
| 560 | JJ2=JJ2+JJD2 | FFT00278 |
| 570 | JJ3=JJ3+JJD3 | FFT00279 |
| | IF (IFSET) 580,600,600 | FFT00280 |
| 580 | DO 590 I=1,NX | FFT00281 |
| 590 | A(2*I)=-A(2*I) | FFT00282 |
| 600 | RETURN | FFT00283 |
| 610 | MT=MAX0(M(1),M(2),M(3))-2 | FFT00284 |
| | MT=MAX0(2,MT) | FFT00285 |
| | IF (MT-20) 630,630,620 | FFT00286 |
| 620 | IFERR=1 | FFT00287 |
| | GO TO 600 | FFT00288 |
| 630 | IFERR=0 | FFT00289 |
| | NT=2**MT | FFT00290 |
| | NTV2=NT/2 | FFT00291 |
| | THETA=.7853981634 | FFT00292 |
| | JSTEP=NT | FFT00293 |
| | JDIF=NTV2 | FFT00294 |
| | S(JCIF)=SIN(THETA) | FFT00295 |
| | DO 660 L=2,MT | FFT00296 |
| | THETA=THETA/2. | FFT00297 |
| | JSTEP2=JSTEP | FFT00298 |
| | JSTEP=JDIF | FFT00299 |
| | JDIF=JSTEP/2 | FFT00300 |
| | S(JCIF)=SIN(THETA) | FFT00301 |
| | JC1=NT-JDIF | FFT00302 |
| | S(JC1)=COS(THETA) | FFT00303 |
| | JLAST=NT-JSTEP2 | FFT00304 |
| | IF (JLAST-JSTEP) 660,640,640 | FFT00305 |
| 640 | GO 650 J=JSTEP,JLAST,JSTEP | FFT00306 |
| | JC=NT-J | FFT00307 |
| | JD=J+JDIF | FFT00308 |
| 650 | S(JD)=S(J)*S(JC1)+S(JDIF)*S(JC) | FFT00309 |
| 660 | CONTINUE | FFT00310 |

```

C
C   SET UP INV(J) TABLE
   MTLEXP=NTV2
   LM1EXP=1
   INV(1)=0
   DO 680 L=1,MT
   INV(LM1EXP+1)=MTLEXP
   DO 670 J=2,LM1EXP
   JJ=J+LM1EXP
670  INV(JJ)=INV(J)+MTLEXP
   MTLEXP=MTLEXP/2
680  LM1EXP=LM1EXP*2
   IF (IFSET) 20,600,20
   END
FFT00311
FFT00312
FFT00313
FFT00314
FFT00315
FFT00316
FFT00317
FFT00318
FFT00319
FFT00320
FFT00321
FFT00322
FFT00323
FFT00324

      SUBROUTINE FFTRS(X,DX,DF,M,INV,S,IFS,IFER)
C
C   FOURIER TRANSFORM OF REAL SYMMETRIC DATA STORED AS COMPLEX ARRAY X
C
C   DIMENSION X(1),M(1),INV(1),S(1)
C
C   DF=1./DX/2.**M(1) $ X(1)=X(1)*DX $ X(2)=0. $ I1=3
C   I2=2**(M(1)+1)-1 $ M1=2**(M(1)-1)
C   DO 10 I=1,M1
C   X(I1)=(X(I1)+X(I2))*DX $ X(I2)=X(I1)
C   X(I1+1)=0. $ X(I2+1)=0. $ I1=I1+2
10  I2=I2-2
C   IF=IABS(IFS)
C   CALL FFT(X,M,INV,S,IF,IFER)
C   RETURN $ END
FFTRS001
FFTRS002
FFTRS003
FFTRS004
FFTRS005
FFTRS006
FFTRS007
FFTRS008
FFTRS009
FFTRS010
FFTRS011
FFTRS012
FFTRS013
FFTRS014
FFTRS015

      PROGRAM INCPD(INPUT,OUTPUT,PUNCH,TAPES=INPUT,TAPE6=OUTPUT,TAPE7=
1PUNCH)
C   A PROGRAM TO GENERATE AND FOURIER TRANSFORM ONE-DIMENSIONAL CHARACTER-
C   ISTIC FUNCTIONS OF THE GENERAL NON-GAUSSIAN MODEL. CORRELATION
C   MATRICIES AND FUNCTIONAL MATRIX ARE READ FROM DATA CARDS.
C   PROGRAM PRINTS PROBABILITY DENSITY FUNCTION IN BOTH STANDARDIZED
C   AND NONSTANDARDIZED FORM IN INCREMENTS OF DX*SR FROM ZERO TO
C   200*DX*SR. RESULTS ARE PUNCHED ON CARDS IF LOGICAL VARIABLE PNCH
C   IS SET TRUE.
C
C   INPUT DATA
C   TITLE (8A10) DESCRIPTIVE TITLE TO ACCOMPANY OUTPUT
C   NDIM (I1) SIZE OF MATRICIES (.LE.8)
C   SG2 (F10.5) VARIANCE OF GAUSSIAN PROCESS
C   A ((A(I,J),J=1,NDIM),I=1,NDIM) BY ROWS (UP TO 8 COLUMNS)
C   CORRELATION MATRIX OF FIRST GAUSSIAN VECTOR
C   B (SAME FORMAT AS A) CORRELATION MATRIX OF 2ND GAUSSIAN VECTOR
C   C (SAME FORMAT AS A) FUNCTIONAL MATRIX
C
COMMON /CCR/ A(8,8),B(8,8),C(8,8),NDIM,ISIZE,DETA,DETB,
1A1(8,8),B1(8,8),C4(8,8),A2(8,8),B2(8,8)
COMMON / / S(128),INV(128),PD(1026)
DIMENSION M(3),PHI(1026),TITLE(8)
EQUIVALENCE (PD(1),PHI(1)) $ LOGICAL PNCH
DATA PNCH/.F./
DATA M,IFS/5,9,0,1/
INCPD001
INCPD002
INCPD003
INCPD004
INCPD005
INCPD006
INCPD007
INCPD008
INCPD009
INCPD010
INCPD011
INCPD012
INCPD013
INCPD014
INCPD015
INCPD016
INCPD017
INCPD018
INCPD019
INCPD020
INCPD021
INCPD022
INCPD023
INCPD024
INCPD025
INCPD026

```



```

      IF (IFER.NE.0) WRITE (6,200) IFER
200  FORMAT (1H0,'----ERROR IN FFT, ERROR FLAG =',I2,'/',5X,'RESULTS INVA
      1LID FOR THIS CASE')
      IF (PNCH) WRITE (7,219) TITLE
219  FORMAT (8A10)
      WRITE (6,220) TITLE
220  FORMAT (1H1,5X,8A10)
      WRITE (6,221)
221  FORMAT (1H0,5X,'NORMALIZED',4X,'STOIZED',4X,'DISTRIBUTION',5X,
      1'UNNORMALIZED',3X,'NONSTOIZED',/,7X,'VARIABLE',3X,'PROBABILITY',
      24X,'FUNCTION',9X,'VARIABLE',5X,'PROBABILITY',/,7X,'X/SIGMA X',
      34X,'DENSITY',27X,'X',10X,'DENSITY',/)
      DX1=DX*SR2
      CD=.5 $ X=0. $ X1=0.
      LINCNT=0
      N=INT(10./DX)+2
      IF (N.GT.NPTS/4) N=NPTS/4 $ N=N*2
      JMAX=N
      DO 250 J=1,N,2
      IF (PD(J).LT.0.) GO TO 255
250  CONTINUE
      GO TO 265
255  JMAX=J
      DO 260 I=J,N,2
260  PD(I)=0.0
265  CONTINUE
      IF (.NOT.PNCH) N=JMAX
      DO 300 J=1,N,2
      UPD=PD(J)/SR2
      WRITE (6,222) X,PD(J),CD,X1,UPD
222  FORMAT (1H ,7X,F6.3,4X,E10.3,3X,E12.5,6X,E10.3,4X,E10.3)
      IF (PNCH) WRITE (7,223) X,PD(J),CD,X1,UPD
223  FORMAT (5E1E.8)
      LINCNT=LINCNT+1
C RESULTS ARE PRINTED AT 51 LINES PER PAGE
      IF (LINCNT.LT.51) GO TO 290
      IF (J.GE.N-1) GO TO 300
      WRITE (6,220) TITLE
      WRITE (6,221)
      WRITE (6,222) X,PD(J),CD,X1,UPD
      LINCNT=1
290  IF (J-3) 291,292,293
291  CD=CD+(C1*PD(J+4)+C2*PD(J+2))*DX
      GO TO 294
292  CD=CD+(C1*PD(J)+C2*PD(J-2))*DX
      GO TO 294
293  CD=CD+(C1*PD(J-4)+C2*PD(J-2))*DX
294  CD=CD+(C1*PD(J+6)+C2*PD(J+4)+C3*(PD(J)+PD(J+2)))*DX
      X=X+DX
300  X1=X1+DX1
      GO TO 15
400  STOP $ END

```

```

INCPD090
INCPD091
INCPD092
INCPD093
INCPD094
INCPD095
INCPD096
INCPD097
INCPD098
INCPD099
INCPD100
INCPD101
INCPD102
INCPD103
INCPD104
INCPD105
INCPD106
INCPD107
INCPD108
INCPD109
INCPD110
INCPD111
INCPD112
INCPD113
INCPD114
INCPD115
INCPD116
INCPD117
INCPD118
INCPD119
INCPD120
INCPD121
INCPD122
INCPD123
INCPD124
INCPD125
INCPD126
INCPD127
INCPD128
INCPD129
INCPD130
INCPD131
INCPD132
INCPD133
INCPD134
INCPD135
INCPD136
INCPD137
INCPD138
INCPD139
INCPD140
INCPD141

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```

SUBROUTINE INRPDT(V,I,J,KK)
C
C SUBROUTINE TO COMPUTE THE INNER PRODUCT OF TWO EIGENFUNCTIONS
C
C E = ARRAY OF EIGENVALUES
C EFP = ARRAY OF F EIGENFUNCTIONS (EIGENFUNCTIONS OF 1ST VARIABLE)
C EFQ = ARRAY OF G EIGENFUNCTIONS (EIGENFUNCTIONS OF 2ND VARIABLE)
C F1,F2 = TEMPORARY STORAGE ARRAYS
C I,J = ITH AND JTH EIGENFUNCTIONS ARE TO BE USED IN COMPUTATION
C KK = + IF P EIGENFUNCTIONS ARE TO BE USED IN COMPUTATION
C      - IF Q EIGENFUNCTIONS ARE TO BE USED
C NPOLY = DEGREE OF INTERPOLATING POLYNOMIALS
C NPT = NUMBER OF ABSCISSAE POINTS
C V = VALUE OF INNER PRODUCT RETURNED BY SUBROUTINE
C X = ARRAY OF ABSCISSAE
C
COMMON /ARRAYS/ E(8),EFP(45,8),EFQ(45,8),X(45),F1(45),F2(45),
INPT,NPOLY,C(4),D(4)
C
NC=NPOLY+1 $ IF (KK) 10,100,15
C
C INNER PRODUCT (Q(I),Q(J))
10 DO 11 L=1,NPT
F1(L)=EFQ(L,I)
11 F2(L)=EFQ(L,J)
GO TO 20
C
C INNER PRODUCT (F(I),P(J))
15 DO 16 L=1,NPT
F1(L)=EFP(L,I)
16 F2(L)=EFP(L,J)
C
20 V=0.0 $ K=NPT-1
DO 40 JJ=1,K
CALL COEF(JJ,F1,C) $ CALL COEF(JJ,F2,D)
X1=X(JJ) $ X2=X(JJ+1)
DO 40 II=1,NC
X12=X1**II $ X22=X2**II
CII=C(II) $ CL=FLOAT(II)
DO 40 LL=1,NC
V=V+CII*D(LL)*(X22-X12)/CL $ CL=CL+1. $ X12=X12*X1
40 X22=X22*X2
RETURN
C
100 WRITE (6,101)
101 FORMAT (1H0,5X,*FATAL ERROR IN INRPDT, KK=0*)
STOP $ END

```

```

SUBROUTINE INVR(A,N,DETERM,ISIZE,JSIZE)
C
C SUBROUTINE TO INVERT MATRIX A AND COMPUTE ITS DETERMINANT
C
C DIMENSION IFIVOT(25),A(ISIZE,JSIZE),INDEX(25,2),PIVOT(25)
C EQUIVALENCE (IROW,JROW),(ICOLUJ,JCOLUJ),(AMAX,T,SWAP)
C
10 DETERM=1.0
15 DO 20 J=1,N
20 IPIVOT(J)=0
30 DO 550 I=1,N
C

```

```

C     SEARCH FOR FIVOT ELEMENT
40  AMAX=0.  $  ICOLUM=0
45  DO 105 J=1,N
50  IF (IPIVOT(J)-1) 60, 105, 60
60  DO 100 K=1,N
70  IF (IPIVOT(K)-1) 80, 100, 740
80  IF (ABS(AMAX)-ABS(A(J,K)))85,100,100
85  IROW=J $  ICOLUM=K $  AMAX=A(J,K)
100 CONTINUE
105 CONTINUE
110 IPIVOT(ICCLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
130 IF (IROW-ICOLUM) 140, 260, 140
140 DETERM=-DETERM
150 DO 200 L=1,N
160 SWAP=A(IROW,L) $  A(IROW,L)=A(ICOLUM,L)
200 A(ICOLUM,L)=SWAP
260 INDEX(I,1)=IROW
270 INDEX(I,2)=ICOLUM $  PIVOT(I)=A(ICOLUM,ICOLUM)
320 DETERM=DETERM*PIVOT(I)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
330 A(ICOLUM,ICCLUM)=1.0
340 DO 350 L=1,N
350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
C
C     REDUCE NON-FIVOT ROWS
380 DO 550 L1=1,N
390 IF (L1-ICOLUM) 400, 550, 400
400 T=A(L1,ICCLUM)
420 A(L1,ICCLUM)=0.0
430 DO 450 L=1,N
440 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
450 CONTINUE
550 CONTINUE
C
C     INTERCHANGE COLUMNS
600 DO 710 I=1,N
610 L=N+1-I
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUM=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=A(K,JROW) $  A(K,JROW)=A(K,JCOLUM) $  A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN $  END

```

```

PROGRAM LEVXNG(OUTPUT,PUNCH,TAPE6=OUTPUT,TAPE7=PUNCH)
C
C PROGRAM TO COMPUTE SPECIAL CASE LEVEL CROSSING FREQUENCIES OF THE NON-
C GAUSSIAN MODEL FOR GIVEN LIST OF R PARAMETER VALUES.
C
C INPUT DATA - NONE (PROGRAM IS CONTROLLED THROUGH DATA STATEMENTS)
C
COMMON /CCR/ C1(8,8),C2(8,8),C3(8,8),C4(8,8),NN,ISIZE,DETA,DETB,
1C(320)
DIMENSION X(8192),S(1024),INV(1024),M(3),DF(2),DX(2),TITLE(8)
DIMENSION RARRAY(6)

```

```

COMPLEX SUM,A(64,64) $ LOGICAL PNCH
EQUIVALENCE(X(1),A(1,1))
C IFS AND M ARE VARIABLES USED BY FFT
DATA IFS,M/1,6,6,0/
C RARRAY CONTAINS THE R PARAMETER VALUES TO BE USED
DATA RARRAY/1.,0.,.5,.75,1.333333333,2./
C NCASE IS THE NUMBER OF R PARAMETER VALUES TO BE USED, .LE.6
DATA NCASE/1/
C DXOS IS INCREMENT OF TABULATION (IN STANDARD DEVIATIONS)
C NTAB IS NUMBER OF TABULATED VALUES DESIRED
DATA DXOS,NTAB/.2,41/
C ISIZE IS SIZE OF MATRICIES USED BY INVR
DATA ISIZE/8/
C PNCH IS LOGICAL VARIABLE, IF SET TRUE RESULTS ARE PUNCHED ON CARDS
DATA PNCH/.F./
C C11,C12, AND C13 ARE CONSTANTS USED IN NUMERICAL INTEGRATION OF THE
C JOINT DENSITY FUNCTION
DATA C11,C12,C13/7.6388888889E-03,-6.4583333333E-02,5.5694444444E-
101/
DATA TWOPI2/19.739208802/
C
DO 140 NRATIO=1,NCASE
R=RARRAY(NRATIO)
C GENERATE TITLE ARRAY TO ACCOMPANY OUTPUT
ENCCDE(80,5,TITLE)R
5 FORMAT (* LEVEL CROSSING FREQUENCY OF THE NON-GAUSSIAN MOD
1EL, R=*,F7.3,* *)
C CONSTRUCT CORRELATION AND FUNCTIONAL DEPENDENCE MATRICIES
NN=2 $ IF (R.LE.0.)NN=0
C1(1,1)=C2(1,1)=R/SQRT(1.+R**2)
C1(2,1)=C1(1,2)=C2(1,2)=C2(2,1)=0.
C3(1,1)=1. $ C3(2,2)=C3(2,1)=C3(1,2)=0.
C4(1,1)=C4(2,2)=0. $ C4(1,2)=C4(2,1)=1.
S1G2=1./(1.+R**2)
C
C CODING USED HERE IS FOR U-GUST COMPONENT UNIVERSAL CURVES
C1(2,2)=C1(1,1)/2. $ C2(2,2)=C2(1,1)/2. $ S2G2=S1G2
C
C CODING FOR THE V-W GUST UNIVERSAL CURVES WOULD BE
C1(2,2)=C1(2,2)/2. $ C2(2,2)=C2(1,1) $ S2G2=1.5*S1G2
C
S12=S1G2+C1(1,1)*C2(1,1)
S22=S2G2+C1(1,1)*C2(2,2)+C1(2,2)*C2(1,1)
IF (NN.NE.0) WRITE (6,30) ((C1(I,J),J=1,2),I=1,2),((C2(I,J),J=1,2)
1,I=1,2)
30 FORMAT (1H1,5X,*COVARIANCE MATRICIES OF FIRST AND SECONO*,/,
16X,*GAUSSIAN VECTORS ARE*,/,(6X,2E10.3))
IF (NN.NE.0) WRITE (6,31) ((C3(I,J),J=1,2),I=1,2),((C4(I,J),J=1,2)
1,I=1,2)
31 FORMAT (1H0,5X,*FUNCTIONAL RELATIONSHIP MATRICIES FOR FIRST*,/,
16X,*AND SECONO TRANSFORM VARIABLES ARE*,/,(6X,2E10.3))
WRITE (6,35) R
35 FORMAT (1H0,5X,*R PARAMETER =*,F6.3)
IF (PNCH) WRITE (7,40) R
40 FORMAT (*R =*,F6.3)
C
C GENERATE CHARACTERISTIC FUNCTION
56 CALL INVR(ANS,F1,F2)
WRITE (6,54) DETA,DETB
54 FORMAT (1H0,5X,*DETERMINANTS OF COVARIANCE MATRICIES*,/,(6X,*DETA =
1*,E12.5,/,(6X,*DETB =*,E12.5)
57 DX(1)=.3*SQRT(S12) $ DX(2)=.3*SQRT(S22)
M1=2**M(1) $ M2=2**M(2)

```

```

LEVXG012
LEVXG013
LEVXG014
LEVXG015
LEVXG016
LEVXG017
LEVXG018
LEVXG019
LEVXG020
LEVXG021
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LEVXG066
LEVXG067
LEVXG068
LEVXG069
LEVXG070
LEVXG071
LEVXG072
LEVXG073
LEVXG074

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DF(1)=1./DX(1)/M1 $ DF(2)=1./DX(2)/M2 LEVXG075
WRITE (6,102) DX,DF LEVXG076
102 FORMAT (1H0,5X,*X INCREMENTS: DX(1) =*,E14.7,* DX(2) =*,E14.7,/, LEVXG077
16X,*F INCREMENTS: DF(1) =*,E14.7,* DF(2) =*,E14.7) LEVXG078
DFOF=DF(1)*CF(2) LEVXG079
S1G2=-S1G2*THOPI2 $ S2G2=-S2G2*THOPI2 LEVXG080
A(1,1)=CMPLX(1.,0.) $ F1=DF(1) $ F2=0. LEVXG081
F11=FLOAT(1-M1)*DF(1) LEVXG082
DO 200 I=2,M1 LEVXG083
CALL CF2(ANS,F1,F2) $ ANS=ANS*EXP(S1G2*F1**2) LEVXG084
CALL CF2(ANS1,F11,F2) $ ANS=ANS+ANS1*EXP(S1G2*F11**2) LEVXG085
A(I,1)=CMPLX(ANS,0.) $ F11=F11+DF(1) LEVXG086
200 F1=F1+DF(1) LEVXG087
F1=0. $ F2=DF(2) $ F22=FLOAT(1-M2)*DF(2) LEVXG088
DO 300 J=2,M2 LEVXG089
CALL CF2(ANS,F1,F2) $ ANS=ANS*EXP(S2G2*F2**2) LEVXG090
CALL CF2(ANS1,F1,F22) $ ANS=ANS+ANS1*EXP(S2G2*F22**2) LEVXG091
A(1,J)=CMPLX(ANS,0.) $ F22=F22+DF(2) LEVXG092
300 F2=F2+DF(2) LEVXG093
F11=FLOAT(M1)*DF(1) $ F1=0. $ M22=M2/2+1 LEVXG094
DO 410 I=2,M1 LEVXG095
F1=F1+DF(1) $ F11=F11-DF(1) LEVXG096
F2=0. $ F22=FLOAT(M2)*DF(2) LEVXG097
DO 400 J=2,M22 LEVXG098
F2=-F2+DF(2) $ CALL CF2(ANS,F1,F2) LEVXG099
ANS=ANS*EXP(S1G2*F1**2+S2G2*F2**2) LEVXG100
F2=-F2 $ CALL CF2(ANS1,F11,F2) LEVXG101
ANS=ANS+ANS1*EXP(S1G2*F11**2+S2G2*F2**2) LEVXG102
F22=-F22+DF(2) $ CALL CF2(ANS1,F1,F22) LEVXG103
ANS=ANS+ANS1*EXP(S1G2*F1**2+S2G2*F22**2) LEVXG104
F22=-F22 $ CALL CF2(ANS1,F11,F22) LEVXG105
ANS=ANS+ANS1*EXP(S1G2*F11**2+S2G2*F22**2) LEVXG106
A(I,J)=CMPLX(ANS,0.) LEVXG107
400 A(M1+2-I,M2+2-J)=CMPLX(ANS,0.) LEVXG108
410 CONTINUE LEVXG109
WRITE (6,61) (A(J,1),J=1,M1) LEVXG110
WRITE (6,62) (A(1,J),J=1,M2) LEVXG111
WRITE (6,62) (A(J,J),J=1,M2) LEVXG112
61 FORMAT (1H1,5X,*FIRST ROW, FIRST COLUMN, AND DIAGONAL OF*,/, LEVXG113
16X,*JOINT CHARACTERISTIC FUNCTION*,/, (5X,8E10.3)) LEVXG114
62 FORMAT (1H ,/, (5X,8E10.3)) LEVXG115
C LEVXG116
C FOURIER TRANSFORM TO OBTAIN PROBABILITY DENSITY LEVXG117
DO 71 J=1,8192,2 LEVXG118
71 X(J)=X(J)*DFOF LEVXG119
CALL FFT(X,M,INV,S,IFS,IFER) LEVXG120
IF (IFER.EQ.0) GO TO 73 LEVXG121
WRITE (6,72) IFER $ STOP LEVXG122
72 FORMAT (1H0,5X,*FATAL ERROR IN FFTRS2, IFER =*,I5) LEVXG123
C LEVXG124
C INTEGRATE TO OBTAIN LEVEL CROSSINGS LEVXG125
73 C1X=C11*DX(2) $ C2X=C12*DX(2) $ C3X=C13*DX(2) LEVXG126
WRITE (6,74) (A(J,1),J=1,M1) LEVXG127
WRITE (6,62) (A(1,J),J=1,M2) LEVXG128
WRITE (6,62) (A(J,J),J=1,M2) LEVXG129
74 FORMAT (1H1,5X,*FIRST ROW, FIRST COLUMN, AND DIAGONAL OF*,/, LEVXG130
16X,*JOINT PROBABILITY DENSITY*,/, (5X,8E10.3)) LEVXG131
X2=C. $ M21=M2/2+1 $ M11=M1/2+3 LEVXG132
DO 80 J=1,M21 LEVXG133
DO 75 I=1,M11 LEVXG134
75 A(I,J)=A(I,J)*X2 LEVXG135
80 X2=X2+DX(2) LEVXG136
M21=M2/2-2 LEVXG137

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DO 86 I=1,M11
SUM=C1X*(-A(I,3)+A(I,4)-A(I,2)+A(I,5))+C2X*(A(I,1)-A(I,2)+A(I,3)+
1A(I,4))+C3X*(A(I,1)+2.*A(I,2)+A(I,3))
DO 85 J=1,M21
85 SUM=SUM+C1X*(A(I,J)+A(I,J+5))+C2X*(A(I,J+1)+A(I,J+4))+C3X*(A(I,J+2
1)+A(I,J+3))
X(I)=FLCAT(I-1)*DX(I)
86 S(I)=REAL(SUM)
WRITE (6,87) (X(I),S(I),I=1,M11)
87 FORMAT (1H1,5X,*COMPUTED VALUES OF X AND N(X)*,/, (4X,2E15.7))
C
C INTERPOLATE TO FIND LEVEL CROSSINGS AT SPECIFIED VALUES OF X
X1=0. $ X2=0. $ DIVISR=S(1)
DX1=DXOS*SQRT(S12) $ M21=M2/2+1 $ NINDEX=1
WRITE (6,90) TITLE
90 FORMAT (1H1,36X,*LEVEL CROSSINGS*,/,5X,8A10,/,/,16X,*DIMENSIONAL
1NON-DIMENSIONAL CROSSINGS PER CROSSINGS PER*,/,19X,*LEVEL*,10X,
2*LEVEL*,9X,*UNIT TIME*,4X,*ZERO CROSSING*,/,21X,*X*,10X,*X/SIGMA
3*,/)
DO 105 I=1,NTAB
91 IF (X(NINDEX)-X1) 92,99,93
92 NINDEX=NINDEX+1 $ IF (NINDEX-M21) 91,91,150
93 NINDEX=NINDEX-1
H=(X1-X(NINDEX))/DX1
FI3=S(NINDEX) $ FI4=S(NINDEX+1) $ FI5=S(NINDEX+2)
FI6=S(NINDEX+3) $ IF (NINDEX-2) 94,96,97
94 IF (NINDEX-1) 150,95,96
95 FI1=FI5 $ FI2=FI4 $ GO TO 98
96 FI1=FI3 $ FI2=S(1) $ GO TO 98
97 FI1=S(NINDEX-2) $ FI2=S(NINDEX-1)
98 F1=-FI1*H*(H**2-1.)*(H-2.)*(H-3.)/120.+FI2*H*(H-1.)*(H**2-4.)*
1(H-3.)/24.-FI3*(H**2-1.)*(H**2-4.)*(H-3.)/12.+FI4*H*(H+1.)*(H**2
2-4.)*(H-3.)/12.-FI5*H*(H**2-1.)*(H+2.)*(H-3.)/24.+FI6*H*(H**2-4.
3*(H**2-1.)/120.
GO TO 100
99 F1=S(NINDEX)
100 F2=F1/DIVISR
WRITE (6,120) X1,X2,F1,F2
IF (PUNCH) WRITE (7,121) X2,F2,F1
IF (F2.LE.1.E-10) GO TO 140
X1=X1+DX1
105 X2=X2+DXOS
120 FORMAT (1H ,15X,E10.3,5X,E10.3,6X,E10.3,5X,E10.3)
121 FORMAT (3E20.10)
140 CONTINUE
150 STOP $ END

```

```

PROGRAM POIST(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=
1PUNCH)
C
C A PROGRAM TO GENERATE PROBABILITY DISTRIBUTIONS FOR THE NON-GAUSSIAN
C MODEL OR THE RESPONSE OF A LINEAR SYSTEM TO THE MODEL.
C
C INPUT DATA:
C TITLE(I),I=1,8 (8A10) DESCRIPTIVE TITLE TO ACCOMPANY OUTPUT
C SR2 (E20.10) SYSTEM RESPONSE VARIANCE FOR UNIT VARIANCE INPUT
C SR2=1.0 FOR TURBULENCE MODEL DISTRIBUTION CALCULATIONS
C NEV (I2) NUMBER OF MCN-GAUSSIAN EIGENVALUES TO BE USED (.LE.20)
C NEV=1 FOR TURBULENCE MODEL DISTRIBUTION CALCULATIONS
C EV(I),I=1,NEV (E20.10) NON-GAUSSIAN EIGENVALUES FOR RESPONSE TO

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```

LEVXG138
LEVXG139
LEVXG140
LEVXG141
LEVXG142
LEVXG143
LEVXG144
LEVXG145
LEVXG146
LEVXG147
LEVXG148
LEVXG149
LEVXG150
LEVXG151
LEVXG152
LEVXG153
LEVXG154
LEVXG155
LEVXG156
LEVXG157
LEVXG158
LEVXG159
LEVXG160
LEVXG161
LEVXG162
LEVXG163
LEVXG164
LEVXG165
LEVXG166
LEVXG167
LEVXG168
LEVXG169
LEVXG170
LEVXG171
LEVXG172
LEVXG173
LEVXG174
LEVXG175
LEVXG176
LEVXG177
LEVXG178
LEVXG179
LEVXG180
LEVXG181
LEVXG182
LEVXG183
POIST001
POIST002
POIST003
POIST004
POIST005
POIST006
POIST007
POIST008
POIST009
POIST010
POIST011
POIST012
POIST013

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C      UNIT VARIANCE MODIFIED BESSEL FUNCTION INPUT. EV(1)=1. FOR      PDIST014
C      TURBULENCE MODEL DISTRIBUTION CALCULATIONS.                    PDIST015
C      R (F10.5) SIGMA RATIO OF THE TURBULENCE MODEL (R.GE.0, R=0     PDIST016
C      FOR GAUSSIAN MODEL, R=INF FOR MODIFIED BESSEL FUNCTION MODEL,  PDIST017
C      IN ORDER TO RETAIN COMPUTATIONAL ACCURACY R SHOULD BE LESS    PDIST018
C      THAN 2)                                                       PDIST019
C                                                                    PDIST020
C IF THE SUM OF THE NON-GAUSSIAN EIGENVALUES SQUARED IS FOUND TO BE  PDIST021
C MORE THAN 5 PERCENT GREATER THAN THE SYSTEM RESPONSE VARIANCE, SR2, PDIST022
C EXECUTION IS ENDED.                                             PDIST023
C                                                                    PDIST024
C IF THE SUM IS FOUND TO BE LESS THAN THE RESPONSE VARIANCE, AN      PDIST025
C INDEPENDENT GAUSSIAN PROCESS IS INTRODUCED TO CORRECT THE DEFICIT. PDIST026
C                                                                    PDIST027
C PROGRAM PRINTS THE RESPONSE PROBABILITY DENSITY FUNCTION IN BOTH   PDIST028
C NORMALIZED AND UNNORMALIZED FORM IN INCREMENTS OF DX*SR2 FROM ZERO PDIST029
C TO 200*DX*SR2. DX IS GIVEN IN DATA STATEMENT BELOW.           PDIST030
C RESULTS ARE PUNCHED ON CARDS IF LOGICAL PARAMETER PNCH IS SET TRUE. PDIST031
C                                                                    PDIST032
C      COMMON / / S(120),INV(120),PD(1026)                          PDIST033
C      DIMENSION M(3),PHI(1026)                                     PDIST034
C      DIMENSION TITLE(8),EV(20),EV2(20)                          PDIST035
C      EQUIVALENCE (PD(1),PHI(1))                                 PDIST036
C      LOGICAL PNCH                                              PDIST037
C RESULTS ARE PUNCHED ON CARDS IF PNCH IS TRUE                    PDIST038
C      DATA PNCH/.F./                                           PDIST039
C      DATA PI22/39.478417604/                                   PDIST040
C      DATA M,IFS/9,0,0,1/                                       PDIST041
C MINIMUM VALUE OF M(1) FOR DX OF .05 AND XMAX OF 10. IS 9 (512)  PDIST042
C      DATA DX/.05/                                             PDIST043
C DX*SR2 IS THE INCREMENT OF EVALUATION                           PDIST044
C      DATA C1,C2,C3/7.6388888889E-03,-6.4583333333E-02,5.5694444444E-01/ PDIST045
C C1,C2,C3 ARE CONSTANTS USED IN NUMERICAL INTEGRATION OF PROB. DENSITY PDIST046
C                                                                    PDIST047
C READ INPUT DATA                                               PDIST048
C      15 READ (5,1) TITLE,SR2,NEV                                PDIST049
C          IF (EOF,5) 400,16                                       PDIST050
C      16 READ (5,2) (EV(I),I=1,NEV)                               PDIST051
C          READ (5,3) R                                             PDIST052
C          1 FORMAT (8A10,/,E20.10,/,I2)                          PDIST053
C          2 FORMAT (E20.10)                                       PDIST054
C          3 FORMAT (F10.5)                                         PDIST055
C                                                                    PDIST056
C WRITE INPUT DATA                                             PDIST057
C      WRITE (6,11) TITLE,SR2,NEV,(EV(I),I=1,NEV)                PDIST058
C      WRITE (6,12) R                                             PDIST059
C      11 FORMAT (1H1,5X,8A10,/,/,6X,*VARIANCE =*,F10.5,/,6X,*NUMBER OF NON- PDIST060
C          1GAUSSIAN EIGENVALUES =*,I2,/,6X,*EIGENVALUES ARE**,/, (12X,E15.8)) PDIST061
C      12 FORMAT (1H0,5X,*SIGMA RATIO OF TURBULENCE MODEL =*,F10.5) PDIST062
C                                                                    PDIST063
C COPY EIGENVALLES TO WORKING ARRAY, CHECK SUM OF SQUARES      PDIST064
C      SE2=0.                                                     PDIST065
C      DO 20 I=1,NEV                                             PDIST066
C          EV2(I)=EV(I)**2                                         PDIST067
C      20 SE2=SE2+EV2(I)                                          PDIST068
C          IF (SE2.LE.SR2) GO TO 100                                PDIST069
C          IF (SE2.LT.SR2*1.05) GO TO 90                          PDIST070
C                                                                    PDIST071
C      SUM OF SQUARES IS TOO LARGE                                PDIST072
C      WRITE (6,30) SR2,SE2                                       PDIST073
C      30 FORMAT (1H0,5X,*-----ERROR IN DATA, SUM OF SQUARED EV(I) MORE THAN PDIST074
C          1 5 PERCENT GREATER THAN SR2*,/,6X,*SR2 =*,E12.5,/,6X,*SUM OF EV(I) PDIST075
C          2 SQUARED =*,E12.5)                                     PDIST076

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GO TO 15
C
C SUM OF SQUARED EIGENVALUES EXCEEDS SYSTEM VARIANCE BY LESS THAN 5 PCNT
90 SE2=SE2/SR2
DO 92 I=1,NEV
EV2(I)=EV2(I)/SE2
92 EV(I)=SQRT(EV2(I))
WRITE (6,94) SE2,(EV(I),EV2(I),I=1,NEV)
94 FORMAT (1H0,5X,*SUM OF EIGENVALUES SQUARED EXCEEDS VARIANCE BY A FACTOR OF*,F8.5,/,6X,*EIGENVALUES WILL BE SCALED TO GIVE CORRECT VARIANCE*,/,6X,*SCALED EIGENVALUES AND SQUARED EIGENVALUES ARE*,/,3(6X,2E15.7))
SE2=SR2
C
C SCALE GAUSSIAN AND NON-GAUSSIAN VARIANCES ACCORDING TO SIGMA RATIO OF TURBULENCE MODEL
100 RC=R**2/(1.+R**2)
SES2=SE2*RC $ SE2G=(SR2-SE2)*RC
EVG2=SR2/(1.+R**2)+SE2G
C EVG2 IS THE VARIANCE OF THE GAUSSIAN PORTION OF THE SYSTEM RESPONSE INCLUDING CORRECTION FOR NEGLECTED NON-GAUSSIAN EIGENVALUES
C
SC=1./(EVG2+SES2) $ SE=SES2*SC $ SG=EVG2*SC
WRITE (6,130) SE2,SR2,SES2,EVG2,SE,SG
130 FORMAT (1H0,5X,*SUM OF EIGENVALUES SQUARED =*,E15.7,/,/,6X,1*FOR VARIANCE OF*,E15.7,/,6X,*NON-GAUSSIAN VARIANCE =*,E15.7,/,6X,2*GAUSSIAN VARIANCE INCLUDING CORRECTION FOR*,/,6X,*NEGLECTED EIGENVALUES =*,E15.7,/,/,6X,*FOR TOTAL VARIANCE OF UNITY*,/,6X,*NON-GAUSSIAN VARIANCE =*,E15.7,/,6X,*GAUSSIAN VARIANCE INCLUDING CORRECTION FOR*,/,6X,*NEGLECTED EIGENVALUES =*,E15.7)
SC=SC*RC
DO 135 I=1,NEV
135 EV2(I)=EV2(I)*SC
WRITE (6,137) (EV2(I),I=1,NEV)
137 FORMAT (1H0,5X,*SQUARED EIGENVALUES SCALED TO GIVE CORRECT NON-GAUSSIAN*,/,6X,*CONTRIBUTION TO UNIT VARIANCE:*,/,6X,E15.7))
C GENERATE RESPONSE CHARACTERISTIC FUNCTION
EVG2=SG/2. $ F=0. $ DF=1./DX/2.**M(1) $ NPTS=2**M(1)+1
DO 150 I=1,NPTS,2
F2=PI22*F**2 $ CFR=1.
DO 140 J=1,NEV
140 CFR=CFR/(1.+EV2(J)*F2)
CFR=SQRT(CFR) $ CFR=CFR*EXP(-EVG2*F2)
PHI(I)=CFR $ PHI(I+1)=0.
150 F=F+DF
C
C TRANSFORM CHARACTERISTIC FUNCTION
CALL FFTRS(PHI,DF,DX,M,INV,S,IFS,IFER)
IFS=2
IF (IFER.EQ.0) GO TO 210
WRITE (6,200) IFER $ STOP
200 FORMAT (1H0,*---ERROR IN FFT, ERROR FLAG =*,I2,/,5X,*RESULTS INVALID FOR THIS CASE*)
C
C INTERPOLATE DENSITY FUNCTION TO OBTAIN FINAL RESULTS
210 IF (PNCH) WRITE (7,219) TITLE
219 FORMAT (8A10)
WRITE (6,220) TITLE
220 FORMAT (1H1,5X,8A10)
WRITE (6,221)
221 FORMAT (1H0,5X,*NORMALIZED*,4X,*STANDARDIZED*,4X,*DISTRIBUTION*,5X,1*UNNORMALIZED*,3X,*NONSTANDARDIZED*,/,7X,*VARIABLE*,3X,*PROBABILITY*,24X,*FUNCTION*,9X,*VARIABLE*,5X,*PROBABILITY*,/,7X,*X/SIGMA X*,

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34X,*DENSITY*,27X,*X*,10X,*DENSITY*,/)
DX1=DX*SR2 $ CD=.5 $ X=0. $ X1=0.
LINCNT=0 $ N=INT(10./DX+.99)+1 $ IF (N.GT.NPTS/4) N=NPTS/4
DO 300 I=1,N
J=2*I-1 $ UPD=PD(J)/SR2
WRITE (6,222) X,PD(J),CD,X1,UPD
222 FORMAT (1H,7X,F6.3,4X,E10.3,3X,E12.5,6X,E10.3,4X,E10.3)
IF (PNCH) WRITE (7,223) X,PD(J),CD,X1,UPD
223 FORMAT (5E16.8)
LINCNT=LINCNT+1
C LISTING PPOUCED AT 51 LINES PER PAGE
IF (LINCNT.LT.51) GO TO 290 $ IF (I.EQ.N) GO TO 15
WRITE (6,220) TITLE
WRITE (6,221)
WRITE (6,222) X,PD(J),CD,X1,UPD
LINCNT=1
290 IF (I-2) 291,292,293
291 CD=CD+(C1*PD(J+4)+C2*PD(J+2))*DX $ GO TO 294
292 CD=CD+(C1*PD(J)+C2*PD(J-2))*DX $ GO TO 294
293 CD=CD+(C1*PD(J-4)+C2*PD(J-2))*DX
294 CD=CD+(C1*PD(J+6)+C2*PD(J+4)+C3*(PD(J)+PD(J+2)))*DX
X=X+DX
300 X1=X1+DX1
GO TO 15
400 STOP $ END

```

POIST140
 POIST141
 POIST142
 POIST143
 POIST144
 POIST145
 POIST146
 POIST147
 POIST148
 POIST149
 POIST150
 POIST151
 POIST152
 POIST153
 POIST154
 POIST155
 POIST156
 POIST157
 POIST158
 POIST159
 POIST160
 POIST161
 POIST162
 POIST163
 POIST164

```

PROGRAM RLEVX(INPUT,OUTPUT,PUNCH,TAPES=INPUT,TAPE6=OUTPUT,TAPE7=
1PUNCH)
C
C PROGRAM TO COMPUTE LEVEL CROSSINGS/UNIT TIME OF LINEAR SYSTEM RESPONSE
C TO NON-GAUSSIAN MODEL. CROSSINGS/DISTANCE CAN BE OBTAINED BY DIVISION
C BY MEAN TRUE AIRSPEED.
C
C INPUT DATA
C TITLE (8A10) DESCRIPTIVE TITLE TO ACCOMPANY OUTPUT
C NPT (I2) NUMBER OF ABCISSAE POINTS AT WHICH EIGENFUNCTIONS ARE
C TABULATED
C NR1 (I1) NUMBER OF EIGENSOLUTIONS DEFINING FIRST VARIABLE
C E(I),(X(J),EFP(J,I),EFQ(J,I),J=1,NPT) (45X,E20.11,/, (3E25.14))
C EIGENVALUE, ABCISSAE POINTS, P EIGENFUNCTION, AND Q EIGENFUNCTION
C RESPECTIVELY AS PUNCHED BY GENERATING PROGRAM. REPEATED FOR
C EACH OF THE NR1 EIGENSOLUTIONS. (I=1,NR1)
C S12 (F10.5) TCTAL VARIANCE OF FIRST VARIABLE
C NR2 (I1) NUMBER OF EIGENSOLUTIONS DEFINING SECOND VARIABLE
C E(I),(X(J),EFP(J,I),EFQ(J,I),J=1,NPT) (45X,E20.11,/, (3E25.14))
C EIGENVALUE, ABCISSAE POINTS, P EIGENFUNCTION, AND Q EIGENFUNCTION
C RESPECTIVELY AS PUNCHED BY GENERATING PROGRAM. REPEATED FOR
C EACH OF THE NR2 EIGENSOLUTIONS (I=1,NR2)
C S22 (F10.5) TCTAL VARIANCE OF SECOND VARIABLE
C R (F10.5) SIGMA RATIO OF TURBULENCE MODEL
C
C NR1+NR2 MUST BE .LE.8, OTHERWISE REDIMENSIONING IS REQUIRED.
C RESULTS ARE PUNCHED ON CARDS IF LOGICAL VARIABLE PNCH IS SET TRUE.
C
COMMON /ARRAYS/ E(8),EFP(45,8),EFQ(45,8),XA(45),G1(45),G2(45),
1NPT,NPOLY,COEFA(8)
COMMON /CCR/ C1(8,8),G2(8,8),C3(8,8),C4(8,8),NN,ISIZE,OETA,DET8,
1C(320)
DIMENSION X(8192),S(1024),INV(1024),M(3),DF(2),DX(2),A(64,64),
1TITLE(8)

```

RLEVX001
 RLEVX002
 RLEVX003
 RLEVX004
 RLEVX005
 RLEVX006
 RLEVX007
 RLEVX008
 RLEVX009
 RLEVX010
 RLEVX011
 RLEVX012
 RLEVX013
 RLEVX014
 RLEVX015
 RLEVX016
 RLEVX017
 RLEVX018
 RLEVX019
 RLEVX020
 RLEVX021
 RLEVX022
 RLEVX023
 RLEVX024
 RLEVX025
 RLEVX026
 RLEVX027
 RLEVX028
 RLEVX029
 RLEVX030
 RLEVX031
 RLEVX032
 RLEVX033
 RLEVX034

```

COMPLEX A,SUM $ LOGICAL PNCH
EQUIVALENCE (X(1),E(1),A(1,1))
C PNCH SET .F. SUPPRESSES PUNCHED OUTPUT
DATA PNCH/.F./
C IFS AND M ARE VARIABLES USED BY SUBROUTINE FFT
DATA IFS,M/1,6,6,0/
DATA TWCP12/19.739208802/
C C11,C12,C13 ARE CONSTANTS USED IN NUMERICAL INTEGRATION OF JOINT
C PROBABILITY DENSITY FUNCTION
DATA C11,C12,C13/7.6388888889E-03,-6.4583333333E-02,5.5694444444E-
10/
C DXOS IS TABULATION INCREMENT (IN STD. DEVIATIONS OF THE RESPONSE)
C NTAB IS NUMBER OF TABULATED VALUES TO BE COMPUTED
DATA DXOS,NTAB/.2,41/
C ISIZE IS DIMENSION OF MATRICES USED BY INVERSION ROUTINE INVR
DATA ISIZE/8/
C
C READ INPUT DATA
1 FORMAT (I2,/,I1)
2 FORMAT (45X,E20.11,/, (3E25.14))
3 FORMAT (E20.11,/,I1)
4 FORMAT (E20.11,/,F10.5)
5 FORMAT (8A10)
READ (5,5) TITLE
READ (5,1) NPT,NR1
IF (NR1.LE.0) GO TO 7
DO 6 I=1,NR1
6 READ (5,2) E(I), (XA(J),EFP(J,I),EFQ(J,I),J=1,NPT)
7 READ (5,3) S12,NR2
J=NR1+1 $ NN=NR1+NR2 $ IF (NR2.LE.0) GO TO 9
DO 8 I=J,NN
8 READ (5,2) E(I), (XA(J),EFP(J,I),EFQ(J,I),J=1,NPT)
9 READ (5,4) S22,R
C
C SCALE EIGENVALUES AND COMPUTE GAUSSIAN VARIANCES
CALL SCALE(E,NR1,NR2,S12,S22,S1G2,S2G2,R,TITLE)
C
C GENERATE COVARIANCE MATRICES
IF (NN.LE.0) GO TO 57
DO 20 I=1,NN
C1(I,I)=1. $ C2(I,I)=1. $ I1=I+1
DO 20 J=I1,NN
IF ((I-NR1)*(J-NR1)) 10,11,12
10 CALL INRPDT(C1(I,J),I,J,1) $ C1(J,I)=C1(I,J)
CALL INRPDT(C2(I,J),I,J,-1) $ C2(J,I)=C2(I,J)
GO TO 20
11 IF (I.EQ.NR1) GO TO 10
12 C1(J,I)=0. $ C1(I,J)=0. $ C2(J,I)=0. $ C2(I,J)=0.
20 CONTINUE
WRITE (6,21) TITLE
21 FORMAT (1H1,5X,*COVARIANCE MATRIX FOR P VARIABLES*,/,6X,8A10)
DO 25 I=1,NN
25 WRITE (6,30) (C1(I,J),J=1,NN)
30 FORMAT (1H0,5X,10F11.6)
WRITE (6,31) TITLE
31 FORMAT (1H0,5X,*COVARIANCE MATRIX FOR Q VARIABLES*,/,6X,8A10)
DO 35 I=1,NN
35 WRITE (6,30) (C2(I,J),J=1,NN)
C
C GENERATE FUNCTIONAL DEPENDENCE MATRICES
36 IF (NR1.LE.0) GO TO 51
I=1
WRITE (6,40) TITLE,I

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RLEVX035
RLEVX036
RLEVX037
RLEVX038
RLEVX039
RLEVX040
RLEVX041
RLEVX042
RLEVX043
RLEVX044
RLEVX045
RLEVX046
RLEVX047
RLEVX048
RLEVX049
RLEVX050
RLEVX051
RLEVX052
RLEVX053
RLEVX054
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RLEVX067
RLEVX068
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RLEVX083
RLEVX084
RLEVX085
RLEVX086
RLEVX087
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RLEVX089
RLEVX090
RLEVX091
RLEVX092
RLEVX093
RLEVX094
RLEVX095
RLEVX096
RLEVX097

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40 FORMAT (1H0,5X,*FUNCTIONAL DEPENDENCE MATRIX*,/,6X,8A10,/,6X,*VARIRLEVX098
TABLE NO.*,I2) RLEVX099
DO 50 I=1,NR1 RLEVX100
C3(I,I)=E(I) $ C4(I,I)=0. $ I1=I+1 RLEVX101
DO 45 J=I1,NN RLEVX102
C3(I,J)=0. $ C3(J,I)=0. $ C4(I,J)=0. $ C4(J,I)=0. RLEVX103
45 CONTINUE RLEVX104
50 WRITE (6,30) (C3(I,J),J=1,NR1) RLEVX105
51 IF (NR2.LE.0) GO TO 56 RLEVX106
I=2 $ M1=NR1+1 RLEVX107
WRITE (6,40) TITLE,I RLEVX108
DO 55 I=M1,NN RLEVX109
C3(I,I)=0. $ C4(I,I)=E(I) $ I1=I+1 RLEVX110
DO 53 J=I1,NN RLEVX111
C3(I,J)=0. $ C3(J,I)=0. $ C4(I,J)=0. $ C4(J,I)=0. RLEVX112
53 CONTINUE RLEVX113
55 WRITE (6,30) (C4(I,J),J=M1,NN) RLEVX114
IF (NR1.LE.0) GO TO 56 $ ERRD=0. RLEVX115
DO 65 I=1,NR1 RLEVX116
DO 65 J=M1,NN RLEVX117
65 ERRD=ERRD+C1(I,J)*C2(I,J)*C3(I,I)*C4(J,J) RLEVX118
ERRD=ERRD/SCRT(S12*S22) RLEVX119
WRITE (6,66) ERRD RLEVX120
66 FORMAT (1+0,5X,*CORRELATION COEFFICIENT OF RESPONSE AND ITS FIRST RLEVX121
1 DERIVATIVE =*,E10.3,/,6X,*THIS COEFFICIENT SHOULD BE MUCH LESS THARLEVX122
2N 1.0*) RLEVX123

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C

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C GENERATE CHARACTERISTIC FUNCTION RLEVX124
56 CALL INVR(ANS,F1,F2) RLEVX125
WRITE (6,54) DETA,DETB RLEVX126
54 FORMAT (1H0,5X,*DETERMINANTS OF COVARIANCE MATRICIES*,/,6X,*DETA =RLEVX128
1*,E12.5,/,6X,*DETB =*,E12.5) RLEVX129
57 DX(1)=.3*SQRT(S12) $ DX(2)=.3*SQRT(S22) RLEVX130
M1=2**M(1) $ M2=2**M(2) RLEVX131
DF(1)=1./DX(1)/M1 $ DF(2)=1./DX(2)/M2 RLEVX132
WRITE (6,102) DX,DF RLEVX133
102 FORMAT (1H0,5X,*INCREMENTS*,/,6X,*DX(1) =*,E14.7,5X,*DX(2) =*, RLEVX134
1E14.7,/,6X,*DF(1) =*,E14.7,5X,*DF(2) =*,E14.7) RLEVX135
DFDF=DF(1)*DF(2) RLEVX136
S1G2=-S1G2*THOPI2 $ S2G2=-S2G2*THOPI2 RLEVX137
A(1,1)=CMPLX(1.,0.) $ F1=DF(1) $ F2=0. RLEVX138
F11=FLOAT(1-M1)*DF(1) RLEVX139
DO 200 I=2,M1 RLEVX140
CALL CF2(ANS,F1,F2) $ ANS=ANS*EXP(S1G2*F1**2) RLEVX141
CALL CF2(ANS1,F11,F2) $ ANS=ANS+ANS1*EXP(S1G2*F11**2) RLEVX142
A(I,1)=CMPLX(ANS,0.) $ F11=F11+DF(1) RLEVX143
200 F1=F1+DF(1) RLEVX144
F1=0.0 $ F2=DF(2) $ F22=FLOAT(1-M2)*DF(2) RLEVX145
DO 300 J=2,M2 RLEVX146
CALL CF2(ANS,F1,F2) $ ANS=ANS*EXP(S2G2*F2**2) RLEVX147
CALL CF2(ANS1,F1,F22) $ ANS=ANS+ANS1*EXP(S2G2*F22**2) RLEVX148
A(1,J)=CMPLX(ANS,0.) $ F22=F22+DF(2) RLEVX149
300 F2=F2+DF(2) RLEVX150
F11=FLOAT(M1)*DF(1) $ F1=0. $ M22=M2/2+1 RLEVX151
DO 400 I=2,M1 RLEVX152
F1=F1+DF(1) $ F11=F11+DF(1) RLEVX153
F2=0. $ F22=FLOAT(M2)*DF(2) RLEVX154
DO 400 J=2,M2 RLEVX155
F2=-F2+DF(2) $ CALL CF2(ANS,F1,F2) RLEVX156
ANS=ANS*EXP(S1G2*F1**2+S2G2*F2**2) RLEVX157
F2=-F2 $ CALL CF2(ANS1,F11,F2) RLEVX158
ANS=ANS+ANS1*EXP(S1G2*F11**2+S2G2*F2**2) RLEVX159
F22=-F22+DF(2) $ CALL CF2(ANS1,F1,F22) RLEVX160

```

```

ANS=ANS+ANS1*EXP(S1G2*F1**2+S2G2*F22**2)
F22=-F22 $ CALL CF2(ANS1,F11,F22)
ANS=ANS+ANS1*EXP(S1G2*F11**2+S2G2*F22**2)
A(I,J)=CMPLX(ANS,0.)
400 A(M1+2-I,M2+2-J)=CMPLX(ANS,0.)
410 CONTINUE
WRITE (6,61) (A(J,1),J=1,M1)
WRITE (6,62) (A(1,J),J=1,M2)
WRITE (6,62) (A(J,J),J=1,M2)
61 FORMAT (1H1,5X,*FIRST ROW, FIRST COLUMN, AND DIAGONAL OF JOINT CHARACTERISTIC FUNCTION*,/(5X,8E10.3))
62 FORMAT (1H ,/(5X,8E10.3))
C
C FOURIER TRANSFORM TO OBTAIN PROBABILITY DENSITY
DO 71 J=1,8192,2
71 X(J)=X(J)*DFDF
CALL FFT(X,M,INV,S,IFS,IFER) $ IF (IFER.EQ.0) GO TO 73
WRITE (6,72) IFER
72 FORMAT (1H0,5X,*FATAL ERROR IN FFTS2, IFER =*,I5)
STOP
C
C INTEGRATE TO OBTAIN LEVEL CROSSINGS
73 C1X=C11*DX(2) $ C2X=C12*DX(2) $ C3X=C13*DX(2)
WRITE (6,74) (A(J,1),J=1,M1)
WRITE (6,62) (A(1,J),J=1,M2)
WRITE (6,62) (A(J,J),J=1,M2)
74 FORMAT (1H1,5X,*FIRST ROW, FIRST COLUMN, AND DIAGONAL OF JOINT PROBABILITY DENSITY FUNCTION*,/(5X,8E10.3))
X2=0. $ M21=M2/2+1 $ M11=M1/2+3
DO 80 J=1,M21
DO 75 I=1,M11
75 A(I,J)=A(I,J)*X2
80 X2=X2+DX(2) $ M21=M2/2-2
DO 86 I=1,M11
SUM=C1X*(-A(I,3)+A(I,4)-A(I,2)+A(I,5))+C2X*(A(I,1)-A(I,2)+A(I,3)+A(I,4))+C3X*(A(I,1)+2.*A(I,2)+A(I,3))
DO 85 J=1,M21
85 SUM=SUM+C1X*(A(I,J)+A(I,J+5))+C2X*(A(I,J+1)+A(I,J+4))+C3X*(A(I,J+2)+A(I,J+3))
X(I)=FLOAT(I-1)*DX(1)
86 S(I)=REAL(SUM)
WRITE (6,87) (X(I),S(I),I=1,M11)
87 FORMAT (1H1,5X,*COMPUTED VALUES OF X AND N(X)*,/(4X,2E15.7))
C
C INTERPOLATE TO FIND LEVEL CROSSINGS AT SPECIFIED VALUES OF X
X1=0. $ X2=0. $ DIVISR=S(1)
DX1=DXOS*SQRT(S12) $ M21=M2/2+1 $ NINDEX=1
WRITE (6,90) TITLE
90 FORMAT (1H1,36X,*LEVEL CROSSINGS*,/(5X,8A10.//,16X,*DIMENSIONAL NON-DIMENSIONAL CROSSINGS PER CROSSINGS PER*,/(19X,*LEVEL*,10X,2*LEVEL*,9X,*UNIT TIME*,4X,*ZERO CROSSING*,/(21X,*X*,10X,*X/SIGMA X*,/(3X,/)
DO 105 I=1,NTAB
91 IF (X(NINDEX)-X1) 92,99,93
92 NINDEX=NINDEX+1 $ IF (NINDEX-M21) 91,91,150
93 NINDEX=NINDEX-1 $ H=(X1-X(NINDEX))/DX(1)
FI3=S(NINDEX) $ FI4=S(NINDEX+1) $ FI5=S(NINDEX+2)
FI6=S(NINDEX+3) $ IF (NINDEX-2) 94,96,97
94 IF (NINDEX-1) 150,95,96
95 FI1=FI5 $ FI2=FI4 $ GO TO 98
96 FI1=FI3 $ FI2=S(1) $ GO TO 98
97 FI1=S(NINDEX-2) $ FI2=S(NINDEX-1)
98 F1=-FI1*H*(H**2-1.)*(H-2.)*(H-3.)/120.+FI2*H*(H-1.)*(H**2-4.)*

```

RLEVX161
RLEVX162
RLEVX163
RLEVX164
RLEVX165
RLEVX166
RLEVX167
RLEVX168
RLEVX169
RLEVX170
RLEVX171
RLEVX172
RLEVX173
RLEVX174
RLEVX175
RLEVX176
RLEVX177
RLEVX178
RLEVX179
RLEVX180
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RLEVX192
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RLEVX196
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RLEVX200
RLEVX201
RLEVX202
RLEVX203
RLEVX204
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RLEVX208
RLEVX209
RLEVX210
RLEVX211
RLEVX212
RLEVX213
RLEVX214
RLEVX215
RLEVX216
RLEVX217
RLEVX218
RLEVX219
RLEVX220
RLEVX221
RLEVX222
RLEVX223

```

1(H-3.)/24.-FI3*(H**2-1.)*(H**2-4.)*(H-3.)/12.+FI4*H*(H+1.)*(H**2
2-4.)*(H-3.)/12.-FI5*H*(H**2-1.)*(H+2.)*(H-3.)/24.+FI6*H*(H**2-4.)
3*(H**2-1.)/120.
GO TO 100
99 F1=S(NINDEX)
100 F2=F1/DIVISR
WRITE (6,120) X1,X2,F1,F2
IF (F1.LE.1.E-11) GO TO 150
X1=X1+OX1
105 X2=X2+OXOS
120 FORMAT (1H ,15X,E10.3,5X,E10.3,6X,E10.3,5X,E10.3)
150 STOP $ END

```

```

RLEVX224
RLEVX225
RLEVX226
RLEVX227
RLEVX228
RLEVX229
RLEVX230
RLEVX231
RLEVX232
RLEVX233
RLEVX234
RLEVX235

```

```

SUBROUTINE SCALE(E,N1,N2,S12,S22,S1G2,S2G2,R,TITLE)
C
C SUBROUTINE TO SCALE EIGENVALUES AND CALCULATE GAUSSIAN VARIANCES
C
C E = EIGENVALUE ARRAY
C N1 = NO. OF EIGENVALUES DESCRIBING FIRST VARIABLE
C N2 = NO. OF EIGENVALUES DESCRIBING SECOND VARIABLE
C S12 = VARIANCE OF 1ST VARIABLE IN RESPONSE TO UNIT VARIANCE INPUT
C S22 = VARIANCE OF 2ND VARIABLE IN RESPONSE TO UNIT VARIANCE INPUT
C S1G2 = GAUSSIAN VARIANCE OF 1ST VARIABLE
C S2G2 = GAUSSIAN VARIANCE OF 2ND VARIABLE
C
C DIMENSION E(1),TITLE(8)
C
S=S12 $ N=N1 $ N0=1
DO 100 I=1,2
SE2=0. $ IF (N.GT.N0) GO TO 9 $ S2G2=S
IF (I.EQ.1) S1G2=S2G2 $ GO TO 60
9 DO 10 J=N0,N
10 SE2=SE2+E(J)**2
IF (SE2.LE.S1G2) $ IF (SE2.LE.S*1.05) GO TO 30
WRITE (6,15) SE2,S,I,N $ STOP
15 FCRPAT (1H0,5X,*FATAL ERROR IN SCALE, SE2 EXCEEDS S BY MORE THAN 5
1 PERCENT*,/,6X,*SE2 =*,E15.8,/,6X,*S =*,E15.8,/,6X,*I =*,I2,/,6X,*
2N =*,I2)
30 SE2=SQRT(S/SE2)
DO 35 J=N0,N
35 E(J)=E(J)*SE2 $ SE2=S
40 S2G2=S-SE2*(R**2)/(1.+R**2)
IF (I.EQ.1) S1G2=S2G2 $ SE2=(R**2)/(1.+R**2) $ SE2=SQRT(SE2)
DO 50 J=N0,N
50 E(J)=E(J)*SE2
60 N0=N1+1 $ N=N1+N2
100 S=S22
IF (N.LE.0) E(1)=0.
WRITE (6,110) TITLE $ J=1 $ WRITE (6,115) J,S1G2,(E(I),I=1,N1)
110 FORMAT (1H1,5X,*RESULTS OF SCALING*,/,6X,8A10)
J=2 $ WRITE (6,115) J,S2G2,(E(I),I=N0,N) $ WRITE (6,120) R
115 FCRPAT (1H0,5X,*VARIABLE NO.*,I2,/,6X,*GAUSSIAN VARIANCE INCLUDING
1 CORRECTION FOR NEGLECTED EIGENVALUES =*,E12.5,/,6X,*SCALED EIGENV
2ALUES*,/,10X,E12.5))
120 FCRPAT (1H0,5X,*SIGMA RATIO OF TURBULENCE MODEL =*,F6.3)
SE2=0.
DO 125 I=1,N1
125 SE2=SE2+E(I)**2
S=SE2+S1G2 $ I=1
WRITE (6,130) I,S12,SE2,S1G2,S

```

```

SCALE001
SCALE002
SCALE003
SCALE004
SCALE005
SCALE006
SCALE007
SCALE008
SCALE009
SCALE010
SCALE011
SCALE012
SCALE013
SCALE014
SCALE015
SCALE016
SCALE017
SCALE018
SCALE019
SCALE020
SCALE021
SCALE022
SCALE023
SCALE024
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SCALE028
SCALE029
SCALE030
SCALE031
SCALE032
SCALE033
SCALE034
SCALE035
SCALE036
SCALE037
SCALE038
SCALE039
SCALE040
SCALE041
SCALE042
SCALE043
SCALE044
SCALE045
SCALE046
SCALE047

```

```

130 FORMAT (1H0,5X,*VARIANCE CHECK, VARIABLE NO.*,I2,/,6X,*CORRECT TOTSCALE048
1AL VARIANCE =*,E12.5,/,6X,*SUM CF SCALED EIGENVALUES SQUARED =*,E1SCALE049
22.5,/,6X,*GAUSSIAN VARIANCE =*,E12.5,/,6X,*TOTAL VARIANCE =*,E12.5SCALE050
3)
SE2=0. SCALE051
DO 135 I=N0,N SCALE052
135 SE2=SE2+E(I)**2 $ S=SE2+S2G2 $ I=2 SCALE053
WRITE (6,130) I,S22,SE2,S2G2,S SCALE054
140 RETURN $ END SCALE055
SCALE056

```

APPENDIX B

TABULATED FUNCTIONS

The purpose of this appendix is to present tabulated values of certain results which were presented in graphic form within the main body of this report. Specific results to be presented are:

Table B1 - standardized probability densities of the non-gaussian turbulence model for various values of the probability distribution parameter R

Table B2 - probability distribution functions of the non-gaussian turbulence model for various values of the probability distribution parameter R

Table B3 - universal level crossing frequencies of the non-gaussian turbulence model longitudinal component for various values of the probability distribution parameter R

Table B4 - universal level crossing frequencies of the non-gaussian turbulence model vertical and lateral components for various values of the probability distribution parameter R .

Table B1.--Standardized probability densities of the non-gaussian
turbulence model for various values of the
probability distribution parameter R .

(Graphical presentation of most of those data is given
in figure 15 on page 61 of this report.)

$$\hat{p}(x|R)$$

| x | R | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| | 0.0 | 0.5 | 0.75 | 1.0 | 1.33 | 2.0 |
| 0.0 | 3.989E-01 | 4.073E-01 | 4.234E-01 | 4.455E-01 | 4.792E-01 | 5.478E-01 |
| .2 | 3.910E-01 | 3.986E-01 | 4.132E-01 | 4.330E-01 | 4.623E-01 | 5.182E-01 |
| .4 | 3.683E-01 | 3.738E-01 | 3.843E-01 | 3.976E-01 | 4.156E-01 | 4.409E-01 |
| .6 | 3.332E-01 | 3.359E-01 | 3.407E-01 | 3.457E-01 | 3.496E-01 | 3.434E-01 |
| .8 | 2.897E-01 | 2.894E-01 | 2.884E-01 | 2.854E-01 | 2.774E-01 | 2.514E-01 |
| 1.0 | 2.420E-01 | 2.391E-01 | 2.335E-01 | 2.248E-01 | 2.096E-01 | 1.786E-01 |
| 1.2 | 1.942E-01 | 1.898E-01 | 1.814E-01 | 1.699E-01 | 1.530E-01 | 1.266E-01 |
| 1.4 | 1.497E-01 | 1.447E-01 | 1.356E-01 | 1.241E-01 | 1.093E-01 | 9.100E-02 |
| 1.6 | 1.109E-01 | 1.063E-01 | 9.802E-02 | 8.838E-02 | 7.748E-02 | 6.662E-02 |
| 1.8 | 7.895E-02 | 7.527E-02 | 6.885E-02 | 6.194E-02 | 5.510E-02 | 4.953E-02 |
| 2.0 | 5.399E-02 | 5.153E-02 | 4.728E-02 | 4.309E-02 | 3.952E-02 | 3.724E-02 |
| 2.2 | 3.547E-02 | 3.420E-02 | 3.195E-02 | 2.997E-02 | 2.865E-02 | 2.822E-02 |
| 2.4 | 2.239E-02 | 2.208E-02 | 2.138E-02 | 2.094E-02 | 2.098E-02 | 2.151E-02 |
| 2.6 | 1.358E-02 | 1.392E-02 | 1.425E-02 | 1.474E-02 | 1.549E-02 | 1.648E-02 |
| 2.8 | 7.915E-03 | 8.603E-03 | 9.511E-03 | 1.046E-02 | 1.150E-02 | 1.266E-02 |
| 3.0 | 4.432E-03 | 5.240E-03 | 6.373E-03 | 7.476E-03 | 8.585E-03 | 9.764E-03 |
| 3.2 | 2.384E-03 | 3.160E-03 | 4.295E-03 | 5.375E-03 | 6.432E-03 | 7.548E-03 |
| 3.4 | 1.232E-03 | 1.896E-03 | 2.912E-03 | 3.884E-03 | 4.834E-03 | 5.848E-03 |
| 3.6 | 6.119E-04 | 1.135E-03 | 1.987E-03 | 2.817E-03 | 3.643E-03 | 4.539E-03 |
| 3.8 | 2.919E-04 | 6.812E-04 | 1.362E-03 | 2.050E-03 | 2.751E-03 | 3.530E-03 |
| 4.0 | 1.338E-04 | 4.103E-04 | 9.382E-04 | 1.495E-03 | 2.081E-03 | 2.749E-03 |

Table B2.--Probability distribution functions of the non-gaussian turbulence model for various values of the probability distribution parameter R .

(Graphical presentation of most of these data is given in figure 15 on page 61 of this report.)

$$P(-\sigma x|R)^{\dagger}$$

| x | R | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| | 0.0 | 0.5 | 0.75 | 1.0 | 1.33 | 2.0 |
| 0.0 | 5.000E-01 | 5.000E-01 | 5.000E-01 | 5.000E-01 | 5.000E-01 | 5.000E-01 |
| .2 | 4.207E-01 | 4.191E-01 | 4.160E-01 | 4.117E-01 | 4.053E-01 | 3.924E-01 |
| .4 | 3.446E-01 | 3.416E-01 | 3.360E-01 | 3.284E-01 | 3.171E-01 | 2.959E-01 |
| .6 | 2.743E-01 | 2.705E-01 | 2.633E-01 | 2.538E-01 | 2.404E-01 | 2.174E-01 |
| .8 | 2.119E-01 | 2.078E-01 | 2.003E-01 | 1.906E-01 | 1.776E-01 | 1.582E-01 |
| 1.0 | 1.587E-01 | 1.550E-01 | 1.481E-01 | 1.396E-01 | 1.291E-01 | 1.156E-01 |
| 1.2 | 1.151E-01 | 1.121E-01 | 1.067E-01 | 1.003E-01 | 9.305E-02 | 8.537E-02 |
| 1.4 | 8.076E-02 | 7.878E-02 | 7.511E-02 | 7.108E-02 | 6.704E-02 | 6.384E-02 |
| 1.6 | 5.480E-02 | 5.380E-02 | 5.189E-02 | 5.000E-02 | 4.854E-02 | 4.823E-02 |
| 1.8 | 3.593E-02 | 3.578E-02 | 3.535E-02 | 3.511E-02 | 3.542E-02 | 3.671E-02 |
| 2.0 | 2.275E-02 | 2.322E-02 | 2.385E-02 | 2.472E-02 | 2.605E-02 | 2.810E-02 |
| 2.2 | 1.390E-02 | 1.474E-02 | 1.602E-02 | 1.750E-02 | 1.930E-02 | 2.160E-02 |
| 2.4 | 8.198E-03 | 9.188E-03 | 1.075E-02 | 1.246E-02 | 1.438E-02 | 1.666E-02 |
| 2.6 | 4.661E-03 | 5.643E-03 | 7.237E-03 | 8.931E-03 | 1.076E-02 | 1.288E-02 |
| 2.8 | 2.555E-03 | 3.429E-03 | 4.892E-03 | 6.436E-03 | 8.085E-03 | 9.987E-03 |
| 3.0 | 1.350E-03 | 2.069E-03 | 3.324E-03 | 4.659E-03 | 6.091E-03 | 7.757E-03 |
| 3.2 | 6.871E-04 | 1.245E-03 | 2.271E-03 | 3.386E-03 | 4.599E-03 | 6.036E-03 |
| 3.4 | 3.369E-04 | 7.491E-04 | 1.559E-03 | 2.468E-03 | 3.480E-03 | 4.704E-03 |
| 3.6 | 1.591E-04 | 4.517E-04 | 1.075E-03 | 1.803E-03 | 2.638E-03 | 3.671E-03 |
| 3.8 | 7.235E-05 | 2.735E-04 | 7.433E-04 | 1.321E-03 | 2.003E-03 | 2.868E-03 |
| 4.0 | 3.167E-05 | 1.663E-04 | 5.156E-04 | 9.687E-04 | 1.523E-03 | 2.243E-03 |
| 4.2 | 1.335E-05 | 1.016E-04 | 3.585E-04 | 7.117E-04 | 1.159E-03 | 1.756E-03 |
| 4.4 | 5.412E-06 | 6.230E-05 | 2.498E-04 | 5.236E-04 | 8.830E-04 | 1.376E-03 |
| 4.6 | 2.112E-06 | 3.840E-05 | 1.744E-04 | 3.857E-04 | 6.735E-04 | 1.079E-03 |
| 4.8 | 7.933E-07 | 2.370E-05 | 1.219E-04 | 2.844E-04 | 5.141E-04 | 8.472E-04 |
| 5.0 | 2.867E-07 | 1.470E-05 | 8.530E-05 | 2.100E-04 | 3.928E-04 | 6.655E-04 |

$$^{\dagger} P(\sigma x) = 1 - P(-\sigma x)$$

Table B3.--Universal level crossing frequencies of the non-gaussian turbulence model longitudinal component for various values of the probability distribution parameter R .
(Graphical presentation of these data is given in figure 16 on page 74 of this report.)

$$\hat{N}_u(x|R)$$

| x | R | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| | 0.0 | 0.5 | 0.75 | 1.0 | 1.33 | 2.0 |
| 0.0 | 1.591E-01 | 1.597E-01 | 1.606E-01 | 1.618E-01 | 1.637E-01 | 1.674E-01 |
| .2 | 1.506E-01 | 1.563E-01 | 1.569E-01 | 1.576E-01 | 1.587E-01 | 1.603E-01 |
| .4 | 1.469E-01 | 1.468E-01 | 1.465E-01 | 1.459E-01 | 1.448E-01 | 1.417E-01 |
| .6 | 1.329E-01 | 1.322E-01 | 1.307E-01 | 1.285E-01 | 1.249E-01 | 1.172E-01 |
| .8 | 1.156E-01 | 1.142E-01 | 1.116E-01 | 1.081E-01 | 1.027E-01 | 9.300E-02 |
| 1.0 | 9.653E-02 | 9.481E-02 | 9.152E-02 | 8.721E-02 | 8.126E-02 | 7.221E-02 |
| 1.2 | 7.747E-02 | 7.564E-02 | 7.221E-02 | 6.796E-02 | 6.262E-02 | 5.595E-02 |
| 1.4 | 5.973E-02 | 5.809E-02 | 5.506E-02 | 5.153E-02 | 4.755E-02 | 4.355E-02 |
| 1.6 | 4.425E-02 | 4.301E-02 | 4.075E-02 | 3.831E-02 | 3.591E-02 | 3.415E-02 |
| 1.8 | 3.150E-02 | 3.077E-02 | 2.944E-02 | 2.815E-02 | 2.717E-02 | 2.691E-02 |
| 2.0 | 2.154E-02 | 2.133E-02 | 2.088E-02 | 2.057E-02 | 2.063E-02 | 2.128E-02 |
| 2.2 | 1.415E-02 | 1.436E-02 | 1.462E-02 | 1.503E-02 | 1.575E-02 | 1.687E-02 |
| 2.4 | 8.934E-03 | 9.432E-03 | 1.017E-02 | 1.102E-02 | 1.207E-02 | 1.339E-02 |
| 2.6 | 5.417E-03 | 6.063E-03 | 7.051E-03 | 8.111E-03 | 9.281E-03 | 1.064E-02 |
| 2.8 | 3.157E-03 | 3.833E-03 | 4.896E-03 | 5.995E-03 | 7.154E-03 | 8.465E-03 |
| 3.0 | 1.768E-03 | 2.395E-03 | 3.409E-03 | 4.446E-03 | 5.524E-03 | 6.738E-03 |
| 3.2 | 9.508E-04 | 1.484E-03 | 2.383E-03 | 3.307E-03 | 4.271E-03 | 5.367E-03 |
| 3.4 | 4.916E-04 | 9.164E-04 | 1.672E-03 | 2.465E-03 | 3.306E-03 | 4.276E-03 |
| 3.6 | 2.441E-04 | 5.654E-04 | 1.177E-03 | 1.841E-03 | 2.560E-03 | 3.409E-03 |
| 3.8 | 1.166E-04 | 3.496E-04 | 8.310E-04 | 1.376E-03 | 1.984E-03 | 2.718E-03 |
| 4.0 | 5.354E-05 | 2.168E-04 | 5.880E-04 | 1.030E-03 | 1.539E-03 | 2.168E-03 |
| 4.2 | 2.351E-05 | 1.348E-04 | 4.167E-04 | 7.714E-04 | 1.194E-03 | 1.730E-03 |
| 4.4 | 1.004E-05 | 8.425E-05 | 2.959E-04 | 5.782E-04 | 9.265E-04 | 1.380E-03 |
| 4.6 | 4.090E-06 | 5.278E-05 | 2.102E-04 | 4.336E-04 | 7.192E-04 | 1.102E-03 |
| 4.8 | 1.580E-06 | 3.315E-05 | 1.495E-04 | 3.253E-04 | 5.585E-04 | 8.793E-04 |
| 5.0 | 6.109E-07 | 2.089E-05 | 1.064E-04 | 2.442E-04 | 4.338E-04 | 7.020E-04 |
| 5.2 | 2.206E-07 | 1.318E-05 | 7.581E-05 | 1.833E-04 | 3.370E-04 | 5.605E-04 |
| 5.4 | 7.410E-08 | 8.328E-06 | 5.402E-05 | 1.377E-04 | 2.619E-04 | 4.476E-04 |
| 5.6 | 2.668E-08 | 5.274E-06 | 3.852E-05 | 1.034E-04 | 2.035E-04 | 3.574E-04 |
| 5.8 | 8.484E-09 | 3.341E-06 | 2.748E-05 | 7.773E-05 | 1.582E-04 | 2.855E-04 |
| 6.0 | 2.424E-09 | 2.117E-06 | 1.960E-05 | 5.842E-05 | 1.230E-04 | 2.281E-04 |
| 6.2 | 8.545E-10 | 1.344E-06 | 1.399E-05 | 4.392E-05 | 9.563E-05 | 1.822E-04 |
| 6.4 | 2.359E-10 | 8.534E-07 | 9.990E-06 | 3.302E-05 | 7.436E-05 | 1.456E-04 |
| 6.6 | 5.528E-11 | 5.419E-07 | 7.134E-06 | 2.483E-05 | 5.784E-05 | 1.163E-04 |
| 6.8 | 2.059E-11 | 3.446E-07 | 5.096E-06 | 1.868E-05 | 4.500E-05 | 9.301E-05 |
| 7.0 | 4.792E-12 | 2.191E-07 | 3.641E-06 | 1.405E-05 | 3.502E-05 | 7.439E-05 |

Table B4.--Universal level crossing frequencies of the non-gaussian turbulence model vertical and lateral components for various values of the probability parameter R .

(Graphical presentation of these data is given in figure 17 on page 80 of this report.)

$$\hat{N}_w(x|R), \hat{N}_v(x|R)$$

| x | R | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| | 0.0 | 0.5 | 0.75 | 1.0 | 1.33 | 2.0 |
| 0.0 | 1.949E-01 | 1.955E-01 | 1.965E-01 | 1.979E-01 | 2.000E-01 | 2.042E-01 |
| .2 | 1.911E-01 | 1.914E-01 | 1.920E-01 | 1.928E-01 | 1.938E-01 | 1.954E-01 |
| .4 | 1.799E-01 | 1.797E-01 | 1.792E-01 | 1.784E-01 | 1.768E-01 | 1.728E-01 |
| .6 | 1.628E-01 | 1.618E-01 | 1.599E-01 | 1.571E-01 | 1.525E-01 | 1.429E-01 |
| .8 | 1.415E-01 | 1.399E-01 | 1.366E-01 | 1.321E-01 | 1.254E-01 | 1.134E-01 |
| 1.0 | 1.182E-01 | 1.161E-01 | 1.120E-01 | 1.066E-01 | 9.924E-02 | 8.806E-02 |
| 1.2 | 9.488E-02 | 9.260E-02 | 8.834E-02 | 8.308E-02 | 7.647E-02 | 6.825E-02 |
| 1.4 | 7.316E-02 | 7.112E-02 | 6.736E-02 | 6.299E-02 | 5.807E-02 | 5.313E-02 |
| 1.6 | 5.420E-02 | 5.266E-02 | 4.985E-02 | 4.682E-02 | 4.386E-02 | 4.167E-02 |
| 1.8 | 3.857E-02 | 3.767E-02 | 3.601E-02 | 3.440E-02 | 3.318E-02 | 3.284E-02 |
| 2.0 | 2.638E-02 | 2.611E-02 | 2.554E-02 | 2.514E-02 | 2.520E-02 | 2.598E-02 |
| 2.2 | 1.733E-02 | 1.758E-02 | 1.788E-02 | 1.838E-02 | 1.923E-02 | 2.059E-02 |
| 2.4 | 1.094E-02 | 1.155E-02 | 1.244E-02 | 1.347E-02 | 1.474E-02 | 1.635E-02 |
| 2.6 | 6.635E-03 | 7.421E-03 | 8.624E-03 | 9.914E-03 | 1.134E-02 | 1.300E-02 |
| 2.8 | 3.866E-03 | 4.692E-03 | 5.988E-03 | 7.328E-03 | 8.742E-03 | 1.034E-02 |
| 3.0 | 2.165E-03 | 2.931E-03 | 4.170E-03 | 5.435E-03 | 6.751E-03 | 8.231E-03 |
| 3.2 | 1.164E-03 | 1.816E-03 | 2.914E-03 | 4.043E-03 | 5.220E-03 | 6.556E-03 |
| 3.4 | 6.021E-04 | 1.122E-03 | 2.045E-03 | 3.014E-03 | 4.040E-03 | 5.225E-03 |
| 3.6 | 2.990E-04 | 6.920E-04 | 1.439E-03 | 2.251E-03 | 3.130E-03 | 4.165E-03 |
| 3.8 | 1.429E-04 | 4.279E-04 | 1.016E-03 | 1.683E-03 | 2.426E-03 | 3.322E-03 |
| 4.0 | 6.557E-05 | 2.653E-04 | 7.192E-04 | 1.259E-03 | 1.881E-03 | 2.650E-03 |
| 4.2 | 2.880E-05 | 1.650E-04 | 5.098E-04 | 9.434E-04 | 1.459E-03 | 2.114E-03 |
| 4.4 | 1.229E-05 | 1.031E-04 | 3.619E-04 | 7.071E-04 | 1.133E-03 | 1.687E-03 |
| 4.6 | 5.009E-06 | 6.460E-05 | 2.572E-04 | 5.303E-04 | 8.794E-04 | 1.347E-03 |
| 4.8 | 1.935E-06 | 4.057E-05 | 1.829E-04 | 3.978E-04 | 6.829E-04 | 1.075E-03 |
| 5.0 | 7.482E-07 | 2.557E-05 | 1.302E-04 | 2.986E-04 | 5.304E-04 | 8.582E-04 |
| 5.2 | 2.702E-07 | 1.614E-05 | 9.275E-05 | 2.242E-04 | 4.121E-04 | 6.852E-04 |
| 5.4 | 9.075E-08 | 1.019E-05 | 6.610E-05 | 1.684E-04 | 3.202E-04 | 5.472E-04 |
| 5.6 | 3.267E-08 | 6.455E-06 | 4.713E-05 | 1.265E-04 | 2.489E-04 | 4.370E-04 |
| 5.8 | 1.039E-08 | 4.089E-06 | 3.362E-05 | 9.508E-05 | 1.935E-04 | 3.491E-04 |
| 6.0 | 2.968E-09 | 2.591E-06 | 2.399E-05 | 7.146E-05 | 1.504E-04 | 2.789E-04 |
| 6.2 | 1.047E-09 | 1.645E-06 | 1.712E-05 | 5.373E-05 | 1.170E-04 | 2.228E-04 |
| 6.4 | 2.890E-10 | 1.044E-06 | 1.222E-05 | 4.040E-05 | 9.095E-05 | 1.780E-04 |
| 6.6 | 6.774E-11 | 6.633E-06 | 8.729E-06 | 3.038E-05 | 7.074E-05 | 1.423E-04 |
| 6.8 | 2.526E-11 | 4.218E-07 | 6.236E-06 | 2.285E-05 | 5.504E-05 | 1.137E-04 |
| 7.0 | 5.911E-12 | 2.682E-07 | 4.455E-06 | 1.719E-05 | 4.283E-05 | 9.098E-05 |